ATM 316 In-class #3 Solution
Fall, 2020 – Fovell

The gradient wind equation, after solving the quadratic, is

\[ V = -\frac{fR}{2} \pm \sqrt{\frac{f^2R^2}{4} - R\frac{1}{\rho} \frac{\partial p}{\partial n}}. \]

You are in the NH \((f > 0)\). By convention, \(R > 0\) is CCW. Identify the two roots resulting when \(R > 0\) and \(\frac{1}{\rho} \frac{\partial p}{\partial n} < 0\) (the bottom left quadrant from my chart). For each root, you should be demonstrating that it is one of the following: regular low, regular high, anomalous low, anomalous high, or unphysical.

**Ans:** Given \(R > 0\) means we have CCW flow. Given \(\frac{1}{\rho} \frac{\partial p}{\partial n} < 0\) means pressure decreases in the \(+\hat{n}\) direction, thus we have CCW flow around low pressure. This means

- \(-R\frac{1}{\rho} \frac{\partial p}{\partial n} > 0\),
- then \(\left[ \frac{f^2R^2}{4} - R\frac{1}{\rho} \frac{\partial p}{\partial n} \right] > \frac{f^2R^2}{4}\) by itself,
- then \(\left[ \frac{f^2R^2}{4} - R\frac{1}{\rho} \frac{\partial p}{\partial n} \right]^{1/2} > \frac{fR}{2}\) by itself, and is positive since \(f\) and \(R\) are positive.

Now consider

\[ V = -\frac{fR}{2} \pm \text{(the term we developed above)}. \]

The positive root yields

\[ V_+ = -\frac{fR}{2} + \text{(term} > \frac{fR}{2}\text{)} > 0. \]

This is the regular low. For the negative root, we have

\[ V_- = -\frac{fR}{2} - \text{(term} > \frac{fR}{2}\text{)} < 0. \]

Both terms individually result in negative numbers, so \(V_- < 0\) and is unphysical.