1. The geostrophic wind at a particular location and altitude is 10 m/s. At a second northern hemisphere location, which has exactly the same pressure gradient as measured in Pa per km, the actual wind speed is observed to be faster. What is different about the second location, relative to the first one? Name at least three different reasons, and fully explain/justify each one. Bonus for including and explaining a fourth reason. 6 pts. + 2 pts. bonus

Ans: We are keeping the pressure gradient constant in this thought experiment. Keeping in mind that the geostrophic wind equation in terms of the pressure gradient (when written in natural coordinates) is

\[ V_g = -\frac{1}{\rho f} \frac{\partial p}{\partial n}, \]

we see that the second location with the same pressure gradient...

- ...could be closer to the equator, as this would cause \( f \) to decrease and thus \( V_g \) to increase;
- ...could be at a higher altitude, have a warmer temperature, and/or have higher absolute humidity, as this would mean \( \rho \) is smaller and thus \( V_g \) is larger;
- ...could have CW curvature and thus be in gradient wind balance, as then \( V > V_g \);
- ...could have a much smaller horizontal length scale or radius of curvature (CW or CCW) with the same pressure gradient, so that cyclostrophic balance applies, as then \( V > V_g \) as well.

Aside: Friction is not part of geostrophic balance, and I’m asking for wind speeds that are supergeostrophic, so nominally none of these answers should directly invoke friction. However, if you know about the Ekman spiral, you appreciate that the wind can oscillate between subgeostrophic and supergeostrophic in the planetary boundary layer (PBL) based on the diurnally-driven cycle of PBL mixing. During the day, strong mixing slows the wind to subgeostrophic values as parcels in a deep layer of the PBL are brought to the surface and experience frictional drag. As mixing disappears overnight, however, the Ekman solution reveals that wind speeds will overshoot their geostrophic values and become supergeostrophic for a period of time. So, another valid answer to this question would be: the second location could be within the PBL near the surface after the afternoon collapse of boundary layer mixing.

2. Physically or mathematically explain why clockwise gradient flow around synoptic-scale low pressure (the anomalous low) in the Northern Hemisphere has to result in very high wind speeds. 3 pts.

Ans: CW flow around synoptic-scale low pressure in the Northern Hemisphere means that both PGF and Coriolis are acting in the same direction; i.e., towards the low. This, the third force in the gradient wind equation has to balance the combination of both PGF and Coriolis terms, which means it needs to be large. As this term is pointing outward from the center of spin, we will label it as a centrifugal force, although this does not really matter to this conclusion. The main point is that whether we interpret the term as centripetal or centrifugal, this acceleration is proportional to the (square of the) velocity, so this implies the wind speed must be quite large. (We verified this numerically in HW 6.)
Mathematically, we could approach this question in the following manner: We start with the gradient wind equation
\[ \frac{V^2}{R} + fV = -\frac{1}{\rho} \frac{\partial p}{\partial n}, \]
identifying the acceleration terms in order as centrifugal, Coriolis and PGF per unit mass. \( V \geq 0 \) at all times, as it is a real wind speed, \( f > 0 \) in the NH, and \( \hat{n} \) is positive to the left of the flow. For the regular low (CCW around low pressure in the NH), we appreciate that \( R > 0 \) and \( \frac{\partial p}{\partial n} < 0 \), so the three terms (centrifugal, Coriolis and PGF) are all positive. This equation means that PGF is balancing the sum of centrifugal and Coriolis. If we rewrite to put Coriolis on the right hand side, it implies the centrifugal term is the small difference between the larger, opposing PGF and Coriolis terms. As a consequence, the wind speed \( V \) is not large.

Next consider the anomalous low, in which \( R < 0 \) and \( \frac{\partial p}{\partial n} > 0 \). The centrifugal term is now negative (owing to \( R \)) and the PGF term is negative as well. The Coriolis term is still positive. We can rearrange this as
\[ \frac{V^2}{R} = -fV - \frac{1}{\rho} \frac{\partial p}{\partial n}, \]
so that all three terms are negative. This means that centrifugal acceleration, alone on the left hand side, has to balance the sum of Coriolis and PGF. For the same \( R \) as the regular low, this implies a much larger wind speed \( V \), because the terms on the right hand side share a sign and thus are additive rather than subtractive.

3. The cyclostrophic balance is counter-clockwise (CCW) or clockwise (CW) flow around low pressure. Using the equation in your answer, fully explain why there cannot be high pressure in the center of the circulation. **4 pts.**

**Ans:** The cyclostrophic balance is obtained from the gradient wind equation by neglecting the Coriolis term:
\[ \frac{V^2}{R} = -\frac{1}{\rho} \frac{\partial p}{\partial n} \]
Solving for \( V \) yields
\[ V = \sqrt{-\frac{1}{R \rho} \frac{\partial p}{\partial n}}. \]
The quantity under the radical has to be positive, and the radius of curvature \( R \) can have two signs. Keeping in mind that the \( \hat{n} \) direction is positive to the left of the motion, consider \( R > 0 \), which means CCW flow. Since \( \rho > 0 \), this means \( \frac{\partial p}{\partial n} \) has to be negative. In other words, the pressure has to decrease in the \( +\hat{n} \) direction. That puts low pressure in the center of this CCW circulation.

Now consider CW flow, or \( R < 0 \). Now \( \frac{\partial p}{\partial n} \) has to be positive, which means pressure has to increase in the \( +\hat{n} \) direction. With CW flow, the \( +\hat{n} \) direction points away from the center of the circulation. Thus, again there needs to be low pressure in the center.

4. You are given station elevation \( Z \) geopotential meters above sea level, station temperature \( T_s \), and station pressure \( p_2 \). If you presume the lapse rate in the fictional air layer between station elevation and sea-level is equal to the standard tropospheric lapse rate of 6.5\(^\circ\)C/km, you can use the hypsometric equation to obtain SLP, \( p_1 \). Suppose instead you presume that
layer is *isothermal* (constant temperature). Would the SLP you estimate be higher, lower, or the same, and why? **4 pts.**

**Ans:** Replacing the standard lapse rate with an isothermal one necessarily means the layer average temperature of this fictional column of air would be *colder*. Pressure decreases with height faster in colder air, which means it increases faster as one approaches the surface. That means the SLP would have to be *higher*.

5. A rocket is fired directly eastward in the Northern Hemisphere, traveling at high velocity, but we observe it turn to the right following its motion. Name all reasons why. **4 pts.**

**Ans:** There are two reasons. Owing to its high velocity, the rocket is acted upon by both the Coriolis and sphericity accelerations, both of which cause the rocket to curve to the right following the motion in this case.

6. Use some reasonable assumptions to estimate how quickly density decreases with height (in kg/m\(^3\) per km) in a shallow layer near sea-level in midlatitudes when the atmosphere is dry, isothermal, and hydrostatic. Check your sign. **4 pts.**

**Ans:** You should start with hydrostatic balance and the ideal gas law (IGL):

\[
\frac{dp}{dz} = -\rho g \quad \text{and} \quad p = \rho R_d T.
\]

Since our layer is isothermal, we can differentiate the IGL to obtain

\[
dp = R_d T d\rho,
\]

which with the hydrostatic equation becomes

\[
-\rho g = R_d T \frac{d\rho}{dz}.
\]

You can use the IGL to avoid logs, and rearrange, to obtain

\[
-\frac{pg}{R_d T} = R_d T \frac{d\rho}{dz} \quad \Rightarrow \quad \frac{d\rho}{dz} = -\frac{pg}{(R_d T)^2}
\]

\[
\frac{dp}{dz} = -0.00014 \text{ kg/m}^3 \text{ per m}
\]

\[
\frac{dp}{dz} = -0.14 \text{ kg/m}^3 \text{ per km}
\]

This result was obtained presuming/using \(p = 1000 \text{ mb} = 100000 \text{ Pa}, T = 300 \text{ K}, g = 10 \text{ m/s}^2,\) and \(R_d = 287 \text{ J/kg/K}.)\) Note the minus sign! Density decreases with altitude.