Lin, Farley, and Orville (1983) = LFO

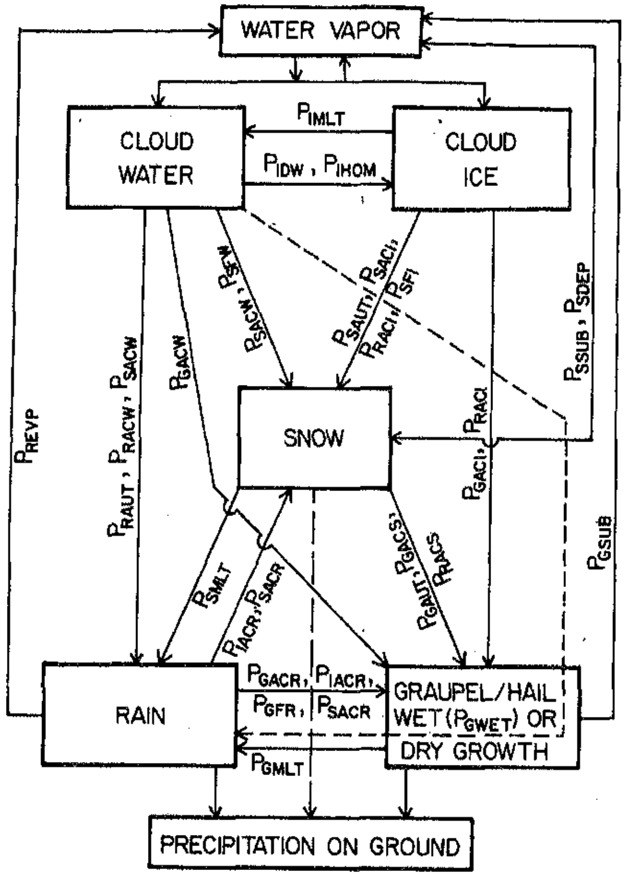

FIG. 1. Cloud physics processes simulated in the model with the snow field included. See Table 1 for an explanation of the symbols.

1) PARAMETERIZATION

Exponential size distributions are hypothesized for the precipitation particles:

$$n_R(D) = n_{0R} \exp(-\lambda_R D_R), \tag{1}$$

$$n_S(D) = n_{0S} \exp(-\lambda_S D_S), \tag{2}$$

$$n_G(D) = n_{0G} \exp(-\lambda_G D_G), \tag{3}$$

where n_{0R}, n_{0S} and n_{0G} are the intercept parameters of the rain, snow and hail size distributions, respectively. The n_{0R} is given by Marshall-Palmer (1948) as 8×10^{-2} cm^{-4}. According to the measurements of Gunn and Marshall (1958), n_{0S} is given as 3×10^{-2} cm^{-4}. Observations by Federer and Waldvogel (1975) of hail distributions lead to a value of $\sim 4 \times 10^{-4}$ cm^{-4} for n_{0G}. D_R, D_S and D_G are diameters of the rain, snow and hail particles, respectively. The slope parameters of the rain, snow and hail size distributions (λ_R, λ_S and λ_G, respectively) are determined by multiplying (1), (2) and (3) by particle mass and integrating over all diameters and equating the resulting quantities to the appropriate water contents; they may be written as

$$\lambda_R = \left(\frac{\pi \rho_W n_{0R}}{\rho l_R}\right)^{0.25}, \tag{4}$$

$$\lambda_S = \left(\frac{\pi \rho_S n_{0S}}{\rho l_S}\right)^{0.25}, \tag{5}$$

$$\lambda_G = \left(\frac{\pi \rho_G n_{0G}}{\rho l_G}\right)^{0.25}, \tag{6}$$

where l_W, l_S and l_G are densities of water, snow and hail, respectively. The density of snow is assumed to be 0.1 g cm^{-3} in this study. The symbols l_R, l_S and l_G are mixing ratios of rain, snow and hail, respectively.

The terminal velocities for a precipitating particle of diameter D_R, D_S or D_G are

TABLE 1. Key to Fig. 1

Symbol	Meaning
P_{IMLT}	Melting of cloud ice to form cloud water, $T \geq T_0$.
P_{IDW}	Depositional growth of cloud ice at expense of cloud water.
P_{IHOM}	Homogeneous freezing of cloud water to form cloud ice.
P_{IACR}	Accretion of rain by cloud ice; produces snow or graupel depending on the amount of rain.
P_{RACI}	Accretion of cloud ice by rain; produces snow or graupel depending on the amount of rain.
P_{RAUT}	Autoconversion of cloud water to form rain.
P_{RACW}	Accretion of cloud water by rain.
P_{REVP}	Evaporation of rain.
P_{RACS}	Accretion of snow by rain; produces graupel if rain or snow exceeds threshold and $T < T_0$.
P_{SACW}	Accretion of cloud water by snow; produces snow if $T < T_0$ or rain if $T \geq T_0$. Also enhances snow melting for $T \geq T_0$.
P_{SACR}	Accretion of rain by snow. For $T < T_0$, produces graupel if rain or snow exceeds threshold; if not, produces snow. For $T \geq T_0$, the accreted water enhances snow melting.
P_{SACI}	Accretion of cloud ice by snow.
P_{SAUT}	Autoconversion (aggregation) of cloud ice to form snow.
P_{SFW}	Bergeron process (deposition and riming)—transfer of cloud water to form snow.
P_{SFI}	Transfer rate of cloud ice to snow through growth of Bergeron process embryos.
P_{SDEP}	Depositional growth of snow.
P_{SSUB}	Sublimation of snow.
P_{SMLT}	Melting of snow to form rain, $T \geq T_0$.
P_{GAUT}	Autoconversion (aggregation) of snow to form graupel.
P_{GFR}	Probabilistic freezing of rain to form graupel.
P_{GACW}	Accretion of cloud water by graupel.
P_{GACI}	Accretion of cloud ice by graupel.
P_{GACR}	Accretion of rain by graupel.
P_{GACS}	Accretion of snow by graupel.
P_{GSUB}	Sublimation of graupel.
P_{GMLT}	Melting of graupel to form rain, $T \geq T_0$. (In this regime, P_{GACW} is assumed to be shed as rain.)
P_{GWET}	Wet growth of graupel; may involve P_{GACS} and P_{GACI} and must include P_{GACW} or P_{GACR}, or both. The amount of P_{GACW} which is not able to freeze is shed to rain.

Terminal velocities

LFO

curve-fitting

$$U_{DR} = aD_R^b\left(\frac{\rho_0}{\rho}\right)^{1/2}, \quad (7)$$

$$U_{DS} = cD_S^d\left(\frac{\rho_0}{\rho}\right)^{1/2}, \quad (8)$$

first principles →

$$U_{DG} = \left(\frac{4g\rho_G}{3C_D\rho}\right)^{1/2} D_G^{1/2}. \quad (9)$$

The terminal velocity U_{DR} of rain is suggested by Liu and Orville (1969) who performed a least squares analysis of Gunn and Kinzer's data (1949). The constants a and b are 2115 $\mathrm{cm}^{1-b}\ \mathrm{s}^{-1}$ and 0.8, respectively. The terminal velocity U_{DS} of snow is based on the relations suggested by Locatelli and Hobbs (1974). Specifically, U_{DS} is that appropriate for graupel-like snow of hexagonal type, with the constants c and d being 152.93 $\mathrm{cm}^{1-d}\ \mathrm{s}^{-1}$ and 0.25, respectively. The square root factor involving air density allows for increasing fallspeeds with increasing altitude, similar to Foote and du Toit (1969). The terminal velocity U_{DG} of hail is proposed by Wisner *et al.* (1972), with the drag coefficient C_D assumed to be 0.6.

Following Srivastava (1967), we define mass-weighted mean terminal velocities as

$$U = \int U_D l(D) dD/l, \quad (10)$$

where U_D is the terminal velocity of a precipitating particle of diameter D, $l(D)$ is the mixing ratio of a precipitating particle of diameter D, and l is the mixing ratio of a precipitating field. Applying (10) to each precipitating field, we obtain the mass-weighted mean terminal velocities of rain, snow and hail:

$$U_R = \frac{a\Gamma(4+b)}{6\lambda_R^b}\left(\frac{\rho_0}{\rho}\right)^{1/2}, \quad (11)$$

$$U_S = \frac{c\Gamma(4+d)}{6\lambda_S^d}\left(\frac{\rho_0}{\rho}\right)^{1/2}, \quad (12)$$

$$U_G = \frac{\Gamma(4,5)}{6\lambda_G^{0.5}}\left(\frac{4g\rho_G}{3C_D\rho}\right)^{1/2}. \quad (13)$$

The mass-weighted mean terminal velocities of rain, snow and hail are shown in Fig. 2.

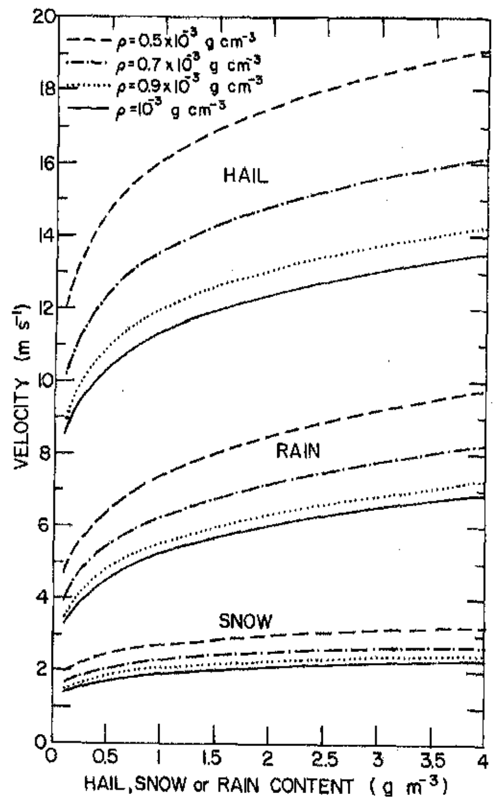

FIG. 2. Mass-weighted mean terminal velocities for rain, snow and hail. The four curves from 9 to 19 m s^{-1} are for hail. The four curves from 3 to 10 m s^{-1} are for rain. The remaining four curves are for snow.

$\bar{\rho}_G \uparrow$

$V_T \uparrow$

2) WATER CONSERVATION EQUATIONS

Four conservation equations are considered here:

$$\frac{\partial q}{\partial t} = -\mathbf{V}\cdot\nabla q + \nabla\cdot K_h\nabla q - P_R - P_S - P_G, \quad (14)$$

$$\frac{\partial l_R}{\partial t} = -\mathbf{V}\cdot\nabla l_R + \nabla\cdot K_m\nabla l_R + P_R + \frac{1}{\rho}\frac{\partial}{\partial z}(U_R l_R \rho), \quad (15)$$

$$\frac{\partial l_S}{\partial t} = -\mathbf{V}\cdot\nabla l_S + \nabla\cdot K_m\nabla l_S + P_S + \frac{1}{\rho}\frac{\partial}{\partial z}(U_S l_S \rho), \quad (16)$$

$$\frac{\partial l_G}{\partial t} = -\mathbf{V}\cdot\nabla l_G + \nabla\cdot K_m\nabla l_G + P_G + \frac{1}{\rho}\frac{\partial}{\partial z}(U_G l_G \rho), \quad (17)$$

where $q = l_{CW} + l_{CI} + r$; l_{CW}, l_{CI}, l_R, l_S, l_G and r are the mixing ratios for cloud water, cloud ice, rain, snow, hail and water vapor, respectively; and P_R, P_S and P_G are the production terms for rain, snow and hail. These terms will be considered in more detail in the next several subsections. Only the final form of the microphysical equations will be presented here. For a more detailed explanation of the derivations, the reader is referred to Wisner *et al.* (1972) or Chang (1977).

The last terms in (15), (16) and (17) are the fallout terms. All of the first terms on the right-hand side are advection terms; the second terms are diffusion terms.

LFO

c. Production term for snow

We have noted earlier that ice crystals originally grow by deposition until reaching a size where aggregation and riming become important, leading to the formation of snow crystals and snowflakes. Within the model, the processes considered to generate snow are the collision and aggregation of the smaller cloud ice particles, contact freezing of small raindrops, and depositional growth and riming of ice crystals. Once generated, the snow continues to grow by accretion and deposition. Sublimation and melting reduce the snow content.

The total production term for snow may be written for two temperature regimes.

(i) If the temperature is below 0°C ($T < T_0$)

$$\begin{aligned} P_S = {} & P_{\mathrm{SAUT}} + P_{\mathrm{SACI}} + P_{\mathrm{SACW}} + P_{\mathrm{SFW}} + P_{\mathrm{SFI}} \\ & + P_{\mathrm{RACI}}(\delta_3) + P_{\mathrm{IACR}}(\delta_3) - P_{\mathrm{GACS}} - P_{\mathrm{GAUT}} \\ & - P_{\mathrm{RACS}}(1-\delta_2) + P_{\mathrm{SACR}}(\delta_2) \\ & + P_{\mathrm{SSUR}}(1-\delta_1) + P_{\mathrm{SDEP}}(\delta_1). \end{aligned} \tag{18}$$

(ii) If the temperature is above 0°C ($T \geq T_0$)

$$P_S = P_{\mathrm{SMLT}} - P_{\mathrm{GACS}}, \tag{19}$$

where δ_1, δ_2 and δ_3 are defined as

$$\left.\begin{array}{l} T < T_0 \\ \quad \delta_1 = \begin{cases} 1, & \text{for } \; l_{CW} + l_{CI} > 0 \\ 0, & \text{otherwise} \end{cases} \\ \quad \delta_2 = \begin{cases} 1, & \text{for } \; l_R \text{ and } l_S < 10^{-4} \text{ g g}^{-1} \\ 0, & \text{otherwise} \end{cases} \\ \quad \delta_3 = \begin{cases} 1, & \text{for } \; l_R < 10^{-4} \text{ g g}^{-1} \\ 0, & \text{otherwise} \end{cases} \\ T \geq T_0 \\ \qquad \delta_1 = \delta_2 = \delta_3 = 0 \end{array}\right\} . \tag{20}$$

Each production term will be discussed in detail below and typical values for most given later in Fig. 3.

1) ICE CRYSTAL AGGREGATION

The aggregation rate of ice crystals to form snow is assumed to follow parameterization concepts originally proposed by Kessler (1969), to simulate the collision-coalescence process for cloud droplets. It may be written as

$$\boxed{P_{\mathrm{SAUT}} = \alpha_1 (l_{CI} - l_{I0}),} \tag{21}$$

where α_1 is a rate coefficient (s^{-1}), which is temperature dependent, and l_{I0} is a threshold amount for aggregation to occur. In this study, we set l_{I0} to be 10^{-3} g g^{-1}. The relationship used for the rate coefficient is

$$\alpha_1 = 10^{-3} \exp[0.025(T - T_0)],$$

which is a crude parameterization of the dependence of aggregation efficiency on crystal structure which, in turn, is temperature dependent.

The physically similar mechanism of aggregation of snow to form graupel, P_{GAUT}, will be described in subsection 3d, along with the other hail production terms.

2) ACCRETION

A variety of accretional growth mechanisms involving the interaction of snow with the other classes of hydrometeors are allowed in the model. There are also accretional processes involving the other classes of hydrometeors which may generate snow. The mathematical formulation for these accretional processes which produce or involve the snow content will now be described.

The accretion of cloud ice by snow is an aggregation process which occurs if the temperature is less than T_0 (273 K). The rate of accretional growth, P_{SACI}, is based on the geometric sweep-out concept integrated over all snow sizes for the assumed snow size distribution (2) which yields

$$P_{\mathrm{SACI}} = \frac{\pi E_{SI} n_{0S} c l_{CI} \Gamma(3+d)}{4\lambda_S^{3+d}} \left(\frac{\rho_0}{\rho}\right)^{1/2}, \tag{22}$$

where E_{SI} is the collection efficiency of the snow for cloud ice. Similar to the rate coefficient for ice crystal aggregation noted for (21), the collection efficiency of snow for cloud ice, E_{SI}, is assumed to be temperature dependent and can be expressed as

$$E_{SI} = \exp[0.025(T - T_0)]. \tag{23}$$

The accretion of cloud water by snow, P_{SACW}, is similar to (22), and is expressed as

$$\boxed{P_{\mathrm{SACW}} = \frac{\pi E_{SW} n_{0S} c l_{CW} \Gamma(3+d)}{4\lambda_S^{3+d}} \left(\frac{\rho_0}{\rho}\right)^{1/2},} \tag{24}$$

where E_{SW} is the collection efficiency of snow for cloud water, which is assumed to be 1 in this model. P_{SACW} will increase the snow content by accreting the cloud water and subsequently freezing it if the temperature is lower than 0°C. If the temperature is warmer than 0°C, P_{SACW} will contribute to the rain content via the assumption that unfrozen water will be shed from the snow particles. This will be described later in subsection 3e. The sensible heat associated with the accreted cloud water will also enhance the melting of snow [see Eq. (32)].

In the following discussion, terminology which is largely an artifact of the hydrometeor classification scheme adapted for this study will be developed and applied. This artificiality is related to the various in-

autoconversion of ice to snow (as in Kessler's cloud to rain)

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4) Sublimation

If hail falls out of a cloudy environment, then sublimation will occur in a subsaturated region. Similar to (31), we obtain the equation

$$P_{\rm GSUB} = \frac{2\pi(S_i - 1)}{\rho(A'' + B'')} n_{0G}\left[0.78\lambda_G^{-2} + 0.31S_c^{1/3}\Gamma(2.75) \times \left(\frac{4g\rho_G}{3C_D\rho}\right)^{1/4} \nu^{-1/2}\lambda_G^{-2.75}\right]. \quad (46)$$

The $(S_i - 1)$ in (46) is negative in the subsaturated region and thus $P_{\rm GSUB}$ is a sink term for hail content.

5) Melting

The melting of hail is based on heat balance considerations as described in Mason (1971) and Wisner *et al.* (1972). The melting rate $P_{\rm GMLT}$ is given by

$$P_{\rm GMLT} = -\frac{2\pi}{\rho L_f}(K_a T_c - L_v\psi\rho\Delta r_s)n_{0G} \times \left[0.78\lambda_G^{-2} + 0.31S_c^{1/3}\Gamma(2.75) \times \left(\frac{4g\rho_G}{3C_D}\right)^{1/4} \nu^{-1/2}\lambda_G^{-2.75}\right] - \frac{C_w T_c}{L_f}(P_{\rm GACW} + P_{\rm GACR}). \quad (47)$$

The discussion of the physics involved in this rate has already been provided in the development of the melting rate for snow, $P_{\rm SMLT}$ [(32)]. The findings of Rasmussen and Pruppacher (1982) noted in the discussion of (32), are especially relevant to (47). In the current formulation, the accreted cloud water is shed as rainwater and represents another source of rain from cloud water.

e. Production term for rain

Similar to the previous sections, we consider the total production rate first. The total production term for rain can be written as:

(i) If the temperature is below 0°C ($T < T_0$):

$$P_R = P_{\rm RAUT} + P_{\rm RACW} - P_{\rm IACR} - P_{\rm SARC} - P_{\rm GACR}\ ({\rm or}\ P'_{\rm GACR}) - P_{\rm GFR} + P_{\rm REVP}(1 - \delta_1). \quad (48)$$

(ii) If the temperature is above 0°C ($T \geqslant T_0$):

$$P_R = P_{\rm RAUT} + P_{\rm RACW} + P_{\rm SACW} + P_{\rm GACW} - P_{\rm GMLT} - P_{\rm SMLT} + P_{\rm REVP}(1 - \delta_1). \quad (49)$$

The terms are described below.

1) Autoconversion

The collision and coalescence of cloud droplets to form raindrops is parameterized using a modified form of the relation suggested by Berry (1968). It may be written as

$$P_{\rm RAUT} = \rho(l_{CW} - l_{W0})^2[1.2 \times 10^{-4} + \{1.569 \times 10^{-12}N_1/[D_0(l_{CW} - l_{W0})]\}]^{-1}, \quad (50)$$

more sophisticated than Kessler's

where N_1 is the number concentration of cloud droplets and D_0 the dispersion, with l_{W0}, a threshold for autoconversion, set equal to 2×10^{-3} g g^{-1}. When the amount of cloud water exceeds l_{W0}, there is a probability of forming raindrops. The introduction of the threshold in (50) is an empirical modification to Berry's original form made to better simulate observations of first echoes. For cold-based clouds typical of the northern High Plains region, we normally turn off $P_{\rm RAUT}$ consistent with observations which indicate the collision-coalescence process is rarely active (Dye *et al.*, 1974). The value of N_1 and D_0 used in this study are consistent with the continental nature of the clouds but, even with the modification, do not provide adequate suppression of the process. Therefore we regard Case 3, which does not allow this process, to be more realistic, especially with regard to precipitation initiation.

2) Accretion

Raindrops, once formed, continue to grow by accretion of cloud water. By applying the geometric sweep-out concept and integrating over all raindrop sizes, this rate is given as

$$\boxed{P_{\rm RACW} = \frac{\pi E_{RW} n_{0R} a l_{CW}\Gamma(3 + b)}{4\lambda_R^{3+b}}\left(\frac{\rho_0}{\rho}\right)^{1/2},} \quad (51)$$

Kessler's pk2 Sec. 8.7 7/8

where the collection efficiency E_{RW} is assumed to be 1. This rate is the same as that used by Wisner *et al.* (1972) and Orville and Kopp (1977), except for a height correction applied to the fallspeed relationship. $P_{\rm RACW}$ always serves as a source term of rain content, independent of temperature regime.

In the temperature region $T < 0$°C, there are three additional accretion terms which provide negative contributions to the rain field; they are $P_{\rm IACR}$, $P_{\rm SACR}$ and $P_{\rm GACR}$ (or $P'_{\rm GACR}$) given respectively by (26), (28) and (42). Two other accretion processes [$P_{\rm SACW}$ (24) and $P_{\rm GACW}$ (40)] provide positive contributions to rain if the temperature is above 0°C. This is another example of shedding in the model.

3) Freezing and melting

The freezing of raindrops $P_{\rm GFR}$ is a source term of hail content and is a sink term for rain content