ATM 316 Equations list

Fall, 2020 – Fovell

- Some equations:

  $\vec{A}\cdot\vec{B} = |\vec{A}||\vec{B}|\cos\theta$

  $\vec{A}\times\vec{B} = |\vec{A}||\vec{B}|\sin\theta\ \hat{n}$ (where $\hat{n}$ perpendicular to $\vec{A}$ and $\vec{B}$)

  $\frac{\partial f(x_0)}{\partial x} \approx \frac{f(x_0+\Delta x)-f(x_0-\Delta x)}{2\Delta x}$

  $\nabla A = \frac{\partial A}{\partial x}\hat{i} + \frac{\partial A}{\partial y}\hat{j} + \frac{\partial A}{\partial z}\hat{k}$

  $\vec{U} = u\hat{i} + v\hat{j} + w\hat{k}$, $\nabla\cdot\vec{U} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$, $\nabla\times\vec{U} = (\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z})\hat{i} - (\frac{\partial w}{\partial x} - \frac{\partial u}{\partial z})\hat{j} + (\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y})\hat{k}$

  $\frac{dB}{dt} = \frac{\partial B}{\partial t}\frac{dt}{dt} + \frac{\partial B}{\partial x}\frac{dx}{dt} + \frac{\partial B}{\partial y}\frac{dy}{dt} + \frac{\partial B}{\partial z}\frac{dz}{dt} = \frac{\partial B}{\partial t} + \vec{U}\cdot\nabla B$

  PGF (per unit mass): $-\frac{1}{\rho}\nabla p$

  $p = \rho R T = \rho R_d T_v$

  $F_{gravity} = -\frac{GMm}{|\vec{r}|^2}\left[\frac{\vec{r}}{|\vec{r}|}\right]$

  $\vec{V}_{tan} = \omega\vec{R}$

  $f = 2\Omega\sin\phi$, $\vec{\Omega} = \Omega\cos\phi\hat{j} + \Omega\sin\phi\hat{k}$

  $(\frac{d\vec{U}}{dt})_{Coriolis} = -2\vec{\Omega}\times\vec{U}$

  $\frac{\partial p}{\partial z} = -\rho g$

  $Ro = \frac{U}{f_0 L}$

  $u_g = -\frac{1}{f\rho}\frac{\partial p}{\partial y} = -\frac{1}{f}\frac{\partial \Phi}{\partial y}$, $v_g = \frac{1}{f\rho}\frac{\partial p}{\partial x} = \frac{1}{f}\frac{\partial \Phi}{\partial x}$

  $\Delta Z = \frac{R_d\bar{T_v}}{g_0}\ln\left[\frac{p_0}{p_1}\right]$

  $d\Phi = g dz$

  $\frac{V^2}{R} + fV = -\frac{1}{\rho}\frac{\partial p}{\partial n}$

  $\frac{V^2}{R} + fV - fV_g = 0$

  Thermal wind relations (overbar here indicates layer mean; $p_0 > p_1$):

  $u_T = -\frac{R_d}{f}\left[\frac{\partial\bar{T_v}}{\partial y}\right]_p \ln\left[\frac{p_0}{p_1}\right]$

  $v_T = \frac{R_d}{f}\left[\frac{\partial\bar{T_v}}{\partial x}\right]_p \ln\left[\frac{p_0}{p_1}\right]$

- Some constants and conversions:

  $R_d = 287$ J kg^{-1} K^{-1}; $g_0 = g = 9.81$ m s^{-2}; $\Omega = 7.292\times10^{-5}$ s^{-1}, $\epsilon = 0.622$;

  1 mb = 100 Pa. Earth radius = 6371 km.