The hypsometric equation is
\[ \Delta Z = \frac{R_d \bar{T}_v}{g_0} \ln \left[ \frac{p_1}{p_2} \right], \]
where \( Z \) is geopotential height in geopotential meters, \( g_0 = 9.81 \text{ m/s}^2 \), \( R_d = 287 \text{ J/kg/K} \), and \( p_1 > p_2 \).

1. Consider the 1000-500 mb layer. At the initial time, the layer mean virtual temperature is 273 K. Diabatic heating causes this layer’s mean virtual temperature to increase by 1.234 \%. By how much does the thickness increase? Express your answer as a percentage.

**Ans:** Note that everything is constant on the right hand side except \( \bar{T}_v \). Thus, mean virtual temperature and thickness are directly proportional. If I increase \( \bar{T}_v \) by 1.234 \%, \( \Delta Z \) increases by the same percentage.

2. Given station elevation \( Z \) geopotential meters above sea level, and station pressure \( p_2 \), if you presume the lapse rate between station elevation and sea-level is equal to the standard tropospheric lapse rate of 6.5\(^\circ\)C/km, you can use the hypsometric equation to obtain SLP \( p_1 \). Suppose instead you presume a larger lapse rate. Would the SLP you estimate be higher, lower, or the same, and why?

**Ans:** You can answer this easily by recognizing that pressure varies with height faster in colder air, and varies slower in warmer air. Thus, how much higher SLP is relative to station pressure \( p_2 \) depends on how warm the air is. If the air is very cold, \( p \) changes rapidly with height, and SLP will be very high (large compared to \( p_2 \)). Making the lapse rate larger means presuming the fake column of air between the station and sea level is warmer, so the SLP will be lower.