The hypsometric equation is
\[
\Delta Z = \frac{R_d T_v}{g_0} \ln \left[ \frac{p_1}{p_2} \right],
\]
where \( Z \) is geopotential height in geopotential meters, \( g_0 = 9.81 \text{ m/s}^2 \), \( R_d = 287 \text{ J/kg/K} \), and \( p_1 > p_2 \).

1. Consider the 1000-500 mb layer. The atmosphere is, and remains, in hydrostatic balance. It has a thickness of 5000 geopotential meters. What can you change about that layer to make the thickness larger?

2. The accurately computed geostrophic wind speed at some location and height level is \( X \text{ m/s} \). But the accurately observed wind at the same location/height has a different magnitude. Name as many things (but at least 4) as you can that can explain this difference. Hint: the Rossby number is \( Ro = \frac{U}{f_0 L} \).
3. The zonal (west-east) geostrophic wind at some location and altitude is 10 m/s. The latitude is \( \phi \), altitude is \( z \), virtual temperature is \( T_v \), pressure is \( p \), and density is \( \rho \). The zonal geostrophic wind is \( u_g = -\frac{1}{f \rho} \frac{\partial p}{\partial y} \), where \( f = 2\Omega \sin \phi \), and \( \Omega \) is Earth’s angular velocity. The ideal gas law is \( p = \rho R_d T_v \). Briefly justify your answers.

(a) If the same pressure difference occurred at the same latitude over the same distance but at a lower altitude, would the geostrophic wind magnitude be the same, larger, or smaller?

(b) If the pressure difference, \( T_v \), \( z \), and \( p \) were the same, but at a latitude closer to the north pole, would the geostrophic wind magnitude be the same, larger, or smaller?

(c) Suppose the latitude, altitude, pressure and pressure difference were the same, but the temperature was higher. Would the geostrophic wind magnitude be the same, larger, or smaller?
4. The atmosphere is precisely in hydrostatic balance. This means there is no vertical motion anywhere. True or false? Briefly justify your answer.

5. Physically or mathematically explain why clockwise gradient flow around synoptic-scale low pressure (the anomalous low) in the Northern Hemisphere has to result in very high wind speeds.

6. Use some reasonable assumptions to estimate how quickly density decreases with height (in kg/m$^3$ per km) in a shallow layer near sea-level in midlatitudes when the atmosphere is dry, isothermal, and hydrostatic. Check your sign!