1. Consider a layer of certain, fixed thickness $\Delta Z$. If the pressure at the bottom of the layer is fixed and the mean layer virtual temperature is increased, how does the pressure at the top of the layer change? Answer with: it increases, it decreases, or it stays the same, and justify your answer. Guesses have no value. 6 pts.

**Ans:** Keeping in mind that pressure decreases with height more slowly in warmer air, if temperature increases, pressure decreases with height more slowly, so the pressure at the layer top would be HIGHER.

2. The geostrophic wind at a particular location and altitude is 10 m/s. At a second northern hemisphere location, which has exactly the same pressure gradient, the actual wind speed is observed to be faster. What is different about the second location, relative to the first one? Name at least three different reasons, and fully explain/justify each one. Bonus for including and explaining a fourth reason. 6 pts. + 2 pts. bonus

**Ans:** We are keeping the pressure gradient constant in this thought experiment. Keeping in mind that the geostrophic wind equation in terms of the pressure gradient (when written in natural coordinates) is

$$V_g = -\frac{1}{\rho f} \frac{\partial p}{\partial n},$$

we see that the second location with the same pressure gradient...

- ...could be closer to the equator, as this would cause $f$ to decrease and thus $V_g$ to increase;
- ...could be at a higher altitude, have a warmer temperature, and/or have higher absolute humidity, as this would mean $\rho$ is smaller and thus $V_g$ is larger;
- ...could have CW curvature and thus be in gradient wind balance, as then $V > V_g$;
- ...could have a much smaller horizontal length scale or radius of curvature (CW or CCW) with the same pressure gradient, so that cyclostrophic balance applies, as then $V > V_g$ as well.

*Aside:* Friction is NOT part of geostrophic balance, and I’m asking for wind speeds that are supergeostrophic, so nominally none of these answers should directly invoke friction. However, if you know about the Ekman spiral, you appreciate that the wind can oscillate between subgeostrophic and supergeostrophic in the planetary boundary layer (PBL) based on the diurnally-driven cycle of PBL mixing. During the day, strong mixing slows the wind to subgeostrophic values as parcels in a deep layer of the PBL are brought to the surface and experience frictional drag. As mixing disappears overnight, however, the Ekman solution reveals that wind speeds will overshoot their geostrophic values and become supergeostrophic for a period of time. So, another valid answer to this question would be: the second location could be within the PBL near the surface after the afternoon collapse of boundary layer mixing. But, that’s beyond the scope of this course.

3. The gradient wind equation, after solving the quadratic, is
\[ V = -\frac{fR}{2} \pm \sqrt{\frac{f^2 R^2}{4} - \frac{R}{\rho} \frac{\partial p}{\partial n}}. \]

You are in the NH. Identify the two roots resulting when \( R > 0 \) and \( \frac{1}{\rho} \frac{\partial p}{\partial n} < 0 \). For each root, you should be demonstrating that it is one of the following: regular low, regular high, anomalous low, anomalous high, or unphysical. Show all your work leading to your conclusions; guesses have no value.

**Ans:** Given \( R > 0 \) means we have CCW flow. Given \( \frac{1}{\rho} \frac{\partial p}{\partial n} < 0 \) means pressure decreases in the \(+\hat{n}\) direction, thus we have CCW flow around low pressure. This means

- \(-R \frac{1}{\rho} \frac{\partial p}{\partial n} > 0,
- \[ \left[ \frac{f^2 R^2}{4} - R \frac{1}{\rho} \frac{\partial p}{\partial n} \right] > \frac{f^2 R^2}{4} \text{ by itself,}
- \[ \left[ \frac{f^2 R^2}{4} - R \frac{1}{\rho} \frac{\partial p}{\partial n} \right]^{1/2} > \frac{fR}{2} \text{ by itself, and is positive since } f \text{ and } R \text{ are positive.}

Now consider
\[ V = -\frac{fR}{2} \pm (\text{the term we developed above}). \]

The positive root yields
\[ V_+ = -\frac{fR}{2} + (\text{term} > \frac{fR}{2}) > 0. \]

This is the regular low. For the negative root, we have
\[ V_- = -\frac{fR}{2} - (\text{term} > \frac{fR}{2}) < 0. \]

Both terms individually result in negative numbers, so \( V_- < 0 \) and is unphysical.

(This question was also asked on In-class exercise # 4.)

4. You are given a situation in which the gradient wind is 10% faster than the geostrophic wind, which is 10 m/s. You are at 40°N. What is the radius of curvature for this flow? Don’t forget to check your sign for reasonableness.

**Ans:** Note this means that \( V = 11 \text{ m/s while } V_g = 10 \text{ m/s. The gradient wind is supergeostrophic, so that means that the radius of curvature } R \text{ had better be negative (CW curvature). I’m going to use the gradient wind balance equation in the form}

\[ \frac{V^2}{R} = f(V_g - V) \]

Solving for \( R \) yields \(-1290747 \text{ m or } -1291 \text{ km. It is negative as expected and required.}

5. The cyclostrophic balance is counter-clockwise (CCW) or clockwise (CW) flow around low pressure. Using the equation in your answer, fully explain why there cannot be high pressure in the center of the circulation. 4 pts.

**Ans:** The cyclostrophic balance is obtained from the gradient wind equation by neglecting the Coriolis term:
\[ \frac{V^2}{R} = -\frac{1}{\rho} \frac{\partial p}{\partial n} \]
Solving for $V$ yields

$$V = \sqrt{-\frac{1}{R\rho} \frac{\partial p}{\partial n}}.$$  

The quantity under the radical has to be positive, and the radius of curvature $R$ can have two signs. Keeping in mind that the $\hat{n}$ direction is positive to the left of the motion, consider $R > 0$, which means CCW flow. Since $\rho > 0$, this means $\frac{\partial p}{\partial n}$ has to be negative. In other words, the pressure has to decrease in the $+\hat{n}$ direction. That puts low pressure in the center of this CCW circulation.

Now consider CW flow, or $R < 0$. Now $\frac{\partial p}{\partial n}$ has to be positive, which means pressure has to increase in the $+\hat{n}$ direction. With CW flow, the $+\hat{n}$ direction points away from the center of the circulation. Thus, again there needs to be low pressure in the center.

6. You are given station elevation $Z$ geopotential meters above sea level, station temperature $T_s$, and station pressure $p_2$. Normally, we presume the lapse rate in the fictional air layer between station elevation and sea-level is equal to the standard tropospheric lapse rate of 6.5$^\circ$C/km and use the hypsometric equation to obtain SLP, $p_1$. Suppose instead you presume that layer is isothermal (constant temperature). Would the SLP you estimate be higher, lower, or the same, and why? 4 pts.

**Ans:** Replacing the standard lapse rate with an isothermal one necessarily means the layer average temperature of this fictional column of air would be colder. Pressure decreases with height faster in colder air, which means it increases faster as one approaches the surface. That means the SLP would have to be higher. See our In-class exercise #3 for a very similar problem.