1. The atmosphere is precisely in hydrostatic balance. This means there is no vertical motion anywhere. True or false? Briefly justify your answer. 3 pts.

**Ans:** False. The vertical equation of motion can be written as

\[
\frac{dw}{dt} = -\frac{1}{\rho} \frac{\partial p}{\partial z} - g.
\]

Hydrostatic balance means the vertical acceleration is zero. In theory, the vertical velocity can be nonzero, but it also needs to be constant.

2. On the synoptic scale, the gradient wind is supergeostrophic for clockwise curvature. Concisely explain why. Draw a picture and label it completely. 3 pts.

**Ans:** Consider a wind in geostrophic balance, flowing between parallel isobars with low pressure to the left and Coriolis acting to the right following the motion, but approaching a region of isobar curvature. Inertia would take this parcel across the curving isobars, towards relatively lower pressure. As the parcel approaches the isobar curvature, note the PGF shifts in orientation relative to the motion, so it remains pointing most directly towards lower pressure.

This creates a component of PGF that is acting in the direction of motion, as illustrated in the figure. This would cause the parcel to accelerate. Its increased speed would enhance the Coriolis force, because that is proportional to wind speed. The imbalance thereby created between the PGF (which remains the same magnitude, as that is determined by isobar spacing) and Coriolis (which has increased) causes the parcel to turn to its right, towards the direction of the Coriolis force is acting. This has the result of keeping the parcel traveling parallel to the curved isobars, but at a supergeostrophic speed.
3. The geostrophic wind at a particular location and altitude is 10 m/s. The latitude is $\phi$, altitude is $z$, virtual temperature is $T_v$, pressure is $p$, and density is $\rho$.

(a) If the same pressure difference occurred at the same latitude over the same distance but at a lower altitude, would the geostrophic wind magnitude be the same, larger, or smaller, and why? 3 pts.

Ans: The geostrophic wind magnitude would be smaller, since the lower altitude means the density would be higher.

(b) If the pressure difference, $T_v$, $z$, and $p$ were the same, but at a latitude closer to the north pole, would the geostrophic wind magnitude be the same, larger, or smaller, and why? 3 pts.

Ans: The geostrophic wind magnitude would be smaller, since $f$ would be larger.

(c) Suppose the latitude, altitude, pressure and pressure difference were the same, but the temperature was higher. Would the geostrophic wind magnitude be the same, larger, or smaller, and why? 3 pts.

Ans: Higher temperature at the same pressure means the density is smaller, which means the geostrophic wind magnitude would be larger.

4. The cyclostrophic balance is counter-clockwise (CCW) or clockwise (CW) flow around low pressure. Using the equation in your answer, fully explain why there cannot be high pressure in the center of the circulation. 5 pts.

Ans: The cyclostrophic balance is obtained from the gradient wind equation by neglecting the Coriolis term:

$$\frac{V^2}{R} = -\frac{1}{\rho} \frac{\partial p}{\partial n}$$

Solving for $V$ yields

$$V = \sqrt{-\frac{1}{R\rho} \frac{\partial p}{\partial n}}.$$

The quantity under the radical has to be positive, and the radius of curvature $R$ can have two signs. Keeping in mind that the $\hat{n}$ direction is positive to the left of the motion, consider $R > 0$, which means CCW flow. Since $\rho > 0$, this means $\frac{\partial p}{\partial n}$ has to be negative. In other words, the pressure has to decrease in the $+\hat{n}$ direction. That puts low pressure in the center of this CCW circulation.

Now consider CW flow, or $R < 0$. Now $\frac{\partial p}{\partial n}$ has to be positive, which means pressure has to increase in the $+\hat{n}$ direction. With CW flow, the $+\hat{n}$ direction points away from the center of the circulation. Thus, again there needs to be low pressure in the center.

5. You are given station elevation $Z$ geopotential meters above sea level, station temperature $T_s$, and station pressure $p_2$. If you presume the lapse rate in the layer between station elevation and sea-level is equal to the standard tropospheric lapse rate of 6.5°C/km, you can use the hypsometric equation to obtain SLP, $p_1$. Suppose instead you presume that layer is isothermal (constant temperature). Would the SLP you estimate be higher, lower, or the same, and why? 5 pts.

Ans: Replacing the standard lapse rate with an isothermal one necessarily means the layer average temperature of this fictional column of air would be colder. Pressure decreases with
height faster in colder air, which means it increases faster as one approaches the surface. That means the SLP would have to be higher.

6. Consider a parcel undergoing circular, horizontal motion influenced only by the Coriolis force, described by these two equations:

$$\frac{du}{dt} = fv, \quad \frac{dv}{dt} = -fu,$$

where $f$ is to be taken as constant. Show that the total kinetic energy of a unit mass parcel, $e = \frac{1}{2}(u^2 + v^2)$, is conserved following the motion. **3 pts.**

**Ans:** We seek to show that $\frac{de}{dt} = 0$.

$$
e = \frac{1}{2} [u^2 + v^2]
\frac{de}{dt} = \frac{1}{2} \frac{d}{dt} [u^2 + v^2]
= \frac{1}{2} \left[ 2u \frac{du}{dt} + 2v \frac{dv}{dt} \right]
= uf v - v fu
= 0.
$$

7. The wind speed is 30 m/s, latitude is 20° N, and length scale is 100 km. Evaluate the applicability of the geostrophic approximation. **3 pts.**

**Ans:** This question can be answered by computing the Rossby number, which evaluates to 6. For the geostrophic approximation to be valid, $Ro$ must be much less than one. Therefore, the geostrophic approximation would be very poor one in this situation.

8. **DO THIS QUESTION LAST.** Suppose you have a Northern Hemisphere counterclockwise flow in gradient wind balance, with wind speed $V_1$, radius of curvature $R=+1000$ m, Coriolis parameter $f = 10^{-4}$ s$^{-1}$, and pressure gradient acceleration $-\frac{1}{\rho} \frac{\partial p}{\partial n} = +10^{-3}$ m/s$^2$. The Earth suddenly stops turning, but the pressure gradient and $R$ do not change. Call the wind speed resulting from this new balance $V_2$. Is $V_2$ larger, smaller, or the same magnitude as $V_1$, and why? **4 pts.**

**Ans:** One way you can approach this: Let $V_1$ be the gradient wind, specified by this equation:

$$\frac{V_1^2}{R} + fV_1 = -\frac{1}{\rho} \frac{\partial p}{\partial n}.$$ 

The Coriolis term suddenly disappears, creating $V_2$ which is in cyclostrophic balance with the same pressure gradient:

$$\frac{V_2^2}{R} = -\frac{1}{\rho} \frac{\partial p}{\partial n}.$$ 

This means we can rewrite the first equation as

$$\frac{V_1^2}{R} + fV_1 = \frac{V_2^2}{R}.$$ 

Multiply through by $R$ and recognize that $f$, $R$ and $V_1$ are positive. This means that $V_2^2 = V_1^2 + a$ positive number. In other words, $V_2 > V_1$. In this particular case, if you plugged in these numbers, you would find $V_1 = 0.95$ m/s and $V_2 = 1$ m/s.