A Double-Moment Multiple-Phase Four-Class Bulk Ice Scheme. Part I: Description

BRAD SCHONBERG FERRIER

Universities Space Research Association, Mesoscale Dynamics and Precipitation Branch, Laboratory for Atmospheres, NASA Goddard Space Flight Center, Greenbelt, Maryland

(Manuscript received 12 November 1992, in final form 14 June 1993)

ABSTRACT

A detailed ice-phase bulk microphysical scheme has been developed for simulating the hydrometeor distributions of convective and stratiform precipitation in different large-scale environmental conditions. The proposed scheme involves 90 distinct microphysical processes, which predict the mixing ratios and the number concentrations of small ice crystals, snow, graupel, and frozen drops/hail, as well as the mixing ratios of liquid water on wet precipitation ice (snow, graupel, frozen drops). The number of adjustable coefficients has been significantly reduced in comparison with other bulk schemes. Additional improvements have been made to the parameterization in the following areas: 1) representing small ice crystals with nonzero terminal fall velocities and dispersive size distributions, 2) accurate and computationally efficient calculations of precipitation collection processes, 3) reformulating the collection equation to prevent unrealistically large accretion rates, 4) more realistic conversion by riming between different classes of precipitation ice, 5) preventing unrealistically large rates of raindrop freezing and freezing of liquid water on ice, 6) detailed treatment of various rime-splitting ice multiplication mechanisms, 7) a simple representation of the Hobbs–Rango ice enhancement process, 8) aggregation of small ice crystals and snow, and 9) allowing explicit competition between cloud water condensation and ice deposition rates rather than using saturation adjustment techniques. For the purposes of conserving the higher moments of the particle distributions, preserving the spectral widths (or slopes) of the particle spectra is shown to be more important than strict conservation of particle number concentration when parameterizing changes in ice-particle number concentrations due to melting, vapor transfer processes (sublimation of dry ice, evaporation from wet ice), and conversion between different hydrometeor species.

The microphysical scheme is incorporated into a nonhydrostatic cloud model in Part II of this study. The model performed well in simulating the radar and microphysical structures of a midlatitude–continental squall line and a tropical–maritime squall system with minimal tuning of the parameterization, even though the vertical profiles of radar reflectivity differed substantially between these storms.

1. Introduction

The spatial distribution of diabatic heating by clouds has a direct impact upon the structure of tropical circulations, as well as those teleconnection patterns affecting midlatitude climate (Hartmann et al. 1984; DeMaria 1985; Lau and Peng 1987; Trenberth et al. 1988). In preparation for the Tropical Rainfall Measuring Mission (TRMM), cloud models are being used to develop algorithms for estimating distributions of diabatic heating in the tropics and subtropics from precipitation profiles retrieved using active (radar) and passive microwave observations from polar orbiting satellites (Simpson et al. 1988; Tao et al. 1990). However, airborne microwave measurements and radiative transfer calculations have shown that passive microwave signatures are highly sensitive to the microphysical structure of thunderstorms (Simpson et al. 1988; Yeh et al. 1990; Mognai et al. 1990; Adler et al. 1991; Smith et al. 1992). If cloud models are to be used in these studies, they must represent with reasonable accuracy the hydrometeor structures of a wide variety of convective storms.

Recently, McCumber et al. (1991, hereafter referred to as M) made a thorough comparison of various two-class (Cotton et al. 1982; Chen 1983) and three-class (Lin et al. 1983, hereafter LFO; Rutledge and Hobbs 1984, hereafter RH) bulk ice schemes using the nonhydrostatic Goddard Cumulus Ensemble (GCE) model (Tao and Soong 1986; Tao and Simpson 1989). They found that the three-class ice parameterizations produced better agreement between simulated and observed structures of a fast-moving GATE squall system, such as the proportion of surface rainfall in the stratiform region and the intensity and structure of the radar bright band. They also noted, however, that different parameterizations must be used in order to simulate the hydrometeor structure of convective systems in different large-scale conditions. For example, the basic hydrometeor structure of tropical and subtropical maritime systems is better simulated using the parameterization of RH, whereas the hydrometeor structure

© 1994 American Meteorological Society
of midlatitude continental storms is more accurately represented using the scheme of LFO (McCumber et al. 1991; Ferrier et al. 1991; Tao et al. 1991; Tao et al. 1993). Furthermore, reasonable agreement between simulated and observed radar reflectivity structures is often obtained only after "trial and error" adjustment of numerous coefficients in the parameterizations.

A new bulk microphysical scheme will be described that can simulate, with improved accuracy compared to other bulk treatments and with minimal adjustment of important coefficients, the diabatic heating and hydrometeor distributions of convective systems in widely varying large-scale environments. An equally important goal of this study is the formulation of a flexible bulk parameterization that can be modified in the future to incorporate findings from 1) observational studies using aircraft and radar data and 2) theoretical and numerical modeling studies using explicit spectral schemes. Substantive improvements in the parameterization of important microphysical processes will be presented. Because of the nature of bulk microphysical schemes, there are several possible ways of representing conversion processes between hydrometeor species. These approaches will be described and the best techniques will be documented. For example, in the formulation of those processes associated with conversion from one hydrometeor species to another, it will be shown that it is more important to preserve the basic spectral characteristics of the particle distributions than to maintain strict conservation of the particle number concentrations. This point is especially important if the modeler is interested in accurately simulating the higher-order moments of the particle distributions, such as those associated with precipitation rates ($D^{-3} - D^{-2} ; D$ is the particle diameter) and radar reflectivities ($D^6$).

### 2. Basic continuity equations

The microphysical parameterization calculates the mixing ratios of water vapor ($q_v$), cloud water in the form of small, nonprecipitating cloud droplets ($q_r$), raindrops ($q_r$), small ice crystals ($q_s$, also referred to as clouds ice), low-density ($\sim 0.1 \text{ g cm}^{-3}$) snow ($q_s$), moderate-density ($\sim 0.4 \text{ g cm}^{-3}$) graupel ($q_g$), and high-density ($\sim 0.9 \text{ g cm}^{-3}$) frozen drops/hail ($q_h$). To first order, the microphysical parameterization combines the main features that contrast the three-class ice schemes of RH (cloud ice, snow, and graupel) and LFO (cloud ice, snow, and hail). However, the model scheme also includes prognostic variables for the number concentrations of all ice hydrometeors ($n_i$ for cloud ice, $n_s$ for snow, $n_g$ for graupel, and $n_h$ for frozen drops), as well as the mixing ratios of liquid water on each of the precipitation ice species during wet growth and melting ($q_{sw}$ for snow, $q_{gw}$ for graupel, and $q_{hw}$ for frozen drops). The inclusion of mixed-phase precipitation ice into the parameterization allows for more accurate radar calculations, and in the future it should be useful in passive radiometric calculations using linked cloud-radiation models (Mugnier et al. 1990; Adler et al. 1991; Smith et al. 1992).

Microphysical continuity equations for the mixing ratios of water vapor and all hydrometeor species, the number concentrations of all classes of ice, and the thermodynamic energy equation are summarized in appendix A. Table 1 contains a brief description of those processes that affect the mixing ratios of all hydrometeor species, while those processes that affect the number concentrations of the various ice categories are described in Table 2. Appendix E contains a complete list

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Source</th>
<th>Sink</th>
<th>Process</th>
</tr>
</thead>
<tbody>
<tr>
<td>QCND</td>
<td>$q_v$</td>
<td>$q_v$</td>
<td>Condensation (QCND &gt; 0) or evaporation (QCND &lt; 0) of cloud water</td>
</tr>
<tr>
<td>QXEV</td>
<td>$q_v$</td>
<td>$q_v$</td>
<td>Evaporation of rain and wet ice (x = r, s, g, h)</td>
</tr>
<tr>
<td>QINT</td>
<td>$q_i$</td>
<td>$q_i$</td>
<td>Nucleation of small ice</td>
</tr>
<tr>
<td>QXDEP</td>
<td>$q_i$</td>
<td>$q_i$</td>
<td>Deposition (QXDEP &gt; 0) or sublimation (QXDEP &lt; 0) of ice (x = i, s, g, h)</td>
</tr>
<tr>
<td>QIFM</td>
<td>$q_i$</td>
<td>$q_i$</td>
<td>Cloud water freezing (QIFM &gt; 0)</td>
</tr>
<tr>
<td>QXFM</td>
<td>$q_i$</td>
<td>$q_i$</td>
<td>or melting of small ice (QIFM &lt; 0)</td>
</tr>
<tr>
<td>QXSHD</td>
<td>$q_i$</td>
<td>$q_i$</td>
<td>Freezing of liquid water on wet ice (QXSHD &gt; 0), melting of precipitation ice (QXSHD &lt; 0)</td>
</tr>
<tr>
<td>QICNVS</td>
<td>$q_i$</td>
<td>$q_i$</td>
<td>Raindrop shedding from ice</td>
</tr>
<tr>
<td>QIHR</td>
<td>$q_i$</td>
<td>$q_i$</td>
<td>Conversion of small ice to snow</td>
</tr>
<tr>
<td>QIFM</td>
<td>$q_i$</td>
<td>$q_i$</td>
<td>Montebello–Rangno freezing of cloud water</td>
</tr>
<tr>
<td>QHMX</td>
<td>$q_i$</td>
<td>$q_i$</td>
<td>Hallet–Mossop ride splintering of ice</td>
</tr>
<tr>
<td>QRAUT</td>
<td>$q_v$</td>
<td>$q_v$</td>
<td>Cloud water autoconversion to rain</td>
</tr>
<tr>
<td>QXACW</td>
<td>$q_v$</td>
<td>$q_v$</td>
<td>Collection of cloud water (x = r, i, s, g, h)</td>
</tr>
<tr>
<td>QXACI</td>
<td>$q_i$</td>
<td>$q_i$</td>
<td>Collection of small ice (x = r, s, g, h)</td>
</tr>
<tr>
<td>QXACS</td>
<td>$q_i$</td>
<td>$q_i$</td>
<td>Collection of snow (x = g, h)</td>
</tr>
<tr>
<td>QIACR</td>
<td>$q_i$</td>
<td>$q_i$</td>
<td>Raindrop freezing by collection of small ice</td>
</tr>
<tr>
<td>QHACR</td>
<td>$q_i$</td>
<td>$q_i$</td>
<td>Collection of rain by frozen drops/hail</td>
</tr>
<tr>
<td>QXACY</td>
<td>$q_i$</td>
<td>$q_i$</td>
<td>Raindrop freezing by collisions with $q_i$ particles to form $q_i$ particles (x = s, g, h; y = s, g, h)</td>
</tr>
<tr>
<td>QRACY</td>
<td>$q_i$</td>
<td>$q_i$</td>
<td>Conversion from $q_i$ particles to $q_i$ particles by raindrop freezing (x = s, g; y = g, h)</td>
</tr>
<tr>
<td>QXACWY</td>
<td>$q_i$</td>
<td>$q_i$</td>
<td>Cloud water riming onto $q_i$ particles to form $q_i$ particles (x = s, g; y = s, g, h)</td>
</tr>
<tr>
<td>QWACY</td>
<td>$q_i$</td>
<td>$q_i$</td>
<td>Conversion from $q_i$ particles to $q_i$ particles by cloud water riming (x = s, g; y = g, h)</td>
</tr>
</tbody>
</table>
Table 2. List of microphysical processes affecting hydrometeor number concentrations.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Source</th>
<th>Sink</th>
<th>Process</th>
</tr>
</thead>
<tbody>
<tr>
<td>NXEP</td>
<td>—</td>
<td>n_e</td>
<td>Evaporation of wet ice ($x = s, g, h$)</td>
</tr>
<tr>
<td>NINT</td>
<td>n_I</td>
<td>—</td>
<td>Initiation of small ice</td>
</tr>
<tr>
<td>NXDEP</td>
<td>n_s</td>
<td>—</td>
<td>Deposition (NXDEP &gt; 0), or sublimation (NXDEP &lt; 0) of ice ($x = i, s, g, h$)</td>
</tr>
<tr>
<td>NIFM</td>
<td>n_I</td>
<td>n_i</td>
<td>Cloud water freezing (NIFM &gt; 0), or melting of small ice (NIFM &lt; 0)</td>
</tr>
<tr>
<td>NXSHD</td>
<td>n_e</td>
<td>—</td>
<td>Shedding of rain by complete melting of precipitation ice ($x = s, g, h$)</td>
</tr>
<tr>
<td>NICNV</td>
<td>—</td>
<td>n_i</td>
<td>Conversion of small ice to snow</td>
</tr>
<tr>
<td>NSCNV</td>
<td>n_s</td>
<td>—</td>
<td>Freezing of cloud water by ice enhancement (Frohns–Rangno)</td>
</tr>
<tr>
<td>NIHR</td>
<td>n_i</td>
<td>—</td>
<td>Halley–Mossop rime splintering of ice ($x = s, g, h$)</td>
</tr>
<tr>
<td>NIHMX</td>
<td>n_i</td>
<td>—</td>
<td>Aggregation of small ice and snow ($x = i, s$)</td>
</tr>
<tr>
<td>NXACX</td>
<td>n_s</td>
<td>—</td>
<td>Collection of small ice ($x = r, s, g, h$)</td>
</tr>
<tr>
<td>NXACI</td>
<td>—</td>
<td>n_i</td>
<td>Collection of snow ($x = g, h$)</td>
</tr>
<tr>
<td>NSBR</td>
<td>n_s</td>
<td>—</td>
<td>Breakup of snow</td>
</tr>
<tr>
<td>NXACS</td>
<td>n_s</td>
<td>—</td>
<td>Collection of snow ($x = g, h$)</td>
</tr>
<tr>
<td>NIACR</td>
<td>n_h</td>
<td>—</td>
<td>Raindrop freezing by collection of small ice</td>
</tr>
<tr>
<td>NRACX</td>
<td>n_s</td>
<td>—</td>
<td>Conversion from $n_s$ particles to $n_r$ particles by raindrop freezing ($x = s, g, y = g, h$)</td>
</tr>
<tr>
<td>NXACRY</td>
<td>n_r</td>
<td>n_s</td>
<td>Conversion from $n_r$ particles to $n_s$ particles by cloud water riming ($x = s, g, y = s, g, h$)</td>
</tr>
</tbody>
</table>

The size distributions for rain and all of the ice species are represented by gamma functions, where

$$n_x(D) = n_o x D^{2x} \exp(-\lambda_x D_x);$$  \hspace{1cm} (3.2)

$n_o$ is the intercept, $\lambda_x$ is the slope, and $\alpha_x$ is the shape parameter of the distribution.\footnote{Note that the shape parameter is defined as $\alpha_x = 1$ in other mathematical representations of the gamma function (e.g., Verlinde et al. 1990).} The shape parameters of the particle distributions are independently specified for each of the hydrometeor categories, but remain constant throughout a given model simulation. Most of the model simulations have assumed exponential ($\alpha_x = 0$) distributions for each of the different ice hydrometeors, whereas the raindrop size spectra are typically represented by either exponential (Marshall and Palmer 1948) or gamma (Willis 1984) distributions. Future development of this scheme will include the number concentrations of raindrops as an additional prognostic variable, where explicit schemes (e.g., Kogan 1991) will be used as a means of improving such bulk approaches as Ziegler (1985). Based upon the modeling of cirrus clouds by Starr and Cox (1985), the functional relationship of (3.2) is also assumed for the particle distributions of small ice crystals, rather than assuming a monodisperse distribution for cloud ice as in other bulk schemes.

Since mixing ratios ($q_x$) and number concentrations ($n_x$) are calculated for each ice category, the slope and intercept of a given particle distribution are, respectively,

$$\lambda_x = \left[ \frac{\Gamma(1 + \alpha_x + d_x c_x n_x)}{\Gamma(1 + \alpha_x)} \right]^{1/d_x},$$  \hspace{1cm} (3.3)

$$n_o = \frac{n_0^{\alpha_x + d_x}}{\Gamma(1 + \alpha_x)};$$  \hspace{1cm} (3.4)

where $m_x(D_x) = c_x D_x^{d_x}$ is the assumed mass–diameter relationship of the dry ice particles, and $\Gamma$ is the gamma function. Precipitation ice is typically assumed to be spherical with $c_x = \pi/6 \cdot \rho_x$ and $d_x = 3$ for $x = s, g,$ and $h$ (snow, graupel, and hail/frozen drops). For cloud ice, $c_i = 0.044$ and $d_i = 3$ for bullet rosettes (Heysmield 1972; Starr and Cox 1985), which is believed to be the dominant small ice habit in thunderstorm anvils (Heysmield and Knollenberg 1972). Although different parameterized rain distributions have been tested, the gamma distributions of Willis (1984) are assumed with $\alpha_x = 2.5$, where (in cgs units)

$$\lambda_x = 3.483 (\rho q_x)^{-0.168};$$  \hspace{1cm} (3.5)

$$n_o = 30.07 (\rho q_x)^{-0.992};$$  \hspace{1cm} (3.6)

3. Particle characteristics

a. Size distributions

Following Williams and Wojtowicz (1982) and Ziegler (1985), the volume of cloud droplets ($v$) are assumed to have an exponential distribution of the form

$$n(v) = (n_w/v_0) \exp(-v/v_0),$$  \hspace{1cm} (3.1)

where $v_0 = \rho q_w/(\rho_l n_w)$ is the mean droplet volume, $n_w$ is the droplet number concentration, and $\rho_l$ is the density of liquid water. In many other bulk schemes cloud droplets are assumed to be monodisperse.

b. Terminal fall speeds

The general relationship used to represent the terminal fall velocities of cloud ice, rain, and precipitation ice (snow, graupel, and hail/frozen drops) is
\[ V_x(D_x) = \gamma a_x D_x \exp(-f_x D_x), \]  
(3.7)
\[ \gamma = (\rho_0/\rho)^{1/2}, \rho \] is the air density, and \( \rho_0 \) is the surface air density. Table 3 lists the values of the coefficients \( a_x \) and \( f_x \) and exponent \( b_x \) assumed for each hydrometeor species, the range of mass-weighted terminal fall velocities (defined later in this subsection), and the reference for each velocity–diameter (\( V–D \)) relationship.

As Potter (1991) recently showed, different fall speed relationships may be derived based upon different assumptions regarding the appropriate diameter of the ice particle. For example, Locatelli and Hobbs (1974) cite \( V–D \) and velocity–mass (\( V–m \)) fall speed relationships for different types of snow and graupel. The particle diameters in their study refer to the maximum particle dimension, which can differ markedly from the mean spherical diameter assumed in the bulk schemes. The snow fall speed coefficients are derived by substituting the assumed mass–diameter relationship for snow in the current scheme of

\[ m_x = \pi/6 \rho_s D_x^3 \]  
(3.8)
with \( \rho_s = 0.1 \text{ g cm}^{-3} \) into

\[ V_x(D_x) = 195.8 m_x^{0.14} \]  
(3.9)
which is the corrected \( V–m \) fall speed relationship (see Potter 1991) for “graupel-like snow of hexagonal type” from Locatelli and Hobbs (1974). This technique is a variation of Potter’s approach, where he used the density of liquid water instead of snow in (3.8) for the purpose of estimating particle fall velocities as functions of their melted diameter. The fall speed relationship for ice crystals was obtained by matching the fall speed of a 200-\( \mu \text{m} \) snow particle with an assumed fall speed of 10 cm s\(^{-1} \) for a 50-\( \mu \text{m} \) ice crystal, resulting in similar mass-weighted fall speeds between cloud ice and snow.

Weighted fall velocities are used to calculate the vertical flux convergence terms for falling hydrometeors. For example, the change in the mixing ratio of a hydrometeor species is

\[ \frac{\partial q_x}{\partial t} = -\frac{1}{\rho} \nabla \cdot (\rho V_q x) + \text{TURB}(q_x) \]
\[ + \frac{1}{\rho} \frac{\partial (\rho q_x[V]_x)}{\partial z} + \frac{\partial q_x}{\partial t}, \]  
(3.10)
where the terms on the right side of (3.10) represent the changes in \( q_x \) by advection, turbulent diffusion, vertical flux convergence, and microphysical sources and sinks, respectively. Equation (3.10) is used to advect dependent variables \( q_x \) and \( q_{aw} \), and \( q_{aw} \) is calculated from \( q_x - q_{aw} \). In calculating the vertical flux convergence of \( q_x \),

\[ [V]_x = \gamma a_x \frac{\Gamma(1 + \alpha_x + d_x + b_x)}{\Gamma(1 + \alpha_x + d_x)} \frac{\lambda_x^{1+\alpha_x+d_x}}{\lambda_x^{1+\alpha_x+d_x+b_x}} \]  
(3.11)
is the mass-weighted terminal velocity of the hydrometeors (Srivastava 1967) using (3.2) and (3.7). A similar prognostic equation is used for the advection of number concentration,

\[ \frac{\partial n_x}{\partial t} = -\nabla \cdot (V n_x) + \text{TURB}(n_x) + \frac{\partial (n_x[V]_x)}{\partial z} + \frac{dn_x}{dt}, \]  
(3.12)
except that

\[ [V]_x = \gamma a_x \frac{\Gamma(1 + \alpha_x + b_x)}{\Gamma(1 + \alpha_x)} \frac{\lambda_x^{1+\alpha_x}}{\lambda_x^{1+\alpha_x+b_x}} \]  
(3.13)
is the terminal velocity of the hydrometeors weighted by number concentration (Srivastava 1978).

c. Densities of wet ice

Although a constant density for dry snow, graupel, and frozen drops is assumed, the density of wet precipitation ice can change as a result of a variable mixture of liquid water and ice. Two different assumptions are made in deriving the densities of wet ice.

For porous snow and graupel, liquid water is assumed to be uniformly soaked throughout the volume of an ice particle, which implies that the volumes are the same between wet (water and ice) and dry (ice only) particles of equal ice mass. The total volume of spherical precipitation particles having a size distribution given by (3.2) is

\[ \text{VOL}_x = \int_0^\infty \pi/6 D_x^3 n_x D_x^{\alpha_x} e^{-\lambda_x D_x} dD_x \]
\[ = \pi/6 \frac{\Gamma(4 + \alpha_x)}{\Gamma(1 + \alpha_x)} n_x \lambda_x^{-3}. \]  
(3.14)

<table>
<thead>
<tr>
<th>Hydrometeor</th>
<th>( a_x )</th>
<th>( b_x )</th>
<th>( f_x )</th>
<th>( [V]_x ) (cm s(^{-1} ))</th>
<th>Reference(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rain</td>
<td>4854</td>
<td>1.0</td>
<td>1.95</td>
<td>170–730</td>
<td>Uplinger (1981)</td>
</tr>
<tr>
<td>Cloud ice</td>
<td>336</td>
<td>.6635</td>
<td>0.0</td>
<td>13–61</td>
<td>See text</td>
</tr>
<tr>
<td>Snow</td>
<td>129.6</td>
<td>.42</td>
<td>0.0</td>
<td>32.5–85.5</td>
<td>See text</td>
</tr>
<tr>
<td>Graupel</td>
<td>351.2</td>
<td>.37</td>
<td>0.0</td>
<td>104–314</td>
<td>Locatelli &amp; Hobbs (1974)</td>
</tr>
<tr>
<td>Frozen drops/hail</td>
<td>1094.3</td>
<td>.6384</td>
<td>0.0</td>
<td>136–922</td>
<td>Böhn (1989), Matson &amp; Huggins (1980)</td>
</tr>
</tbody>
</table>
In the following discussion, variables with the subscripts \( x \) and \( xi \) represent those quantities associated with the water–ice mixture and dry ice, respectively. Since the number concentrations and particle volumes are the same between the wet and dry distributions (i.e., \( n_x = n_{xi} \) and \( \text{VOL}_x = \text{VOL}_{xi} \)), it follows from (3.14) that the slopes of the wet and dry particle distributions are the same (\( \lambda_x = \lambda_{xi} \)). Using (3.3) to represent \( \lambda_x \) and \( \lambda_{xi} \) and noting that \( q_s = q_{si} + q_{osw} \), then the bulk density of wet snow and wet graupel is

\[
\rho_s = \rho_{si} (1 - F_{sw}),
\]

(3.15)

where \( F_{sw} \) is the liquid water fraction of the snow and graupel (see A.12), \( \rho_{si} = 0.1 \text{ g cm}^{-3} (x = s) \), and \( \rho_{pl} = 0.4 \text{ g cm}^{-3} (x = g) \).

In contrast, liquid water is assumed to be uniformly coated around high density frozen drops/hail because there is very little air within the interior of these (dry) particles. The liquid water content is therefore proportional to the difference in volumes between the wet (water and ice) frozen drops and their ice cores, such that

\[
\rho_{q_{osw}} = (\text{VOL}_h - \text{VOL}_{hi}).
\]

(3.16)

The bulk density of wet frozen drops/hail is obtained by using (3.3) and (3.14) to represent \( \text{VOL}_h \) and \( \text{VOL}_{hi} \) (\( \rho_{hi} = 0.9 \text{ g cm}^{-3} \)):

\[
\rho_h = \frac{\rho_{hi}}{1 - (1 - \rho_{hi}/\rho_L)F_{bw}}.
\]

(3.17)

Note that the density and the liquid water fraction of wet precipitation ice are calculated from the mixing ratios of the total water–ice (\( q_s \)) and liquid water constituents (\( q_{osw} \)).

4. Improved microphysical processes

Improvements made to the most important microphysical processes are discussed in this section. Appendix B contains additional discussion and extended derivations of some of these processes, as well as descriptions of the other microphysical processes listed in Tables 1 and 2.

a. Collection of precipitation

The discussion in this subsection is separated into three parts. First, a new method for calculating the collection rates associated with collisions between different classes of precipitation is described that is accurate and computationally efficient. Second, this technique is applied to more complicated three-component freezing processes involving collisions between ice particles and supercooled raindrops. Finally, a modified collection kernel is proposed in situations where hydrometeors are removed rapidly by large accretion rates.

1) Binary (Two Component) Accretion Processes (QXACZ, NXACZ)

The change in the mixing ratio of species \( X \) due to collection of species \( Z \) is

\[
\text{QXACZ} = \rho^{-1} \int \frac{n}{4} E_x (D_x + D_z)^2 \times |V_x - V_z| c_x D_x^6 n_x(D_x)n_z(D_z)dD_x dD_z,
\]

(4.1)

where QXACZ is used to represent the microphysical processes of QSACR, OGACR, QHACR, OGACS, and QHACS. Because this collection kernel has no straightforward analytic solution, simple approximations have been used in previous models to evaluate this integral (Wismer et al. 1972; Flatau et al. 1989; Murakami 1990). Verlinde et al. (1990) performed a detailed mathematical analysis of this collection kernel\(^2\) and described the errors associated with various approximations.

The approach in this parameterization is to solve these equations numerically and store the solutions in lookup tables. Substituting for \( n_x(D_z) \) and \( n_z(D_z) \) using (3.2), as well as for \( V_x(D_z) \) and \( V_z(D_z) \) using (3.7), the collection kernel in (4.1) is

\[
\text{QXACZ} = \frac{n}{4\rho} \gamma c_x n_x n_z \cdot \Lambda_q (\lambda_x, \lambda_z) \cdot \Delta V_q (\lambda_x, \lambda_z),
\]

(4.2)

\[
\Lambda_q = \Lambda_q (\lambda_x, \lambda_z)
\]

\[
= \frac{\Gamma (1 + \alpha_x) \Gamma (6 + \alpha_z)}{\lambda_x^{1+\alpha_x} \lambda_z^{5+\alpha_z}} + 2 \frac{\Gamma (2 + \alpha_x) \Gamma (5 + \alpha_z)}{\lambda_x^{2+\alpha_x} \lambda_z^{5+\alpha_z}} + \frac{\Gamma (3 + \alpha_x) \Gamma (4 + \alpha_z)}{\lambda_x^{3+\alpha_x} \lambda_z^{4+\alpha_z}},
\]

(4.3)

\(^2\) A constant collection efficiency independent of particle diameter was assumed in their study.

<table>
<thead>
<tr>
<th>Hydrometeor</th>
<th>( \alpha_x )</th>
<th>( \lambda_x )</th>
<th>( \lambda_x )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rain</td>
<td>0</td>
<td>10</td>
<td>100</td>
</tr>
<tr>
<td>Snow</td>
<td>2.5</td>
<td>20</td>
<td>150</td>
</tr>
<tr>
<td>Graupel</td>
<td>0</td>
<td>5</td>
<td>100</td>
</tr>
<tr>
<td>Frozen drops</td>
<td>0</td>
<td>5</td>
<td>100</td>
</tr>
</tbody>
</table>
\[
\Delta V_q = \Delta V_q(\lambda_x, \lambda_v) = \Lambda_q^{-1} \int \int \int E_{\omega}(D_x + D_v)^2 |a_x D_x^\nu e^{-f_x} - a_v D_v^\nu e^{-f_v} | D_x^{\alpha_v + \beta_x} e^{-\lambda_x D_x} dD_x D_v^{\alpha_v - \lambda_v D_v} dD_v 
\] (4.4)

is a scaled velocity associated with the difference in the terminal fall speeds of the colliding hydrometeor species, and \(c_z = \pi / 6 \rho_s\) and \(d_z = 3\) are assumed for precipitation. The term \(\Lambda_q\) is equal to the numerator in (4.4) but without the factor associated with the absolute value of the particle fall speed differences. The advantage of (4.2) over (4.1) is that the scaled velocity \(\Delta V_q\) is a function only of the slopes of the collector (\(\lambda_x\)) and collected (\(\lambda_v\)) particle distributions, and \(\Delta V_q\) varies much more gradually over a wide range of \(\lambda_x\) and \(\lambda_v\) than QXACZ. For each collection process, a two-dimensional lookup table is created that contains numerical solutions of \(\Delta V_q\) at 20 different logarithmically spaced values of \(\lambda_x\) and \(\lambda_v\), where

\[
(\lambda_x)_j = \chi^{-1}(\lambda_x)_j, \quad \text{and} \quad \chi = \exp\{[\ln(\lambda_x)_1 - \ln(\lambda_x)_{20}] / 19\}. \quad \text{(4.5)}
\]

Table 4 contains a list of the values of (\(\lambda_x)_1\) and (\(\lambda_x)_20\) for rain (\(\alpha_x = 0\) and \(\alpha_v = 2.5\) distributions), snow, graupel, and frozen drops/hail. Accurate estimates of \(\Delta V_q\) are obtained by bilinear interpolation with respect to the tabulated values of \(\lambda_x\) and \(\lambda_v\). Solutions for \(\Delta V_q\) are calculated by numerically integrating (4.1) over 50 discrete size intervals in \(D_x\) and \(D_v\) with \(D_x \leq 20 / \lambda_x\) and \(D_v \leq 20 / \lambda_v\). Tests indicate that the maximum relative error in calculating QXACZ using this approach is less than 5%, which is far more accurate than those approximations typically used in other bulk schemes, as discussed in Verlinde et al. (1990). The use of lookup tables also makes this technique computationally efficient. In addition, these tables only need to be created once and can be used in many model simulations, so long as the particle fall speeds and the range of \(\lambda\)'s (Tables 3 and 4) are unchanged. The lookup tables will be expanded in the future to include a larger range of \(\lambda\).

In conditions where graupel and frozen drops are growing by cloud water riming above the freezing level, it is possible that the larger collector particles are wet (wet growth), while the smaller collector particles remain dry (dry growth). Since the collection efficiency of snow (\(E_{\omega}\)) depends upon whether the surface of the collector particles is wet or dry, a modification to the collection kernel in (4.2) is described in appendix B for calculating the collection of snow by graupel (QGACS) and frozen drops (QHACS) in the presence of supercooled cloud water.

2) THREE-COMPONENT ACCRETION PROCESSES—THE FREEZING OF RAINDROPS (QXACRY, QRACXY, NRACXY, QIACR, NIACR)

In the parameterizations of LFO and RH, collisional freezing of raindrops was the source of either snow or rimed ice (i.e., graupel in RH, hail in LFO) based upon somewhat arbitrary threshold mixing ratios of rainwater and snow. The experiences of the Goddard cloud modeling group, however, indicate that the use of different threshold mixing ratios can have an important impact upon the microphysical structure of the simulated storms. Precipitation produced by collisional freezing of rain should be classified according to the densities of the resultant ice particles. For example, collisions between populations of rain and snow should simultaneously produce snow (large snowflakes collecting smaller drops), graupel (snow colliding with similar-sized drops), and frozen drops (large drops collecting smaller snow), whereas collisions between graupel and rain populations should produce either graupel (graupel collecting smaller drops) or frozen drops (large drops collecting graupel).

Such a process is considered in the current parameterization by assuming that the liquid water from the raindrop is evenly distributed throughout the volume of the collided ice particle before freezing. Equating the particle masses of an ice particle of diameter \(D_x\) colliding with a raindrop of diameter \(D_r\), then

\[
\pi / 6 (\rho_s D_x^3 + D_r^3) = \pi / 6 \rho_s D_r^3, \quad \text{(4.7)}
\]

where \(\rho_s\) is the density of the newly formed ice-drop mixture (\(\rho_t = 1\) for drops), which is classified as either snow \([\rho_s \leq 0.5 (\rho_t + \rho_s)]\), graupel \([0.5 (\rho_t + \rho_e) < \rho_s < 0.5 (\rho_t + \rho_s)]\), or frozen drops \([\rho_s \geq 0.5 (\rho_t + \rho_e)]\). Substituting these values of \(\rho_s\) into (4.7) gives a range of drop sizes (\(D_1 \leq D_x \leq D_2\)) as a function of the size of the colliding ice particle (\(D_x\)). Table 5 lists the values of \(D_1\) and \(D_2\) for each of the accretion processes. The production of ice (species \(Y\)) by the freezing of raindrops colliding with other ice particles (species \(X\), which can be the same ice class as \(Y\)) then becomes

\[
\text{QXACRY} = \frac{\pi^2}{24 \rho} \gamma_{\alpha_r \eta_{sr} \lambda_q} (\lambda_x, \lambda_v) \cdot \Delta V_q(\lambda_x, \lambda_v), \quad \text{(4.8)}
\]

\[
\Delta V_q(\lambda_x, \lambda_v) = \Lambda_q^{-1} \int_0^\infty D_x^{\nu_x} e^{-\lambda_x D_x} dD_x 
\times \int_{D_1}^{D_2} E_{\omega}(D_x + D_v)^2 |a_x D_x^\nu e^{-f_x} - a_v D_v^\nu e^{-f_v} | D_x^{\alpha_v} e^{-\lambda_v D_v} dD_v, \quad \text{(4.9)}
\]
in which \( \Lambda_q(\lambda_x, \lambda_s) \) is given by (4.3). Conversion of mixing ratio and number concentration, respectively, from one category of ice (species \( X \)) to another (species \( Y \)) as a result of collisional freezing of rain is

\[
\text{QRACXY} = \frac{\pi}{4\rho} \gamma c_x n_{ax} n_{ar} \cdot \Lambda_q(\lambda_r, \lambda_x) \cdot \Delta V_q(\lambda_r, \lambda_s),
\]

(4.10)

\[
\Delta V_q(\lambda_r, \lambda_s) = \Lambda_q^{-1} \int_0^\infty D_x^{u+2} e^{-\lambda D_x} dD_x \\
\times \int_{D_1}^{D_2} E_{ar}(D_x + D_r)^2 |a_r D_r^b| \\
- a_r D_r^b e^{-\lambda D_r} |D_r^p e^{-\lambda D_r} dD_r, \]

(4.11)

\[
\text{NRACXY} = \text{NXACRY}
= \frac{\pi}{4} \gamma n_{ax} n_{ar} \cdot \Lambda_x(\lambda_r, \lambda_x) \cdot \Delta V_x(\lambda_r, \lambda_s),
\]

(4.12)

\[
\Delta V_x(\lambda_r, \lambda_s) = \Lambda_x^{-1} \int_0^\infty D_x^{u+2} e^{-\lambda D_x} dD_x \\
\times \int_{D_1}^{D_2} E_{ar}(D_x + D_r)^2 |a_r D_r^b| \\
- a_r D_r^b e^{-\lambda D_r} |D_r^p e^{-\lambda D_r} dD_r, \]

(4.13)

where \( \Lambda_q(\lambda_x, \lambda_s) \) in (4.11) and \( \Lambda_x(\lambda_r, \lambda_x) \) in (4.13) are given by expressions analogous to (4.3) and (B.4), respectively. Values of \( \Delta V_q \) and \( \Delta V_x \) in (4.9), (4.11), and (4.13) are also represented by two-dimensional lookup tables at the same discrete values of \( \lambda_r \) and \( \lambda_s \) as in Table 4. The tabulated values of \( \Delta V_q \) also conserve hydrometeor mass, such that the following microphysical relationships,

\[
\text{QSACR} = \text{QSACRS} + \text{QSACRG} + \text{QSACRH}
\]

(4.14)

\[
\text{QGACR} = \text{QGACRG} + \text{QGACRH},
\]

(4.15)

are accurate to a precision of three orders of magnitude.

Another three-component freezing process is the production of frozen raindrops due to collisions with small ice crystals (QIACR, NIACR). As in other schemes (e.g., LFO; RH; Cotton et al. 1986, hereafter referred to as COT), the effects of ice crystal fall speeds upon the collection kernel are neglected because raindrop fall speeds are much faster. The errors associated with this assumption are small, given the limited size of the ice crystals. Ice crystals are converted into snow when \( \lambda_r < 50 \text{ cm}^{-1} \) (see section 4f). Future refinements of the scheme will include lookup tables for the collection of ice crystals. It is assumed, however, that raindrops effectively collect only those ice crystals larger than a critical size \( (D_n, \text{ where } E_n = 0 \text{ for } D < D_n \text{ and } E_n = 1 \text{ for } D > D_n) \), such that

\[
\text{QIACR} = \frac{\pi^2}{24\rho} \gamma a_r n_{ax} n_1 \Gamma(6 + \alpha + b_r)(\lambda_r + f_r^{(6+a_r+b_r)}) \\
\times \left\{ 1 - \gamma^* [6 + \alpha_r + b_r(\lambda_r + f_r^{(6+a_r+b_r)})] \right\}, \]

(4.16)

\[
\text{NIACR} = \frac{\pi}{4\rho} \gamma a_r n_{ax} n_1 \Gamma(3 + \alpha_r + b_r)(\lambda_r^{-(3+a_r+b_r)}) \\
\times \left\{ 1 - \gamma^* [3 + \alpha_r + b_r(\lambda_r + f_r^{(3+a_r+b_r)})] \right\}, \]

(4.17)

where \( \gamma^*(x, y) \) is the incomplete gamma function\(^3\) and \( D_n \) is 40 \( \mu \text{m} \) (Lew et al. 1985). This assumption is important because many of the ice crystals that coexist with raindrops are small \( (D_1 < D_n) \), resulting in effective collection efficiencies \( (E_n) \) much smaller than unity.

3) MODIFIED COLLECTION KERNEL FOR RAPID ACCRETION RATES

Although the techniques used to calculate instantaneous accretion rates are much more accurate than in previous parameterizations, unrealistically large changes in hydrometeor mixing ratio \( (\Delta q) \) occasionally occur when multiplying the accretion rates (QXACZ) by the model time step\(^4\) \( \Delta t \), such that the relative change in the hydrometeor mixing ratio,

\[ \gamma(x, y) = \gamma(x, y)/\Gamma(x), \text{ where } \gamma(x, y) = \int_0^x t^{\alpha_r-1} e^{-t} dt. \]

\[ \text{Time steps typically used in the GCE model range from 5 to 10 s.} \]
\[ \Delta q_r/q_t = \frac{\text{QXACZ}: \Delta t}{q_t}, \]  

(4.18)

is much larger than one. Such conditions most often occur in association with collisional freezing of supercooled raindrops in convective cells. Although the temptation is to reduce each of the rates of the microphysical sink terms in proportion to the initial hydrometeor mixing ratio \( q_t \) as is done in other bulk parameterizations, this could result in unrealistically large microphysical rates and biases between different processes. This concern is especially important when partitioning the mass and number concentrations associated with the freezing of raindrops into various classes of precipitation ice.

Consider the mixing ratio of rain frozen in time step \( \Delta t \) due to collection of small ice \( \Delta q_r \),

\[ \Delta q_r = \rho^{-1} \int_0^\infty m_r(D_r)P_n(D_r)n_r(D_r)dD_r, \]  

(4.19)

where \( m_r(D_r) = \pi/6\rho_r D_r^3 \) is the mass of a spherical raindrop, \( n_r(D_r) \) is the drop size distribution given by (3.2), \( P_n(D_r) \) is the probability function for drop freezing,

\[ P_n(D_r) = \min\{1, n_n(n_r, D_r)\}, \]  

(4.20)

and

\[ n_n(n_r, D_r) = \pi/4E n_r D_r^2 V, \Delta t \]  

(4.21)

is the number of ice crystals collected by a drop. The assumption used to derive (4.16) is that a drop collects at most one ice crystal during the time step. However,


\( n_{ri} \) increases rapidly with drop size, such that drops larger than \( D_{ri} \) will collect more than one ice crystal [i.e., \( n_{ri}(n_{i}, D_{ri}) = 1 \)] given that a sufficient number of ice particles are present. Figure 1 shows what portion of a raindrop distribution is frozen due to collection of small ice crystals, as well as how more drops are overcounted using (4.16) and (4.17) as the ice crystal number concentrations increase. Although there is no difference in calculating the rate of drop freezing using QIACR in (4.16) or \( \Delta q_{i}/\Delta t \) in (4.19) when the ice crystal concentrations are frozen small (Fig. 1a), the error associated with (4.16) increases dramatically with higher ice particle concentrations as the threshold drop size \( D_{ri} \) decreases; that is, more of the larger drops are overcounted as a result of each drop collecting more than one ice crystal during the time step (Figs. 1b–d). Detailed calculations indicate that the relative error associated with (4.16) is less than 5%–10% for values of \( \Delta Q_{IACR} \) (\( = \)QIACR \cdot \Delta t/q_{i} \) less than 0.1, whereas essentially all of the rain is frozen when values of \( \Delta Q_{IACR} \) exceed 7.5 for \( \alpha = 0 \) rain distributions and 5.0 for the \( \alpha = 2.5 \) rain distributions of Willis (1984); only for intermediate values of \( \Delta Q_{IACR} \) are revised calculations of QIACR made using (4.19). Thus, final values for QIACR are

\[
\text{QIACR} = \begin{cases} 
\text{QIACR from (4.16),} & \Delta Q_{IACR} < 0.1 \\
q_{i}/\Delta t, & \Delta Q_{IACR} > 7.5 \ (\alpha = 0) \\
\Delta q_{i}/\Delta t \text{ from (4.19), otherwise.} & \Delta Q_{IACR} > 5 \ (\alpha = 2.5)
\end{cases}
\]  

(4.22)

A procedure similar to (4.22) is used to calculate final values for NIACR using either (4.17) or \( \Delta n_{i}/\Delta t \), where

\[
\Delta n_{i} = n_{0i} \int_{0}^{\infty} \min \left( 1, \frac{\pi}{4} E_{n} n_{i} V_{i} D_{i}^{2} \Delta t \right) D_{i}^{2+\alpha_{i}} e^{-\lambda_{i}D_{i}} dD_{i}
\]  

(4.23)

is obtained from (3.2), (4.20), and (4.21).

When large collection rates occur between precipitation species, the mass of species \( Z \) collected by species \( X \) in time step \( \Delta t \) is

\[
\Delta q_{Z} = \frac{\pi}{6} \rho_{Z} n_{0i} \int_{0}^{\infty} P_{Z}(D_{z}) D_{z}^{3+\alpha_{z}} e^{-\lambda_{z}D_{z}} dD_{z} 
\]  

(4.24)

\[
P_{Z}(D_{z}) = \min \left[ 1, n_{Z}(n_{Z}, \lambda_{Z}, D_{z}) \right],
\]

(4.25)

\[
n_{Z}(n_{Z}, \lambda_{Z}, D_{z}) = \frac{\pi}{4} \gamma \Delta m_{Z} \int_{0}^{\infty} E_{Z} D_{z} + D_{z} \times |V_{z} - V_{i}| D_{z}^{2+\alpha_{z}} e^{-\lambda_{z}D_{z}} dD_{z},
\]

(4.26)

where \( E_{Z} \) is the collection efficiency, and \( n_{Z} \) is the number of particles of size \( D_{Z} \) collected by species \( X \). Numerical integration of (4.24) is performed only if the values of QXACZ \cdot \Delta t/q_{i} \) are in the ranges of 0.1–7.5 for the collection of \( \alpha = 0 \) distributed drops, 0.3–5.0 for the collection of \( \alpha = 2.5 \) distributed drops, and 0.5–1.5 for the collection of snow and graupel \( (\alpha = \alpha_{s} = 0) \). A final value for NXACZ is obtained by replacing \( P_{Z}(D_{z}) \) in (4.23) with \( P_{Z}(D_{z}) \). In situations where the freezing of rain by QXACR is modified using (4.24), the rates associated with three-body drop-freezing processes (QXACRY) are adjusted in order to satisfy (4.14) and (4.15).

In the future it should be possible to store numerical solutions for (4.19) and (4.24) in additional lookup tables by scaling the modified collection kernels based on the number concentrations of the collector particles. Successfully developing such a technique would make the microphysical scheme more computationally efficient.

b. Conversion by rime processes (QXACWX, QXACYY, QWACXY, NWACXY)

In the RH scheme and in the modification of the LFO scheme by Farley et al. (1989), snow can be converted to graupel (RH) or hail (LFO) by rapid cloud water riming when the mixing ratios of snow and cloud water exceed independently specified thresholds. Nevertheless, conversion from one ice class to another should be based upon changes in the bulk densities of the rimed particles. Because rime density is a complex function of the cloud droplet size, the ice particle surface temperature, and the impact velocity of the droplets onto the ice particle (Macklin 1962; Pflaum and Pruppacher 1979; Heymsfield and Pflaum 1985), the density characteristics of particles produced from the simplified riming conversion processes in RH and Farley et al. (1989) can differ substantially from rimed ice in real clouds. The effects of variable ice particle densities resulting from accretion of low-density rime upon the microphysical structure of a hailstorm has been studied by Farley (1987) using the explicit model of Farley and Orville (1986) with 20 size categories of precipitation ice.

In the current scheme, conversion between precipitation ice categories is based upon the riming rate and the rime density collected on the ice particles. Conversion by riming can occur from snow into graupel (QSACWG, QWACSG, NWACSG), from graupel into frozen drops/hail (QGACWH, QWACGH, NWACGH), and from frozen drops/hail into graupel (QHACWG, QWACHG, NWACHG); conversion from graupel into snow is not considered because the bulk characteristics of graupel remain essentially unchanged even when rapidly accreting low-density rime (Buser and Aufermann 1973; Farley 1987). For riming to occur, it is assumed that 1) the rime density is similar to that of the converted particle species, and 2) a sufficient amount of rime has accumulated so as to alter the bulk density of the converted
particles. The riming rates onto the converted and unconverted ice, respectively, are

\[ Q_{XACW} = Q_{XACW} \left[ \gamma^* (3 + \alpha_x + b_x, \lambda_x D_{2yv}) - \gamma^* (3 + \alpha_x + b_x, \lambda_x D_{1yv}) \right] \]  
(4.27)

\[ Q_{XACWX} = Q_{XACW} - Q_{XACWY} \]  
(4.28)

where \( Q_{XACW} \) is given by (B.7). Diameter \( D_{1yv} \) is the minimum size in which the rime density is similar to that of the converted particle species, while \( D_{2yv} \) is the size in which the particle mass doubles within time interval \( \Delta t_{\text{lime}} \) (typically assumed to be 120 s), such that only the smaller particles \( (D_x \leq D_{2yv}) \) are considered to have had their densities sufficiently modified. Rime collected on ice particles smaller than \( D_{1yv} \) and larger than \( D_{2yv} \) remains a mass source for the rimed species \( X \) in (4.27), while in (4.28) rime collected on particles in the size range \( D_{1yv} \leq D \leq D_{2yv} \) is a source of mass for the converted species \( Y \). The rate at which rimed hydrometeors are converted from species \( X \) to species \( Y \) is given by

\[ Q_{XACY} = q_s / (\Delta t_{\text{lime}}) \gamma^* (1 + \alpha_x + d_x, \lambda_x D_{2yv}) - \gamma^* (1 + \alpha_x + d_x, \lambda_x D_{1yv}) \]  
(4.29)

\[ NW_{XACY} = n_x / (\Delta t_{\text{lime}}) \gamma^* (1 + \alpha_x, \lambda_x D_{2yv}) - \gamma^* (1 + \alpha_x, \lambda_x D_{1yv}) \]  
(4.30)

The methods used to calculate \( D_{1yv} \) and \( D_{2yv} \) are presented in appendix B.

c. Freezing and melting of precipitation ice (QXFM), shedding of liquid water (QXSHD, NXSHD)

Liquid water is shed as a result of the complete melting of precipitation, or in order to maintain a maximum mass fraction of liquid water on the ice particle of \( F_{\text{swm}} \). The value of \( F_{\text{swm}} \) is typically set to 0.5, which assumes that at most half of the mass of an ice particle is composed of liquid water. The number concentrations of precipitation ice change only when liquid water is shed due to the complete melting of ice particles, where it is assumed that the slopes of the particle distributions \( \lambda_x \) are approximately constant during this process (Koenig and Murray 1976; Kopp et al. 1983; Murakami 1990). The processes associated with the shedding of liquid water, as well as the rates of freezing and melting of precipitation ice, are described in appendix B.

Alternative methods for calculating changes in ice number concentrations by melting were also examined. First, the intercept of the particle distribution \( n_{0i} \) was assumed constant during melting, as in LFO and RH. But as Orville and Kopp (1977) and Kopp et al. (1983) noted, this method produced unrealistically large decreases in the ratio of large to small particle sizes, resulting in much larger melting rates than are represented in the current approach. A more sophisticated technique was then tested where the mass and number concentrations associated with only those smallest ice particles that melt completely within a time step were removed. Since exponential distribution for ice is usually assumed, this method worked in the opposite sense to the constant \( n_{0i} \) approach by removing many more of the smaller particles than the larger particles, resulting in unrealistically large mean particle diameters below the melting level with substantially smaller melting rates than in the constant \( \lambda_x \) approach. These different treatments of melting will be shown in future studies to have a strong impact upon how far the ice falls below the melting level.

d. Initiation of small ice crystals

There are four different modes by which ice crystals can be initiated in the model: deposition/condensation freezing, stochastic/homogeneous freezing of cloud droplets, ice multiplication by rime splintering (e.g., Hallett and Mossop 1974), and the less understood ice enhancement mechanism of Hobbs and Rangno (1985).

1) DEPOSITION/CONDENSATION FREEZING (QINT, NINT)

The nucleation of small ice crystals at temperatures warmer than \(-5^\circ C\) follows COT and Murakami (1990), while at colder temperatures nucleation by deposition and condensation freezing following Meyers et al. (1992) was used:

\[ \text{NINT} = \max (0, w) \cdot \partial n_{0i} / \partial z, \]  
(4.31)

\[ \text{QINT} = \rho^{-1} m_{0i} \text{NINT}, \]  
(4.32)

\[ n_{0i} = \begin{cases} n_{0i1}, & T_c \geq -5^\circ C \\ n_{0i2}, & T_c < -5^\circ C, \end{cases} \]  
(4.33)

\[ n_{0i1} = n_{0i1} \left[ (q_o - q_{iv}) / (q_{sw} - q_{iv}) \right]^{\alpha_1} \exp (\beta_1 T_c), \]  
(4.34)

\[ n_{0i2} = n_{0i2} \exp (\alpha_2 SS_1 - \beta_2), \]  
(4.35)

where \( w \) is the vertical velocity, \( m_{0i} \) is the mass of a nucleated ice crystal (assumed to have an initial diameter of 25 \( \mu m \)), \( q_o \) and \( q_{sw} \) are the saturation mixing ratios with respect to ice and water, respectively, \( SS_1 = q_o / q_{iv} - 1 \) is the supersaturation ratio with respect to ice, \( n_{0i2} = 10^{-3} \text{ cm}^{-3} \cdot \alpha_2 = 12.96, \beta_2 = 0.639 \) (Meyers et al. 1992), \( \alpha_1 = 4.5, \beta_1 = 0.6 \text{ K}^{-1} \) (COT), and \( n_{0i1} = 5 \times 10^{-3} \text{ cm}^{-3} \) in order for \( n_{0i1} = n_{0i2} \) at \(-5^\circ C\). As in Meyers et al. (1992), \( n_{0i1} \) can also be set to zero in order to prevent nucleation by deposition/sorption of ice crystals at temperatures warmer than \(-5^\circ C\). Following Ziegler (1985) and Murakami (1990), the rate of nucleation in (4.31) is assumed to be dominated by vertical advection. Meyers et al. (1992) formulated
their scheme based upon improved measurements of crystal concentrations using continuous flow devices, which were available only at temperature above $-25^\circ\text{C}$. The importance of ice initiation in the upper portions of convective systems supports an obvious need for ice particle and aerosol measurements at much colder temperatures (and higher altitudes).

2) **Freezing of cloud water and melting of cloud ice (QIFM, NIFM)**

Murakami (1990) modified Wisner et al.'s (1972) probabilistic drop-freezing scheme in order to calculate the rate at which cloud droplets freeze stochastically. Probabilistic freezing of raindrops is not considered in this scheme because it is typically several orders of magnitude smaller than collisional drop freezing. A similar approach is adopted here by integrating Eq. (27) in Wisner et al. (1972) over the assumed droplet size distribution of (3.1), yielding

$$\text{NIFM} = \int_0^\infty B' \left[ \exp(A'T_c) - 1 \right] \nu(v) dv,$$

$$\text{QIFM} = \rho^{-1} \int_0^\infty B' \left[ \exp(A'T_c) - 1 \right] \rho v^2 \nu(v) dv.$$

(4.36) (4.37)

The droplet number concentration is assumed constant, except when the mean droplet diameter ($D_w$) reaches a minimum value ($D_{w_{\text{min}}}$), after which $n_w$ changes so as to maintain a mean droplet diameter of $D_{w_{\text{min}}}$. This constraint prevents unrealistically small mean droplet sizes from developing in regions where many of the cloud droplets are removed by riming onto ice rather than evaporation into dry air. Otherwise, assuming a constant droplet number concentration produces mean droplet sizes that are small enough to inhibit the stochastic freezing of cloud droplets at cold temperatures ($T_c < -20^\circ\text{C}$) where it is expected to be most effective. Using the definition for the mean droplet volume, the rate that droplets freeze into ice crystals at temperatures warmer than homogeneous freezing ($T_c > T_{\text{hom}}$) is

$$\text{QIFM} = \frac{2B'}{\rho_l n_w} \left[ \exp(A'T_c) - 1 \right] \rho q_w^2,$$

$$\text{NIFM}^* = \rho_l ^{-1} B' \left[ \exp(A'T_c) - 1 \right] \rho q_w,$$

$$\text{NIFM} = \min \left[ \text{NIFM}^*, n_w / D_l \right].$$

(4.38) (4.39) (4.40)

The rate of cloud water freezing for $T_c \leq T_{\text{hom}}$ is

$$\text{QIFM} = \max \left( 0, q_w / \Delta t + \text{QCND} \right),$$

$$\text{NIFM}^* = \rho_l ^{-1} B' \left[ \exp(A'T_c) - 1 \right] \text{QIFM} \cdot \Delta t,$$

and NIFM is given by (4.40). The rates of droplet freezing given by (4.38)–(4.40) increase rapidly as temperatures decrease toward $T_{\text{hom}}$, such that droplets typically glaciate completely within $\pm 5^\circ\text{C}$ of $T_{\text{hom}}$ (assumed to be $-40^\circ\text{C}$). Although (4.41)–(4.42) are used to ensure that homogeneous glaciation will occur when $T_c \leq T_{\text{hom}}$, it may soon be possible to improve upon this parameterization of homogeneous droplet freezing (Cotton 1993, personal communication).

Melting of small ice crystals is assumed to proceed rapidly, such that

$$\text{QIFM} = \frac{-q_l}{\Delta t},$$

$$\text{NIFM} = \frac{-n_l}{\Delta t}.$$

(4.43) (4.44)

3) **Rime-splitter ice multiplication (QIHMX, NHMX)**

Rime splintering has been hypothesized to be an important secondary ice multiplication process in convective clouds (Hallett and Mossop 1974; Mossop 1976; Hallett et al. 1978; Black and Hallett 1986; Willis and Hallett 1991; Houze et al. 1992). Although the mechanism was parameterized in COT and Ziegler (1988), a thorough investigation of the process was undertaken in this study using the recent laboratory results of Mossop (1985). The number of ice splinters produced depends upon 1) the ratio of small ($\leq 12 \mu m$) to large ($\geq 25 \mu m$) cloud droplets rimed onto the ice particle, 2) the cloud temperature between $-2^\circ\text{C}$ and $-8^\circ\text{C}$, and 3) the fall speeds of ice particles less than 6 m s$^{-1}$. A general derivation of rime splintering by precipitation ice is described in appendix B. Rime splintering will be shown in future studies to be effective in increasing the ice concentrations in late-mature and dissipating convective cells. It may also be important in weak convective clouds, in convective elements embedded within stratiform clouds, and in storms where rimed particles are recycled into updrafts with high water contents.

4) **Ice enhancement (QIHR, NIHR)**

Large number concentrations of small ice particles in maritime clouds have been observed to occur in a two-step process: 1) frozen and unfrozen drops form in concentrations of a few per liter near cloud top, followed within 5–10 min by 2) the onset of small, uniformly sized vapor-grown ice crystals in concentrations of $10–100 \text{ L}^{-1}$ (Hobbs and Rangno 1985, 1990; Rangno and Hobbs 1991). This ice enhancement mechanism occurs in the upper regions of clouds that have cloud-top temperatures colder than $-6^\circ\text{C}$, are wider than 3 km in diameter, and have a broad droplet spectrum. Recently, Barth et al. (1992) prescribed the effects of ice enhancement in their study of the chemistry of rainbands. An independently derived representation of Hobbs and Rangno's aircraft observations is presented in appendix B for the purpose of evaluating its impact upon model simulations of convective systems in different large-scale environments, even though the underlying physical processes associated with these observations are not well understood.
e. Adjustment of deposition rates

All microphysical processes involving the exchange of water vapor between various hydrometeor constituents and the environment (i.e., QXEVVP, QXDEP, QCND, QIDEPE, and QINT) are calculated independently in the current scheme. But as in real clouds, competition between these different processes for the available water vapor supply occurs in the mixed-phase region of a cloud where large numbers of ice particles coexist with cloud droplets. Constraints must be placed on these microphysical processes in order to prevent too much drying (moistening) of the cloud by condensation (evaporation) and deposition (sublimation). In some microphysical models, this problem is overcome through the use of saturation adjustment schemes, which assume that 1) the saturation vapor mixing ratio varies between water and ice in proportion to the relative amount of cloud water and cloud ice, 2) the relative rates of cloud water condensation and cloud ice deposition are a linear function of cloud temperature (e.g., Lord et al. 1984; Tao et al. 1989).

A less restrictive technique is used in the current scheme that adjusts the rates of vapor deposition onto ice only when too much water vapor is removed from (added to) the environment in association with large condensation (evaporation) and deposition (sublimation) rates. It is implemented if 1) \( q_v < q_a \) due to ice deposition and cloud water condensation at \( T_c > T_{hom} \) (\( T_{hom} \) is the homogeneous freezing temperature of \(-40^\circ C\)), 2) \( q_v > q_a \) as a result of ice sublimation and cloud water evaporation at \( T_c > T_{hom} \), or 3) ice deposition occurred at \( T_c < T_{hom} \). The procedure assumes that cloud water condensation is a much more rapid process than deposition onto ice at \( T_{hom} < T_c < 0^\circ C \). Representing the change in water vapor mixing ratio resulting from net condensation and net deposition processes, respectively, by

\[
\text{CND} = \Delta t (\text{QCND} + \text{QREVP})
\]

and

\[
\text{DEP} = \Delta t (\text{QINT} + \text{QIDEPE} + \text{QSDEP} + \text{QGDEP} + \text{QHDEP}),
\]

then the change in the water vapor mixing ratio is

\[
\Delta t = q_v^* \Delta t = q_v - \text{CND} - \Delta t \text{DEP}, \tag{4.47}
\]

where \( \zeta \) is a coefficient (0 \( \leq \zeta \leq 1 \)) used to adjust the various ice deposition rates in (4.46) so that the final water vapor mixing ratio at time \( t + \Delta t \) is at ice saturation (QCND, QREVP, and QXDEP are defined in appendix B). The saturation vapor mixing ratio with respect to ice at time \( t + \Delta t \) is

\[
q_v^* \Delta t = q_v [1 + a_2 \Delta T/(T - 7.66)^2], \tag{4.48}
\]

where \( T \) is the air temperature (deg K) and \( a_2 = 5807.7 \) (Tao et al. 1989), and the change in temperature due to latent heating is

\[
\Delta T = (L_i \Delta t \text{QIFM} + L_c \text{CND} + \zeta \cdot L_i \text{DEP})/C_p. \tag{4.49}
\]

The rate of freezing of cloud droplets into small ice crystals (QIFM) is included in the latent heating calculation because it can be substantial at colder temperatures. Combining (4.47) - (4.49) yields an expression for the coefficient \( \zeta \) used to modify each of the deposition processes in (4.46),

\[
\zeta = \frac{q_v - \text{CND} - q_a \left[ 1 + \frac{a_2 (L_i \Delta t \text{QIFM} + L_c \text{CND})}{C_p (T - 7.66)^2} \right]}{\text{DEP} \left[ 1 + \frac{a_2 L_i q_a}{C_p (T - 7.66)^2} \right]}. \tag{4.50}
\]

This method of constraining only the rates of vapor deposition has advantages over the adjustment techniques used in other microphysical parameterizations. In the proposed method, water vapor mixing ratios in convective updrafts are near water saturation in the presence of cloud water. Larger deposition rates onto ice reduce the supply of water vapor available for condensation onto the cloud droplets as ice concentrations increase with height (decreasing temperature), while at the same time cloud water is being removed rapidly by riming onto the various ice hydrometeors. Eventually enough ice is present to absorb the excess water vapor provided by ascent in the updrafts, such that water vapor mixing ratios fail to reach water saturation and prevent the condensation of cloud water. Because ice grows at the expense of the cloud water in this ice scheme, there is no need for parameterizing the Bergeron–Findeisen process as in LFO and RH. Furthermore, these competitive rates will vary in a dynamically and microphysically consistent manner in response to changes made to the processes parameterized in the scheme (this is also a feature of the COT scheme). This consideration is important when assessing the impact of microphysical sensitivity tests.

f. Conversion of small ice to snow

(QICNV, NICNV, NSCNV)

Small ice crystals are converted into snow as they grow to large enough sizes by deposition and aggregation. An initial technique was adopted that gradually converted the mass and number concentrations of ice crystals larger than a maximum size \( D_{i\text{max}} \) (typically 0.05 cm) into snow over a time interval \( \Delta t_s \) (varied between 60 and 300 s), such that

\[
\text{QICNV} = (q_i/\Delta t_s) [1 - \gamma^s (\alpha_i + d_i + 1, \lambda D_{i\text{max}})], \tag{4.51}
\]

\[
\text{NICNV} = \text{NSCNV} = (n_i/\Delta t_s) [1 - \gamma^s (\alpha_i + 1, \lambda D_{i\text{max}})]. \tag{4.52}
\]
But after conducting numerous model simulations, it was discovered that this conversion method did not conserve the higher moments of the particle spectra, such that the combined radar reflectivities associated with the small ice and snow increased by as much as 10 dBZ as a result of applying (4.51) and (4.52).

An alternative parameterization was derived by 1) conserving radar reflectivity during the conversion process, 2) assuming the number concentrations of cloud ice are approximately constant by converting a few of the largest crystals into snow, and 3) converting from cloud ice to snow by adjusting the slope of the cloud ice distribution \( \lambda_i \) to a minimum value \( \lambda_{ni} \) (50 cm\(^{-1} \)) only when \( \lambda_i < \lambda_{ni} \). The resultant process for converting ice crystals into snow is

\[
QICNVS = \frac{q_i}{\Delta t} [1 - (\lambda_i / \lambda_{ni})^3],
\]

(4.53)

\[
NSCNV = \frac{q_i}{\Delta t} [1 - (\lambda_i / \lambda_{ni})^3] / [1 + (\lambda_i / \lambda_{ni})^3]
\]

(4.54)

and NICNV = 0.

5. Radar reflectivity

Because many studies have comprehensively documented, as a function of space and time, the radar structure of storms in different geographical regions, a reasonably straightforward and comprehensive means of evaluating model performance is to compare simulated and observed radar reflectivity fields. Since the microphysical parameterization allows for variable particle size distributions, as well as calculates the liquid water contents on precipitation ice, a simple method for calculating radar reflectivity using Rayleigh theory is described in appendix C. The approach of Smith et al. (1975) has been used in previous models to calculate the radar reflectivities associated with large, wet (hail) ice (e.g., Orville and Kopp 1977; Ziegler 1985; Banta and Hanson 1987; McCumber et al. 1991); however, these calculations were based on a very limited set of microphysical conditions, in which \( n_{ni} = 0.0003 \text{ cm}^{-4} \) was assumed for the hail distribution with a constant water film 0.5 mm thick around the hail particles. Surprisingly, the reflectivities computed from (C.11) using the hail size spectra in Smith et al. (1975) are within ±0.5 dBZ of their Mie calculations for a 10-cm radar. Nevertheless, future plans are to improve the technique of calculating reflectivities using Mie scattering theory, since errors will increase for smaller-wavelength radars and for ice particles with more complicated ice—water topologies.

6. Issues regarding spectral characteristics of particle size distributions

The equations describing the rate of change of hydrometeor number concentrations due to accretion processes were derived based on conserving the number concentrations of the interacting species. At first this approach seemed straightforward and logical, and it is the physical assumption used to derive NRACYX and NXACRY in (4.12), NIACRY in (4.17), NWA- CXY in (4.30), and NXACS in (B.3). However, GCE model simulations of GATE and TAMEX squall systems revealed that the simulated radar reflectivities in the convective cells above the freezing level were up to 10–15 dBZ higher than observed. Reflectivity maxima exceeded 60 dBZ in the modeled convective cores above the freezing level, which is inconsistent with GATE and TAMEX radar observations of modest peak reflectivities (50–55 dBZ) at lower levels with reflectivities decreasing steadily with height above the freezing level (Zisper and LeMone 1980; Szoke et al. 1986; Szoke and Zisper 1986; Jorgensen and LeMone 1989).

Figure 2 illustrates how the slopes (mean diameters) of the frozen drop spectra decrease (increase) dramatically with respect to the original drop size distributions when the mass and number concentrations of the collected drops in Fig. 1 are substituted into (3.3). This artificial decrease in the slopes of the converted particle spectra is especially large when few ice crystals are present (Figs. 2a and 2b), since, as Fig. 1 shows, only the largest raindrops are frozen [see also Eqs. (4.20) and (4.25)]. The significant increase in the number of particles larger than 4 mm in diameter (Figs. 2b–d) illustrates how the combined radar reflectivity factors \( D^6 \) moment of the parameterized particle distributions can increase substantially as a result of strictly conserving particle number concentrations in the conversion process, while at the same time constraining the particle distributions to remain exponential. Such redistribution of the particle spectra is clearly undesirable, especially since there is a much greater functional dependence of \( \lambda_n \) upon the microphysical rates than number concentration.

The size distributions of the collected drops shown in Fig. 1 exhibit a much larger degree of kurtosis than the exponential distribution associated with the original drop population, such that the spectra of the collected drops would be better represented by gamma distributions with large shape parameters \( \alpha_s > 0 \). The errors in simulated radar reflectivities are due to the constraint of assuming constant shape parameters for each of the precipitation species during a model run. Conserving the number concentrations, mixing ratios, and radar reflectivities \( D^6 \) moment when converting from one precipitation category to another (e.g., the freezing of a few raindrops) requires calculating the variations in the slopes, intercepts, and shape parameters of the interacting particle distributions. Because the current scheme predicts variations in \( \lambda_n \) and \( n_{ni} \) for an assumed constant \( \alpha_s \), errors in reflectivity will occur when conserving the number concentrations and mass mixing ratios of the various precipitation categories. Calculating the changes in
Fig. 2. The unfrozen raindrop size distribution (sum of the collected and uncollected drops) in Fig. 1 is shown by the solid lines in (a)–(d). The dashed lines represent the \( n_0 = 0 \) size distributions of the water-equivalent \( \left( \rho_e = \rho_r \right) \) frozen drops associated with the number concentrations and mixing ratios of the collected drops in Fig. 1, where the radar reflectivity associated with the redistribution of collected and uncollected drops increased by (a) 1.8 dBZ, (b) 6.7 dBZ, (c) 6.7 dBZ, and (d) 3.7 dBZ. The dotted lines represent the size spectra of the water-equivalent frozen drops calculated by assuming the same slopes as for the original, unfrozen drop size distribution, in which the radar reflectivity associated with the redistribution of collected and uncollected drops increased by 0.01 dBZ in (a), decreased by 0.8 dBZ in (b), decreased by 0.6 dBZ in (c), and decreased by 0.08 dBZ in (d).

The shape parameters of different hydrometeor classes is beyond the scope of this paper, and the usefulness of such an endeavor will likely be determined in the future by advances in computer technology and by improvements in explicit, detailed microphysical schemes.

Although some of the improved microphysical processes discussed in the previous section helped to reduce the large discrepancy between the simulated and observed radar fields, it is most important that the spectral characteristics \( i.e., \lambda_p \) of the interacting particle distributions be preserved in the conversion process rather than strictly conserving the number concentrations of the constituent species. This conclusion was also made in determining the final version of other microphysical processes described in section 4 and in appendix B (e.g., NXEVP, NXDEP, NXSHD, NSCNV). Figure 2 shows how the size distributions of the collected, frozen drops are improved by assuming the same slope as for the unfrozen rain.

Consequently, those microphysical processes that involve conversion between hydrometeor categories as a result of accretional processes have been rederived by conserving the slope for the particle distributions:

\[
\begin{align*}
NXACI &= (n_0/q_e) QXACI, \\
NXACS &= (n_0/q_e) QXACS, \\
NWACXY &= (n_0/q_e) QWACXY, \\
NRACXY &= (n_0/q_e) QRACXY, \\
NIACR &= (R_e \cdot n_0/q_e) QIACR, \\
NXACRY &= \frac{[R_e \cdot n_0/q_e] QXACRY^2}{QXACRY + QRACXY}. 
\end{align*}
\]

The first four equations are similar to the relationships for NXDEP and NXEVP [see Eqs. (B.55) and (B.56), respectively]. Furthermore, many of the processes af-
fecting ice number concentrations in the ice schemes of Koenig and Murray (1976) and Murakami (1990) have a similar functional form to (6.1)–(6.4). The last two equations differ in that these processes involve converting from raindrops, which may have an exponential ($\alpha_r = 0$) or gamma ($\alpha_r = 2.5$) distribution, to $\alpha_r = 0$ ice distributions. Appendix D contains a derivation for $R_\alpha$, which is a correction factor used to conserve the radar reflectivity factor when the shape parameter ($\alpha$) changes from one particle distribution to another. For example, $R_\alpha = 1$ in (6.5) and (6.6) when the conversion process occurs between two exponential hydrometeor distributions (e.g., $\alpha_r = \alpha_e = 0$); however, when $\alpha_r = 2.5$ distributed raindrops freeze to form $\alpha_r = 0$ distributed ice, $R_\alpha = 4.181$ in order to conserve the $D_m^3$ moment of the particle distributions ($D_m$ is the equivalent melted diameter of an ice particle, which has a mass equal to that of a raindrop of the same size). For the three-body accretion process in (6.6), an average of the $\lambda$ associated with the colliding particle distributions is used that is weighted by their respective mass conversion rates (QXACRY for the freezing of drops; QRACXY for the accretion of $q$, ice).

7. Conclusions

The following improvements have been made to the proposed bulk microphysical scheme in comparison to other bulk parameterizations:

- Four categories of ice are predicted in the model (small ice crystals, snow, graupel, and frozen drops/hail).
- Number concentrations of each ice class are predicted.
- The liquid water fraction is calculated for each of the precipitation ice species.
- Small ice crystals have a dispersive size distribution with nonzero terminal fall speeds.
- Improvements have also been made in parameterizing the accretion of precipitation, riming, conversion, raindrop freezing, and the freezing of liquid water on ice, ice multiplication and ice enhancement processes, aggregation of small ice crystals and snow, rates of cloud water condensation and ice deposition, and conversion processes between hydrometeor classes.

Calculating the number concentrations of the various ice categories offered unique problems that needed to be overcome in developing the parameterization. The most important problem that was addressed was how to formulate the conversion of particle number concentrations between hydrometeor species, where it was concluded that, in order to conserve the higher particle moments, preserving the slopes of the interacting particle distributions is more important than conserving the number concentrations of the particle species. A future study will assess the sensitivity of the simulated convection to different parameterized drop size distributions, terminal fall speeds, and aggregation efficiencies of ice crystals and snow, ice nucleation at cold temperatures ($T_c < -20^\circ C$), changes in particle number concentrations due to melting and sublimation, and changes in hydrometeor concentrations associated with various conversion processes (including drop freezing and riming conversion) between hydrometeor species.

The double-moment four-class ice scheme has been developed to represent the microphysical structure of storms in different large-scale conditions. Given the current limitations in our knowledge of important characteristics of ice in clouds, it will be shown that the radar structure associated with the convective and stratiform portions of mesoscale convective systems in different environments are reproduced well with minimal tuning of the parameterization. Predicting the liquid water fraction on ice should also allow for improved calculations of active and passive radiometric signatures using linked cloud and radiation models.

The approach adopted in the current scheme has been to improve the parameterization of various accretion and riming conversion processes by storing the solutions to complex, nonanalytic equations in detailed lookup tables. In the future, additional lookup tables will be used extensively to represent many more of the microphysical processes using results from explicit warm rain (e.g., Clark 1973; Soong 1974; Young 1975; Kogan 1991) and ice phase parameterizations (Takahashi 1976; Hall 1980; Farley and Orville 1986), laboratory experiments, and future airborne microphysical observations, such as from TOGA COARE (Tropical Ocean Global Atmosphere Coupled Ocean–Atmosphere Response Experiment). It is envisioned that the current bulk parameterization can be made more realistic using results from detailed observational and theoretical studies, yet remain computationally efficient given the need for more physical processes to be included in numerical models, as well as the use of nested or adaptive grids in combined mesoscale–convective models.

Acknowledgments. The author gratefully acknowledges Drs. Joanne Simpson and Wei-Kuo Tao for their support and encouragement of this research, as well as their thoughtful review of the manuscript. Constructive comments from Drs. William Cotton, Richard Farley, and an anonymous reviewer are also appreciated. The author also thanks Dr. Harold Orville for finding inadequacies in the parameterization of melting ice (which have since been corrected) during his review of Part II. The author is also grateful to Drs. J. Theon and R. Karak for their support of this study under Contract 460-23-54 of the NASA Headquarters Radiation, Dynamics and Hydrology Branch, and by the NASA TRMM project under Contract 460-63-58.
APPENDIX A

Continuity Equations

The prognostic equations for the mixing ratios of all phases of water in the parameterization (i.e., vapor, liquid, ice, and liquid water on ice) are as follows:

$$\frac{dq_v}{dt} = -QCND - QREVP - (1 - \delta)(QSEVP + QGEVP + QHEVP)$$
$$- \delta(QINT + QIDEP + QSDEP + QGDEP + QHDEP), \quad (A.1)$$

$$\frac{dq_w}{dt} = QCND - QRAUT - QRACW - QSACWS - QGACWG - QHACWH$$
$$- QIFM - \delta(QIACW + QIHR + QSACWG + QGACWH + QHACWG), \quad (A.2)$$

$$\frac{dq_i}{dt} = QIFM + \delta(QINT + QIDEP + QIACW + QIHR + QIHMS + QIHMG$$
$$+ QIHMH - QICNVS - QRACI - QSACI - QGACI - QHACI), \quad (A.3)$$

$$\frac{dq_r}{dt} = QREVP + QRAUT + QRACW + QSSHD + QGSHD + QHSHD$$
$$- \delta(QIACR + QSACRS + QSACRG + QSACRH + QGACRG + QGACRH + QHACR), \quad (A.4)$$

$$\frac{dq_k}{dt} = QSACWS - QGACS - QHACS - QSSHD + (1 - \delta)QSEVP + \delta(QSDEP + QICNVS$$
$$+ QSACI + QSACRS - QRACSG - QRACSH - QWACSG - QIHMS), \quad (A.5)$$

$$\frac{dq_h}{dt} = QGACWG + QGACS - QGSHD + (1 - \delta)QGEVP + \delta(QGDEP + QGACI$$
$$+ QGACRG + QSACRG + QRACSG + QSACWG + QWACSG + QHACWG$$
$$+ QWACHG - QRACGH - QWACGH - QIHMG), \quad (A.6)$$

$$\frac{dq_m}{dt} = QHACWH + QHACS - QHSHD + (1 - \delta)QHEVP + \delta(QHDEP + QHACI$$
$$+ QHACR + QIACR + QRACI + QSACRH + QRACSH + QGACRH + QRACGH$$
$$+ QGACWH + QWACGH - QWACHG - QIHMH), \quad (A.7)$$

$$\frac{dq_{sw}}{dt} = QSACW - QSFM - QSSHD - F_{sw}(QGACS + QHACS) + (1 - \delta)QSEVP$$
$$+ \delta[QSACRS - F_{sw}(QRACSG + QRACSH + QWACSG)], \quad (A.8)$$

$$\frac{dq_{gw}}{dt} = QGACW - QGFM - QGSHD + F_{sw} \cdot QGACS + (1 - \delta)QGEVP$$
$$+ \delta[QGACRG + QSACRG + QSACWG + QHACWG + F_{sw}(QRACSG + QWACSG)$$
$$+ F_{sw} \cdot QWACHG - F_{gw}(QRACGH + QWACGH)], \quad (A.9)$$

$$\frac{dq_{lw}}{dt} = QHACW - QHFM - QHSHD + F_{sw} \cdot QHACS + (1 - \delta)QHEVP$$
$$+ \delta[QIACR + QSACRH + QGACRH + QHACR + QGACWH + F_{sw} \cdot QRACSH$$
$$+ F_{sw}(QRACGH + QWACGH) - F_{lw} \cdot QWACHG]. \quad (A.10)$$

The functions \( \delta \) in (A.1) – (A.10) and \( F_{lw} \) in (A.8) – (A.10) are defined as
\[
\delta = \begin{cases} 
1, & T < 0^\circ C \\
0, & \text{otherwise}, 
\end{cases} 
\]
\hspace{1cm} \text{(A.11)}
\[
F_{sw} = q_{sw}/q_s, 
\hspace{1cm} \text{(A.12)}
\]

where the variable \( x \) represents the precipitation ice species of snow, graupel, and hail/frozen drops (\( x = s, g, h \)). Changes in the simulated potential temperature (\( q \)) due to latent heating are calculated using the following thermodynamic energy equation:
\[
\frac{d\theta}{dt} = \frac{L_v}{\Pi C_p} (\text{QCND} + \text{QREV}) 
+ \frac{\delta L_r}{\Pi C_p} (\text{QINT} + \text{QIDEP} + \text{QSDEP} + \text{QGDEP} + \text{QHDEP}) 
+ \frac{L_f}{\Pi C_p} [\text{QIFM} + \text{QSFM} + \text{QGFM} + \text{QHFM} + \delta(\text{QIACW} + \text{QIHR})], 
\hspace{1cm} \text{(A.13)}
\]

where \( \Pi \) is the Exner function \( (\rho_0/\rho)^\kappa \) and \( \kappa = R_v/C_p \).

Finally, prognostic equations for the number concentrations of each ice species are
\[
\frac{dn_s}{dt} = \text{NIFM} + \delta(\text{NINT} + \text{NIDEP} + \text{NIHMS} + \text{NIHMG} + \text{NIHM} + \text{NIHR} 
- \text{NICNV} - \text{NIACI} - \text{NRACI} - \text{NSACI} - \text{NGACI} - \text{NHACI}), 
\hspace{1cm} \text{(A.14)}
\]
\[
\frac{dn_g}{dt} = \text{NSBR} - \text{NSACS} - \text{NGACS} - \text{NHACS} + (1 - \delta)(\text{NSEVP} - \text{NSSHD}) 
+ \delta(\text{NSCNV} + \text{NSDEP} - \text{NRACSG} - \text{NRACSH} - \text{NWACS}), 
\hspace{1cm} \text{(A.15)}
\]
\[
\frac{dn_h}{dt} = (1 - \delta)(\text{NGEVP} - \text{NGSHD}) + \delta(\text{NGDEP} + \text{NSACRG} 
+ \text{NWACS} + \text{NWACH} - \text{NRACGH} - \text{NWACGH}), 
\hspace{1cm} \text{(A.16)}
\]
\[
\frac{dn_n}{dt} = (1 - \delta)(\text{NHEVP} - \text{NHSHD}) + \delta(\text{NHDEP} + \text{NIACR} 
+ \text{NSACRH} + \text{NGACRH} + \text{NWACGH} - \text{NWACH}). 
\hspace{1cm} \text{(A.17)}
\]

**APPENDIX B**

**Description of Microphysical Processes**

a. **Collection of snow (QXACS, NXACS)**

Letting \( D_{sw} \) represent the minimum particle diameter in which the collector particles (i.e., graupel and frozen drops) become wet, such that only those collector particles larger than \( D_{sw} \) are wet \((D \geq D_{sw})\), the resulting collection kernel is
\[
\text{QXACS} = \frac{\pi}{4\rho} c_r n_w n_{sw} \Lambda_s \Delta V_s [(E_{sw})_{dry} (1 - F_{QX})] 
+ (E_{sw})_{wet} F_{QX}, 
\hspace{1cm} \text{(B.1)}
\]

where \( \Lambda_s \) and \( \Delta V_s \) are described in (4.3)–(4.4), \((E_{sw})_{dry}\) and \((E_{sw})_{wet}\) represent the collection efficiencies of snow for dry and wet collector (i.e., graupel or frozen drop) particles, respectively [see Eq. (B.12) for values of \((E_{sw})_{dry}\) and \((E_{sw})_{wet}\)], and
\[
F_{QX} = \left[ \frac{\int_{D_{sw}}^\infty D_s^\alpha e^{-\lambda_s D_s} dD_s \int_0^D (D_s + D)^2}{\int_{D_{sw}}^\infty D_s^\alpha e^{-\lambda_s D_s} dD_s \int_0^D (D_s + D)^2} \right] 
\times \left[ \frac{\int_{D_{sw}}^\infty D_s^\alpha e^{-\lambda_s D_s} dD_s \int_0^D (D_s + D)^2}{\int_{D_{sw}}^\infty D_s^\alpha e^{-\lambda_s D_s} dD_s \int_0^D (D_s + D)^2} \right], 
\hspace{1cm} \text{(B.2)}
\]

is the fractional contribution to the collection kernel made by the wet collector particles. The diameter \( D_{sw} \) is calculated using a heat balance equation similar to Musil (1970). Since \( D_{sw} = 0 \) and \( F_{QX} = 1 \) at \( T_c = 0^\circ C \), QXACS is calculated from (4.2) with \((E_{sw})_{wet}\) as the collection efficiency of snow. Three-dimensional lookup tables for \( F_{QX} \) (i.e., \( F_{QX} \) for QGACS and \( F_{QX} \) for QHACS) were created in the same manner as for \( \Delta V_s \) using the same values of \((\lambda_s)\) as in Table 4; however, ten discrete, logarithmically spaced values of \((D_{sw})\) from 0.05 to 1.0 cm and only five logarithmically spaced values of \((\lambda_s)_i\) from 10 cm\(^{-1}\) to 100 cm\(^{-1}\) are needed in the \( F_{QX} \) table.
Tables of $\Delta V_n$ and $\text{FN}_{nx}$ were also created for the purposes of calculating the changes in the number concentrations of collected snow (NGACS, NHACS), where

$$\text{NXACS} = \pi\, n_{\alpha n_{\alpha}} \cdot \Lambda_n(\lambda_x, \lambda_s) \cdot \Delta V_n(\lambda_x, \lambda_s)[(E_{nx})_{\text{dry}}(1 - \text{FN}_{nx}) + (E_{nx})_{\text{wet}}\text{FN}_{nx}], \quad (B.3)$$

$$\Lambda_n = \Lambda_n(\lambda_x, \lambda_s) = \frac{\Gamma(3 + \alpha_x) \Gamma(1 + \alpha_s)}{\lambda_x^{3+\alpha_x} \lambda_s^{1+\alpha_s}} + 2 \frac{\Gamma(2 + \alpha_x) \Gamma(2 + \alpha_s)}{\lambda_x^{2+\alpha_x} \lambda_s^{2+\alpha_s}} + \frac{\Gamma(1 + \alpha_x) \Gamma(3 + \alpha_s)}{\lambda_x^{1+\alpha_x} \lambda_s^{3+\alpha_s}}, \quad (B.4)$$

$$\Delta V_n = \Delta V_n(\lambda_x, \lambda_s) = \Lambda_n^{-1} \int_{D_{\text{sw}}}^{\infty} D_s^n e^{-\lambda_s D_s} dD_s \int_{0}^{\infty} \left( D_x + D_y \right)^2 |V_x - V_s| D_x^n e^{-\lambda_x D_x} dD_x, \quad (B.5)$$

and

$$\text{FN}_{nx} = \frac{\int_{D_{\text{sw}}}^{\infty} D_s^n e^{-\lambda_s D_s} dD_s \int_{0}^{\infty} \left( D_x + D_y \right)^2 |V_x - V_s| D_x^n e^{-\lambda_x D_x} dD_x}{\int_{0}^{\infty} D_s^n e^{-\lambda_s D_s} dD_s \int_{0}^{\infty} \left( D_x + D_y \right)^2 |V_x - V_s| D_x^n e^{-\lambda_x D_x} dD_x}. \quad (B.6)$$

b. Collection of cloud droplets and small ice (QXACW, QXACI, NXACI)

The collection of cloud water by other hydrometeor species $X$ ($x = i, r, s, g, h$) is

$$\text{QXACW} = \pi/4 E_{\text{sw}} q_w \gamma a_s n_{\alpha x} \Gamma(3 + \alpha_x + b_s)$$

$$\times (\lambda_x + f_x)^{-(3+\alpha_x+b_s)}, \quad (B.7)$$

where $E_{\text{sw}}$ is the mean collection efficiency integrated over all sizes of collector particles and all sizes of cloud droplets. Since accurate calculation of $E_{\text{sw}}$ is not possible through analytic means because the collection efficiency $E_{\text{sw}}(D_{\text{sw}}, D_x)$ is a complex function of the sizes of the droplets and the collector particles (e.g., Hall 1980), $E_{\text{sw}}$ is assumed to be unity for each of the hydrometeor categories.

The collection of cloud ice by precipitation ($x = r, s, g, h$) is given by

$$\text{QXACI} = \pi/4 E_{ri} q_i \gamma a_s n_{\alpha x} \Gamma(3 + \alpha_x + b_s)$$

$$\times (\lambda_x + f_x)^{-(3+\alpha_x+b_s)}, \quad (B.8)$$

$$\text{NXACI} = \pi/4 E_{ni} q_i \gamma a_s n_{\alpha x} \Gamma(3 + \alpha_x + b_s)$$

$$\times (\lambda_x + f_x)^{-(3+\alpha_x+b_s)}, \quad (B.9)$$

where $E_{ri}$ represents the collection efficiency of ice crystals by precipitation particles larger than $D_{\text{sw}}$ (40 $\mu$m) in diameter. The collection efficiencies of small crystals by rain, snow, and rimed ice (graupel and frozen drops/hail), respectively, are

$$E_{ri} = 1 - \gamma^*[3 + \alpha_x + b_s, (\lambda_x + f_s)D_{\text{sw}}], \quad (B.10)$$

$$E_{ri} = E_{ni1} \exp(E_{ni2} T_c) \{1 - \gamma^*[3 + \alpha_x + b_s, \lambda_s D_{\text{sw}}]\}, \quad (B.11)$$

$$E_{xy} = E_{xy1} \exp(E_{xy2} T_c) \{\gamma^*[3 + \alpha_x + b_s, \lambda_x D_{\text{sw}}]\}$$

$$- \gamma^*[3 + \alpha_x + b_s, \lambda_x D_{\text{sw}}] + (E_{xy})_{\text{wet}}$$

$$\times [1 - \gamma^*[3 + \alpha_x + b_s, \lambda_x D_{\text{sw}}]], \quad (B.12)$$

where $E_{ni1} = 0.25$ and $E_{ni2} = 0.05$ in (B.11) were estimated from Kajikawa and Heysfield (1989), $E_{xy1} = E_{xy2} = 0.1$ in (B.12) describe the collection efficiencies of ice crystals and snow by dry graupel and dry frozen drops/hail ($x = g, h; y = i, s$), and $(E_{xy})_{\text{wet}} = 1$ is assumed for the collection of cloud ice and snow by wet, rimed precipitation ice.

As more detailed laboratory results become available, accurate calculations of the integrated collection efficiencies ($E_{\text{sw}}, E_{ri}$) can be incorporated into the parameterization by storing their numerical solutions in lookup tables as functions of the mean droplet diameter ($D_{\alpha}$) and the slope ($\lambda_x$) of the particle spectra.

c. Diameters $D_{1xy}, D_{2xy}$ in riming conversion (QXACWX, QXACWY, WQACXY, NWACXY)

Particle diameters $D_{1xy}, D_{1gh},$ and $D_{1kg}$ [symbolized by $D_{xy}$ in Eqs. (4.27)–(4.30)] are the minimum particle sizes in which the density of the accreted rime is equal to $\rho_{x}$ (conversion from snow to graupel), $\rho_{h}$ (conversion from graupel to frozen drops), and $\rho_{k}$ (conversion from frozen drops to graupel), respectively. A minimum diameter threshold of 0.05 cm is
typically assumed for \( D_{1xy} \). Rime densities were calculated using Heymsfield and Pflaum (1985) and the detailed parameterization of impact velocities by Rasmussen and Heymsfield (1985). Values of \( D_{1xy} \) were solved by numerical iteration, and the solutions stored in three-dimensional lookup tables at discrete values of mean droplet diameter (\( D_{a} \), cloud temperature (\( T_{c} \) in °C), and height [variation of \( \gamma \) with height; see Eq. (3.7)]. Table A1 shows the organization of the lookup tables for \( D_{1x} \), \( D_{1yh} \), and \( D_{1kg} \). These tables need to be created only when using a different input environment or when the vertical resolution in the model is modified (the GCE model allows for variable stretched vertical coordinates).

Particle diameter \( D_{2xy} \) represents the size in which the particle mass (\( m_{x} \)) doubles within time interval \( \Delta t_{\text{time}} \), and it is solved by using the following collection equation,

\[
dm_{x}/dt = \pi/4E_{xw}\gamma\rho_{w}a_{x}D_{2xy}^{1-b}, \quad \text{(B.13)}
\]

and by assuming that the density of the accreted rime is that of the converted particle (\( \rho_{r} \)),

\[
dm_{x}/dt = d(\pi/6\rho_{r}D_{2xy}^{3})/dt. \quad \text{(B.14)}
\]

Combining (B.13) and (B.14), integrating over time interval \( \Delta t_{\text{time}} \), and representing the final size of the rimed particle by \( D_{2xy} \), yields

\[
D_{2xy}^{1-b} - D_{2xy}^{1-b} = \frac{(1 - b)x_{w}\rho_{w}a_{x}(\Delta t)_{\text{time}}}{2\rho_{r}}. \quad \text{(B.15)}
\]

Since the mass of the particle is assumed to have doubled during \( \Delta t_{\text{time}} \),

\[
\pi/6\rho_{r}(D_{2xy}^{3} - D_{2xy}^{3}) = \pi/6\rho_{r}D_{2xy}^{3}. \quad \text{(B.16)}
\]

Solving for \( D_{2xy} \) in (B.16) and substituting into (B.15), a relationship for \( D_{2xy} \) as a function of cloud water content and height (\( \gamma \)) is

\[
D_{2xy} = \left[ \frac{\tau_{xy}(\Delta t)_{\text{time}}\gamma\rho_{w}}{1-b} \right]^{(1-b)^{-1}}. \quad \text{(B.17)}
\]

where

\[
\tau_{xy} = \frac{(1 - b)x_{w}\rho_{w}a_{x}}{2\rho_{r}}[(1 + \rho_{w}/\rho_{r})^{(1-b)/3} - 1]^{-1} \quad \text{(B.18)}
\]

is a constant associated with each riming conversion process (e.g., \( \tau_{xy} \) is the constant for the processes QSACWG, QWACSG, and NWACSG). Rimming conversion occurs only when \( D_{1xy} < D_{2xy} \) and \( D_{2xy} \approx \Phi \).

### Table A1. Data structure of lookup tables for \( D_{1xy} \) in (4.27)–(4.30). See discussion after (B.18) for how \( \Delta t_{h} \) is obtained. The factor \( \Phi_{c} \) is scaled such that \( D_{1xy} \) is the mean cloud droplet diameter for a maximum cloud water content of 5 g m\(^{-3} \).

| \( D_{1xy} \) (\( T_{c}, a, z \)) for QSACWS, QSACWG, QWACSG, NWACSG. |
|----------------|------------------|-------------------|
| \( T_{c} \) = i\( \Delta t_{c} \) for \( i = 1 \) to \( 40 \), \( \Delta t_{c} \approx -0.5^\circ \text{C} (T_{c} \text{ varies from } -0.5^\circ \text{C } \text{ to } -20^\circ \text{C}) \), |
| \( D_{a} \) = \( x^{-1} D_{a} \) (\( D_{a} \)), for \( i = 1 \) to \( 30 \), |
| \( z_{i} \), \( n \) vertical levels between \( 0^\circ \text{C} \) and \( -30^\circ \text{C} \) (typically 7–9 discrete levels, depending upon vertical coordinates used in model). |
| \( D_{1xy} \) (\( T_{c}, a, z \)) for QGACWG, QGACWH, QWACHG, NWACHG. |
| \( T_{c} \) = i\( \Delta t_{c} \) for \( i = 1 \) to \( 40 \), \( \Delta t_{c} \approx -0.5^\circ \text{C} (T_{c} \text{ varies from } -0.5^\circ \text{C } \text{ to } -20^\circ \text{C}) \), |
| \( D_{a} \) = \( x^{-1} D_{a} \) (\( D_{a} \)), for \( i = 1 \) to \( 30 \), |
| \( z_{i} \), \( n \) vertical levels between \( 0^\circ \text{C} \) and \( -30^\circ \text{C} \) (typically 7–9 discrete levels, depending upon vertical coordinates used in model). |
| \( D_{1xy} \) (\( T_{c}, a, z \)) for QHACWG, QHACWH, QWACHG, NWACHG. |
| \( T_{c} \) = i\( \Delta t_{c} \) for \( i = 1 \) to \( 40 \), \( \Delta t_{c} \approx -1^\circ \text{C} (T_{c} \text{ varies from } -5^\circ \text{C } \text{ to } -40^\circ \text{C}) \), |
| \( D_{a} \) = \( x^{-1} D_{a} \) (\( D_{a} \)), for \( i = 1 \) to \( 30 \), |
| \( z_{i} \), \( n \) vertical levels between \( 0^\circ \text{C} \) and \( -45^\circ \text{C} \) (typically 9–11 discrete levels, depending upon vertical coordinates used in model). |

**d. Aggregation of small ice and snow (NXACX), breakup of snow (NSBR)**

Many studies have recognized the importance of ice aggregation, especially in stratiform clouds (e.g., Lo and Passarelli 1982; Stewart et al. 1984; Churchill and Houze 1984; Gordon and Marwitz 1986; Heymsfield 1986; Houze and Churchill 1987; Willis and Heymsfield 1989; Gamache 1990; Houze et al. 1992). The decrease in the number of concentrations of small ice crystals and snow by aggregation is

\[
\text{NXACX} = -\frac{\pi}{8} \gamma a_{r} n_{sx}^{2} \int_{0}^{\infty} E_{sx}(D_{1} + D_{2})^{2} [D_{2xy}^b - D_{2xy}^b] D_{2xy}^{2} \int_{0}^{\infty} e^{-\lambda(D_{1} + D_{2})} dD_{1} dD_{2}. \quad \text{(B.19)}
\]

Using an approach similar to Passarelli (1978), the analytic solution to (B.19) is

\[
\text{NXACX} = \gamma a_{r} E_{sx} n_{sx}^{2} I(\alpha_{s}, b_{s})^{-1}, \quad \text{(B.20)}
\]

\[
I(\alpha_{s}, b_{s}) = \frac{\pi \Gamma(4 + 2\alpha_{s} + b_{s})}{2^{6 + 2\alpha_{s} + b_{s}}} \sum_{k=1}^{\infty} C_{k} \left[ \frac{F^{*}(k + \alpha_{s} + 1)}{k + \alpha_{s}} - \frac{F^{*}(k + \alpha_{s} + b_{s} + 1)}{k + \alpha_{s} + b_{s}} \right], \quad \text{(B.21)}
\]
where \( F^*(x) = F(1, 4 + 2a_x + b_x; x; 1/2) \) is Gauss’ hypergeometric function, \( c_1 = c_2 = 1 \) and \( c_2 = 2 \), and \( I(a_x, b_x) \) is a constant that depends only upon the shape parameter \( a_x \) and full speed exponent \( b_x \) given in Tables 3 and 4, respectively, for small ice and snow. The self-breakup of snow is parameterized implicitly by preventing the slope of the snow distribution from decreasing below a lower limit of \( \lambda_{00} \), where aircraft observations and theoretical studies suggest a value of \( \lambda_{00} = 10 \text{ cm}^{-1} \) (Lo and Passarelli 1982; Mitchell 1988).

e. Freezing rates of liquid water onto precipitation ice (QXFМ)

It was assumed in an earlier version of the model, as well as in other microphysical schemes, that collisional freezing of rain by collection of small ice crystals and snow (QIACR, QSACR) resulted in the complete freezing of the drop. But in simulations where large numbers of ice particles were present, this assumption resulted in the immediate freezing of copious amounts of rain (up to 5 g kg\(^{-1}\)) at supercooled air temperatures warmer than \(-0.5^\circ\text{C}\). (These situations can occur when active turrets rise in regions where ice particles are already present.) Such rapid glaciation rates are unrealistic and are inconsistent with the time required for the complete freezing of raindrops (Pruppacher and Klett 1978). Since other microphysical parameterizations only limit the freezing rates of liquid water collected on hail during wet growth, the rates of drop freezing by collection of cloud ice and snow in these schemes are also likely to be too rapid (e.g., RH, LFO, COT).

The rate of freezing of liquid water onto a population of precipitation ice \((x = s, g, h)\) is

\[
\text{QXFМ} = \min(\text{XFМ1}, \text{XFМ2}), \quad (B.22)
\]

where XFМ1 describes the maximum freezing rate of the precipitation ice, which is controlled by the rate that heat is dissipated to the environment by evaporation and conduction, and XFМ2 is the amount of liquid water available for freezing. The terms XFМ1 and XFМ2 are for snow, graupel, and frozen drops/hail, respectively:

\[
\text{SFM1} = (1 - \Delta t \text{QWACSG}/q_t) \text{QSFZ},
\]

\[
\text{SFM2} = q_{nw}/\Delta t + \text{QSACW + QSACRS} - F_{sw}(\text{QWACSG + QRACSG + QRACSH + QGACS + QHACS}),
\]

\[
\text{GFM1} = (1 - \Delta t \text{QWACGH}/q_h) \text{QGFZ} + \Delta t \{\text{QRFZ \cdot QSACRGQ}/q_t + \text{QSACW} \cdot \text{QWACSG}/q_t + \text{QHACHG} \cdot \text{QWACHG}/q_h\},
\]

\[
\text{GFM2} = q_{nw}/\Delta t + \text{QGACW + QGACRG + QSACRG + QSACWG + QHACWG + F}_{sw}(\text{QWACSG + QRACSG + QGACS}),
\]

\[
\text{HFM1} = (1 - \Delta t \text{QWACGH}/q_h) \text{QHFZ} + \Delta t \{\text{QRFZ \cdot QIACR + QSACRH + QGACR}/q_t + \text{QGFZ \cdot QWACGH}/q_h\},
\]

\[
\text{HFM2} = q_{nw}/\Delta t + \text{QHACW + QHACR + QIACR + QSACRH + QGACWH + QGACR + F}_{sw}(\text{QRACSH + QHACS}) + F_{sw}(\text{QWACGH + QRACG}) - F_{sw}(\text{QWACGH}).
\]

The variables QXFМ \((x = r, s, g, h)\) represent the freezing rates derived from the heat balance relationship between the heat gained by freezing and heat lost by evaporation and conduction (Musil 1970; Pruppacher and Klett 1978):

\[
\text{QRFZ} = \min[q_t/\Delta t, \max(0, X_t \cdot n_{wi} \cdot \text{VENT}_t)], \quad (B.29)
\]

\[
\text{QSFZ} = \max(0, X_t \cdot n_{wi} \cdot \text{VENT}_t + X_2 \cdot \text{QSACI}), \quad (B.30)
\]

\[
\text{QGFZ} = \max[0, X_1 \cdot n_{wi} \cdot \text{VENT}_t + X_2 \cdot (\text{QGACI} + \text{QGACS})], \quad (B.31)
\]

\[
\text{QHFZ} = \max[0, X_t \cdot n_{wi} \cdot \text{VENT}_t + X_2 \cdot (\text{QHACI} + \text{QHACS})], \quad (B.32)
\]

where \(X_t\) and \(X_2\) are defined as

\[
X_t = X_5/(1 + c_w T_c/L_f), \quad (B.33)
\]

\[
X_2 = -C_r T_c/(L_f + c_w \Delta T_c), \quad (B.34)
\]
\[ X_3 = 2\pi/L_f \{ L_s \psi[\varphi_{sw}(0^\circ C) - q_s] - \rho^{-1}K_eT_e, \} \text{ (B.35)} \]

\( \psi \) is the diffusivity of water vapor in air, \( K_e \) is the thermal conductivity of air,
\[ \text{VENT}_x = A_x \frac{\Gamma(2 + \alpha_x)}{\lambda_x^{2 + \alpha_x}} + B_x S_{s}^{1\over 3} \left( \frac{\gamma a_x}{\nu} \right)^{1\over 2} \times \frac{\Gamma(2.5 + \alpha_x + 0.5b_s)}{(\lambda_x + f_x)^{2.5 + \alpha_x + 0.5b_s}} \text{ (B.36)} \]

is the ventilation effects associated with falling precipitation integrated over the particle size distributions using (3.2) and (3.7), \( \nu \) is the kinematic viscosity, \( Sc (=\nu/\rho) \) is the Schmidt number, and \( A_x = 0.78 \) and \( B_x = 0.31 \) are the ventilation coefficients assumed for precipitation (Beard and Pruppacher 1971).

Melting of precipitation is treated in a manner similar to other schemes, where
\[ \text{QXFM} = \min[0, X_{qsw} \cdot \text{VENT}_x - c_w T_s (\text{QXACW} + \text{QXACR})/L_f]. \text{ (B.37)} \]

In order to prevent unrealistic temperature oscillations across the 0°C, the combined freezing and melting rates, respectively, of all hydrometeors were limited near 0°C as follows:
\[ \Delta T_{lo} = T_e + \Delta t_{c}/C_e (\text{QIFM} + \text{QIACW} + \text{QIH}) + \Delta t_{sw}/C_s (\text{QINT} + \text{QIDEP} + \text{QSDEP} + \text{QGDEP} + \text{QHDEP}), \text{ (B.39)} \]
\[ \text{QSF} + \text{QGM} + \text{QHM} \leq -0.5C_p \Delta T_{sw}/L_f, \text{ (B.38)} \]
\[ \Delta T_{mh} = T_e + \Delta t_{c}/C_e (\text{QCN} + \text{QREVP} + \text{QSEVP} + \text{QGEVP} + \text{QHEVP}) + \Delta t_{sw}/C_s (\text{QIFM} + \text{QHM}), \text{ (B.40)} \]

causing gridpoint temperatures near the freezing level to oscillate above and below 0°C at subsequent time steps.

\[ f. \text{ Shedding of liquid water (QXSHD, NXSHD)} \]

The rates that liquid water is shed due to the complete melting of ice at \( T_e > 0^\circ C \) are
\[ \text{QSSHD} = \max[0, q_s/\Delta t + \text{QSCW} + \text{QSCR} + \text{QSEVP} - \text{QGACS} - \text{QHAC}], \text{ (B.42)} \]
\[ \text{QGSHD} = \max[0, q_s/\Delta t + \text{QGACW} + \text{QGACR} + \text{QGGEVP} + \text{QGASC}], \text{ (B.43)} \]
\[ \text{QHSHD} = \max[0, q_s/\Delta t + \text{QHACW} + \text{QHACR} + \text{QHEVP} + \text{QHAC}], \text{ (B.44)} \]

Otherwise, the rates in which excess amounts of liquid water are shed from large ice (at all temperatures) are
\[ \text{QXSHD} = \max[0, q_{sw} - F_{sw}(q^* - q^*_{sw}) + (1 - F_{sw})/\Delta t], \text{ (B.45)} \]

where \( q^* \) and \( q^*_{sw} \) are dummy values at time \( t + \Delta t \) calculated as a result of all of the other microphysical processes, such that
\[ q^* = q_e + \Delta t (d q_e/\Delta t + \text{QXSHD}), \text{ (B.46)} \]
\[ q^*_{sw} = q_{sw} + \Delta t (d q_{sw}/\Delta t + \text{QXSHD}), \text{ (B.47)} \]
and \( d q_e/\Delta t \) and \( d q_{sw}/\Delta t \) are given by (A.5)–(A.7) and (A.8)–(A.10), respectively.

The number concentrations of precipitation ice change when liquid water is shed during melting by assuming that the slopes of the particle distributions are approximately constant:
\[ \text{NSSHD} = (n_s/q_e) \max[0, \text{QSSHD} - \text{QSCW} - \text{QSCR}], \text{ (B.48)} \]
\[ \text{NGSHD} = (n_s/q_e) \max[0, \text{QGSHD} - \text{QGACW} - \text{QGACR} - \text{QGACS}], \text{ (B.49)} \]
\[ \text{NHSHD} = (n_s/q_s) \max[0, \text{QHSHD} - \text{QHACW} - \text{QHACR} - \text{QHACS}], \text{ (B.50)} \]

\[ g. \text{ Evaporation and deposition onto precipitation (QXEVP, NXEVP, QXDEP, NXDEP)} \]

Evaporation from rain and condensation (or evaporation) from melting precipitation ice \( (x = s, g, h) \) are, respectively,
\[ \text{QREVP} = 2\pi S_{sw} S_{sw} \text{VENT}_s/\lambda_{sw}, \text{ (B.51)} \]
\[ \text{QXEVP} = 2\pi S_{sw} S_{sw} \text{VENT}_s/\lambda_{sw}, \text{ (B.52)} \]
where \( \text{VENT}_s \) is given by (B.36) for rain and precipitation ice, \( S_{sw} = q_s/q_{sw} - 1 \) is the subsaturation ratio with respect to water, and
\[ \text{AB}_w = \frac{L_i^2}{K_e R_T^2} + \frac{1}{p q_{sw}}, \text{ (B.53)} \]

The rates of depositional growth or decay of precipitation ice are
\[ \text{QXDEP} = 2\pi S_{sw} S_{sw} \text{VENT}_s/\lambda_{sw}/\lambda_{sw}, \text{ (B.54)} \]
where \( S_{sw} \) is the subsaturation ratio with respect to ice, and
\[ \text{AB}_r = \frac{L_i^2}{K_e R_T^2} + \frac{1}{p q_{sr}}, \text{ (B.53)} \]

The rates in which the number concentrations of precipitation ice are reduced by sublimation (QXDEP...
\( \text{VF}_{i}(D_i) = \begin{cases} 
1 + 0.14 \chi^2_i, & \chi_i < 1.0 \\
0.86 + 0.28 \chi_i, & \chi_i \geq 1.0 
\end{cases} \) (B.62)

is the ventilation coefficients of ice crystals taken from Hall and Pruppacher (1976) with \( \chi_i(D_i) = \frac{Sc^{1/3}}{Re^{1/2}} \) and \( Re = V/D_i/\nu \). Substituting (B.62) into (B.61) yields

\[
\text{VENT}_i = \frac{\Gamma(2 + \alpha_i)}{\lambda^{2+\alpha_i}} \times [0.86 + 0.14 \gamma^*(2 + \alpha_i, \lambda_i D_{eq})] \]

\[
+ 0.14 \varphi_i \frac{\Gamma(3 + \alpha_i + b_i)}{\lambda^{4+\alpha_i+b_i}} \gamma^*(3 + \alpha_i + b_i, \lambda_i D_{eq}) \]

\[
+ 0.28 \varphi_i \frac{\Gamma(2.5 + \alpha_i + 0.5b_i)}{\lambda^{2.5+\alpha_i+0.5b_i}} \gamma^*(2.5 + \alpha_i + 0.5b_i, \lambda_i D_{eq})
\] (B.63)

where \( D_{eq} \) is defined as the size of an ice crystal such that \( \chi_i(D_{eq}) = 1 \), and \( \varphi_i = (\gamma a_i)^{1/2} (\nu^{1/2} \psi)^{-1/3} \). Reductions in the number concentrations of small ice crystals by sublimation (NIDEF < 0) are calculated in a similar manner as for precipitation ice in (B.55) by assuming \( \lambda \) is approximately constant during sublimation.

\( \text{QCND} = \max \left\{ \frac{q_v - q_{w*}}{\Delta t}, \frac{q_v - q_{w*}}{4098.026 L_{eq w}} \right\} \) (B.59)

where \( q_v \) and \( q_{w*} \) are the dummy values of the actual water vapor mixing ratio and the water vapor mixing at water saturation, respectively, calculated at time \( t + \Delta t \) as a result of advection and diffusion. The saturation water vapor mixing ratio is calculated using Teten’s formula. This method does not allow model grid points to be supersaturated with respect to water.

Deposition onto small ice crystals is calculated using an approach similar to (B.54),

\[
\text{QIDEF} = 2\pi SS_{n_v} \text{VENT}_i / \text{AB}. \] (B.60)

However, the expression representing the ventilation effects of small ice crystals (VENT) is more complicated than for precipitation (VENT, is given by Eq. (B.36)). The ventilation effect of small ice integrated over all particle sizes is

\[
\text{VENT}_i = \int_{0}^{\infty} \text{VF}_{i}(D_i) D_i^{2+\alpha_i} e^{-\lambda D_i} dD_i, \] (B.61)

where

\[
\text{NIHM} = \pi/4 \gamma a_i n_m HM_r HM_w \times \int_{0}^{\infty} E_{w} \text{HM}_w D_{eq}^{2+\alpha_i+b_i} e^{-\lambda D_{eq}} dD_{eq}. \] (B.64)

where the quantitative effects of temperature, droplet sizes, and rime fall velocities upon ice splinter production are represented by the functional relationships HM_r, HM_w, and HM_w, respectively. It is assumed in (B.64) that \( f_x = 0 \) for precipitation ice.

For temperatures between \(-2^\circ \) and \(-8^\circ \), the dependence of temperature upon crystal production is

\[
\text{HM}_r = \begin{cases} 
0.5, & -2^\circ < T_c < -4^\circ \ C \\
1.0, & -4^\circ < T_c < -6^\circ \ C \\
0.5, & -6^\circ < T_c < -8^\circ \ C \\
0, & \text{otherwise.} 
\end{cases} \] (B.65)

This functional relationship using air temperature
rather than the surface temperature of the particle, as suggested by Heymsfield and Mossop (1984), is based upon an approximate representation of Fig. 3 in Mossop (1985), as well as taking into account the limited vertical resolution with respect to temperature in all cloud models.

The number of splinters was found by Mossop to vary as a function of the number of large drops rimed onto the falling ice particle, such that

\[ \text{HM}_{\text{w}} = n_{\text{w}} \left[ 10^{-2} + 1.5 \cdot 10^{-3} \log \left( n_{\text{w}} / n_{\text{w}}^* \right) \right], \tag{B.66} \]

in which \( n_{\text{w}}^* \) and \( n_{\text{w}} \) are the number concentrations of small (\( \leq 12 \mu m \)) and large (\( \geq 25 \mu m \)) droplets, respectively, that are rimed onto the precipitation ice particle. Since Mossop (1985) found that

\[ n_{\text{w}} / n_{\text{w}}^* \sim 1/3 \left( n_{\text{w}} / n_{\text{w}}^* \right), \tag{B.67} \]

where \( n_{\text{w}} \) and \( n_{\text{w}}^* \) are the number concentrations of small (\( \leq 12 \mu m \)) and large (\( \geq 25 \mu m \)) droplets, respectively, then substituting (B.67) into (B.66) and assuming that \( n_{\text{w}}^* \sim n_{\text{w}} \) yields

\[ \text{HM}_{\text{w}} = n_{\text{w}} \left[ 9.3 \cdot 10^{-3} + 1.5 \cdot 10^{-3} \log \left( n_{\text{w}} / n_{\text{w}}^* \right) \right]. \tag{B.68} \]

Expressions for \( n_{\text{w}}^* \) and \( n_{\text{w}} \) are then obtained by integrating (3.1) over the appropriate range of droplet sizes:

\[ n_{\text{w}} = n_{\text{w}} \left\{ 1 - \exp \left[ -\frac{\pi \rho_{\text{w}} n_{\text{w}} \left( 12 \times 10^{-4} \right)^3}{6 \rho_{\text{w}}} \right] \right\}, \tag{B.69} \]

\[ n_{\text{w}} = n_{\text{w}} \exp \left[ -\frac{\pi \rho_{\text{w}} n_{\text{w}} \left( 25 \times 10^{-4} \right)^3}{6 \rho_{\text{w}}} \right]. \tag{B.70} \]

The effect of rime fall velocity upon crystal production is parameterized based on Fig. 4a of Mossop (1985) as

\[ \text{HM}_{\text{x}} = \begin{cases} V_x / 200, & 0 < V_x \text{ (cm s}^{-1} \text{)} < 200 \\ 1, & 200 \leq V_x \leq 400 \\ (600 - V_x) / 200, & 400 < V_x \leq 600. \end{cases} \tag{B.71} \]

Substituting (B.71) into (B.64), assuming \( E_{\text{w}} = 1 \), and rearranging terms produces

\[ \text{NIHM}_{\text{x}} = \pi / 4 \gamma a_x n_{\alpha x} \cdot \text{HM}_{\text{x}} \cdot \text{HM}_{\text{w}} \cdot \text{HMV}_{\text{x}}, \tag{B.72} \]

\[ \text{HMV}_{\text{x}} = \frac{2a_x}{200} \left[ \int_{0}^{\text{D}_{\text{hi}1}} D_{x}^{2 \left( 1 + b_x \right) + a_x} e^{-\lambda_{\text{Hi}1} D_{x}} dD_{x} - \int_{\text{D}_{\text{Hi}2}}^{\text{D}_{\text{Hi}1}} D_{x}^{2 \left( 1 + b_x \right) + a_x} e^{-\lambda_{\text{Hi}1} D_{x}} dD_{x} \right] + \int_{\text{D}_{\text{Hi}2}}^{\text{D}_{\text{Hi}2}} D_{x}^{2 + b_x + a_x} e^{-\lambda_{\text{Hi}2} D_{x}} dD_{x} + 3 \int_{\text{D}_{\text{Hi}2}}^{\text{D}_{\text{Hi}2}} D_{x}^{2 + b_x + a_x} e^{-\lambda_{\text{Hi}2} D_{x}} dD_{x}, \tag{B.73} \]

in which the range of ice particle diameters are defined by \( \gamma a_x D_{\text{hi}1} = 200 \text{ cm s}^{-1} \), \( \gamma a_x D_{\text{hi}2} = 400 \text{ cm s}^{-1} \), and \( \gamma a_x D_{\text{hi}3} = 600 \text{ cm s}^{-1} \). Integrating each of the terms in (B.73) and using the modified gamma probability function gives

\[ \text{HMV}_{\text{x}} = \Gamma(3 + 2b_x + a_x) \gamma a_x \lambda_{\text{x}}^{(3 + 2b_x + a_x) \text{HM}_{\text{w}}} / 200 + \Gamma(3 + b_x + a_x) \lambda_{\text{x}}^{(3 + b_x + a_x) \text{HM}_{\text{w}}}, \tag{B.74} \]

where

\[ \text{HM}_{\text{w1}} = \gamma^* [3 + 2b_x + a_x, \lambda_{\text{x}}(200/\gamma a_x)^{1/b_x}] + \gamma^* [3 + 2b_x + a_x, \lambda_{\text{x}}(400/\gamma a_x)^{1/b_x}] - \gamma^* [3 + 2b_x + a_x, \lambda_{\text{x}}(600/\gamma a_x)^{1/b_x}], \tag{B.75} \]

\[ \text{HM}_{\text{w2}} = 3\gamma^* [3 + 2b_x + a_x, \lambda_{\text{x}}(600/\gamma a_x)^{1/b_x}] - \gamma^* [3 + 2b_x + a_x, \lambda_{\text{x}}(200/\gamma a_x)^{1/b_x}] - 2\gamma^* [3 + 2b_x + a_x, \lambda_{\text{x}}(400/\gamma a_x)^{1/b_x}]. \tag{B.76} \]

Assuming a characteristic diameter for the ice splinters (\( D_{\text{sw}} \)) of 0.01 cm, the mass generation of small ice crystals by cloud water riming onto snow, graupel, and hail/frozen drops (\( x = s, g, h \)) is

\[ \text{QIHMX} = \rho^{-1} c_i(D_{\text{sw}})^6 \text{NIHM}_{\text{x}}, \tag{B.77} \]

where \( \text{NIHM}_{\text{x}} \) is given by substituting (B.65), (B.68) – (B.70), and (B.74) – (B.76) into (B.72).

\( k. \) Ice enhancement (\( \text{QIHR}, \text{NIHR} \))

The observations summarized in Hobbs (1990) are the basis for this parameterization, such that

\[ n_{\text{max}} (L^{-1}) = (D_i / 16 \mu m)^7 \tag{B.78} \]

is the maximum ice particle number concentration, and \( D_i \) is a threshold droplet diameter where the number concentration of all droplets larger than \( D_i \) is 3 cm\(^{-3}\). Defining the threshold droplet volume \( v_i = \pi / 6 D_i^3 \), integrating over all droplet volumes larger than \( v_i \) using (3.1), and substituting into (B.78), an expression for the maximum ice crystal number concentration is

\[ n_{\text{max}} (\text{cm}^{-3}) = 1.69 \cdot 10^{17} \left( \frac{\ln \left( n_{\text{w}} / 3 \right)}{n_{\text{w}}} \right)^{7/3}. \tag{B.79} \]

Differentiating (B.79) with respect to time and making the simple assumption that \( \text{NIHR} \) and \( n_{\text{max}} / \Delta \text{TIHR} \) with values of \( \Delta \text{TIHR} = 300 \text{ s} \), then the rate of the Hobbs–Rango ice enhancement mechanism becomes

\[ \text{NIHR} = \frac{1.69 \cdot 10^{17} \rho_{\text{w}} \ln \left( n_{\text{w}} / 3 \right)^{7/3}}{\Delta \text{TIHR}}, \tag{B.80} \]

\[ \text{QIHR} = \rho^{-1} c_i(D_{\text{Hi}1})^6 \text{NIHR}, \tag{B.81} \]

and \( D_{\text{Hi}1} \) is assumed to be 0.01 cm.
APPENDIX C

Radar Reflectivity Calculations

The total equivalent radar reflectivity factor \( Z_{er} \) (in \( \text{mm}^6 \text{ m}^{-3} \)) is calculated from the sum of the reflectivities for all hydrometeor species,

\[
Z_{er} = Z_{er}^r + Z_{er}^i + Z_{er}^a + Z_{er}^h, \tag{C.1}
\]

where the reflectivity for each hydrometeor category \((x = r, i, s, g, h)\) using (3.2) is

\[
Z_{ex} = 10^{12}n_{ex}\int_0^\infty \frac{|K|^2}{|K|^2} D^{5+\alpha_x}\exp(-\lambda_xD_x)dD_x, \tag{C.2}
\]

and \(|K|^2\) and \(|K|^2\) are \(|K|^2\) for ice species \(X\) and liquid water, respectively.

The equivalent radar reflectivity for raindrops is

\[
Z_{er} = 10^{12}\Gamma(7 + \alpha_x)n_{er}\lambda_x^{-(7+\alpha_x)}, \tag{C.3}
\]

with \(|K|^2 = |K|^2\) in (C.2), and \(n_{er}\) and \(\lambda_x\) are defined in section 3a for either exponential or gamma drop size distributions.

For dry ice particles \((x = i, s, g, h)\) the radar reflectivity factor is

\[
Z_{ex} = 10^{12}n_{sum}\int_0^\infty \frac{|K|^2}{|K|^2} D^{5+\alpha_x}\exp(-\lambda_xD_{sum})dD_{sum}, \tag{C.4}
\]

where \(D_{sum}\) is the melted diameter of the particles, \(n_{sum}\) and \(\lambda_x\) are the parameters for the melted equivalent ice distributions, and \(|K|^2 = |K|^2\) = 0.224 (Smith 1984).

Integrating (C.4) over all particle sizes yields

\[
Z_{ex} = 0.224 \times 10^{12}\Gamma(7 + \alpha_x)n_{sum}\lambda_{sum}^{-(7+\alpha_x)}. \tag{C.5}
\]

The number concentration and mass content for a melted ice particle distribution, respectively, are

\[
n_x = \Gamma(1 + \alpha_x)n_{sum}\lambda_{sum}^{-(1+\alpha_x)}, \tag{C.6}
\]

\[
Rq_x = \pi/6\rho_\ell\Gamma(4 + \alpha_x)n_{sum}\lambda_{sum}^{-(4+\alpha_x)}. \tag{C.7}
\]

Combining (C.5)–(C.7) yields a general expression for the radar reflectivity of dry ice particles as a function of their mass and number concentrations:

\[
Z_{ex} = C'_r(\rho q_x)^2/n_x, \tag{C.8}
\]

\[
C'_r = 0.224 \times 10^{12}(6/\pi\rho_\ell)\Gamma(7 + \alpha_x)
\times \Gamma(1 + \alpha_x)/\Gamma(4 + \alpha_x)^2, \tag{C.9}
\]

where \(C'_r = 1.63410^{13}\) for exponential ice distributions \((\alpha_x = 0\) for \(x = i, s, g, h)\).

For simplicity the dielectric factor of wet precipitation ice \((x = s, g, h)\) is calculated from the mass-weighted dielectric factors of water and ice using the theory of Debye (Battan 1973), such that

\[
|K|^2 = [|K|^2(q_e - q_{sw}) + |K|^2 q_{sw}]/q_e. \tag{C.10}
\]

Substituting (C.10) into (C.4) and proceeding in the same manner as in the previous paragraph, the radar backscatter from wet precipitation ice is calculated as

\[
Z_{ex} = C_x\rho^2(0.224q_e + 0.776q_{sw})q_e/n_x, \tag{C.11}
\]

\[
C_x = 10^{12}(6/\pi\rho_\ell)^2\Gamma(7 + \alpha_x)\Gamma(1 + \alpha_x)/\Gamma(4 + \alpha_x)^2, \tag{C.12}
\]

in which \(C_x = 7.295 \times 10^{13}\) for \(\alpha_x = 0\). Equation (C.11) is a general radar relationship that is used for both dry and wet ice particles, since it is equivalent to (C.8) when \(q_{sw} = 0\).

APPENDIX D

Derivation of \(R_2\) in (6.5)–(6.6)

The factor \(R_2\) conserves the \(D^6\) moment \((\text{i.e.}, \text{radar reflectivity})\) when converting from gamma \((\alpha_x \neq 0)\) to exponential drop size distributions. The number concentrations \((n_x)\), mass contents \((\rho q_x)\), and radar reflectivities \((Z_x)\) for rain distributions given by (3.3) are, respectively:

\[
n_x = \Gamma(1 + \alpha_x)n_{sum}\lambda_{sum}^{-(1+\alpha_x)}, \tag{D.1}
\]

\[
rq_x = \pi/6\rho_\ell\Gamma(4 + \alpha_x)n_{sum}\lambda_{sum}^{-(4+\alpha_x)}, \tag{D.2}
\]

\[
Z_x = 10^{12}\Gamma(7 + \alpha_x)n_{sum}\lambda_{sum}^{-(7+\alpha_x)}. \tag{D.3}
\]

Substitution of (D.1) into (D.2) and (D.3) and noting that \(\rho_\ell = 1\) yields

\[
\rho q_x = (\pi/6)[\Gamma(4 + \alpha_x)/\Gamma(1 + \alpha_x)]n_x\lambda_x^{-3}, \tag{D.4}
\]

\[
Z_x = 10^{12}[\Gamma(7 + \alpha_x)/\Gamma(1 + \alpha_x)]n_x\lambda_x^{-6}. \tag{D.5}
\]

A general expression for \(Z_x\) as a function of \(\rho q_x\) and \(n_x\) is then obtained by solving for \(\lambda_x^{-3}\) in (D.4) and incorporating the result in (D.5),

\[
Z_x = (6/\pi)^{3/2}10^{12}[\Gamma(7 + \alpha_x)/\Gamma(1 + \alpha_x)\Gamma(4 + \alpha_x)^2]
\times (\rho q_x)^2/n_x. \tag{D.6}
\]

Changing the shape parameter of the drop size distribution from \(\alpha_x\) to \(\alpha_{er}\) produces a reflectivity, mass content, and number concentration of the new raindrop spectrum of \(Z_{er}, \rho q_{er}\), and \(n_{er}\), respectively. By equating the reflectivity and mass contents of the raindrop spectra \((\text{i.e.}, Z_r = Z_{er} \text{ and } q_r = q_{er})\), then \(n_{er} = R_2n_x\), with

\[
R_2 = \frac{\Gamma(7 + \alpha_{er})\Gamma(1 + \alpha_{er})[\Gamma(4 + \alpha_{er})]^2}{\Gamma(7 + \alpha_x)\Gamma(1 + \alpha_x)\Gamma(4 + \alpha_x)^2}. \tag{D.8}
\]

For \(\alpha_x = 2.5\) and \(\alpha_{er} = 0\) (as assumed for ice) \(R_2 = 4.181\). When \(n_x\) is used instead of \(R_2n_x\) in (6.5) and (6.6), the radar reflectivity will increase artificially by 6.2 dB \((Z_{er}/Z_r = 4.181)\) as a result of conserving number concentration when \(\alpha_r = 2.5\) drop distributions are converted into \(\alpha_{er} = 0\) drop size spectra.
## List of Symbols Not Referenced in Tables 1 and 2

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a_2)</td>
<td>Constant for ice saturation mixing ratio</td>
<td>5807.7</td>
<td>K</td>
</tr>
<tr>
<td>(a_k)</td>
<td>Fall speed constant for graupel</td>
<td>351.2</td>
<td>cm(^{(1-b_g)}) s(^{-1})</td>
</tr>
<tr>
<td>(a_h)</td>
<td>Fall speed constant for frozen drops/hail</td>
<td>1094.3</td>
<td>cm(^{(1-b_h)}) s(^{-1})</td>
</tr>
<tr>
<td>(a_i)</td>
<td>Fall speed constant for ice crystals</td>
<td>1998.5</td>
<td>cm(^{(1-b_i)}) s(^{-1})</td>
</tr>
<tr>
<td>(a_r)</td>
<td>Fall speed constant for raindrops</td>
<td>4854</td>
<td>cm(^{(1-b_r)}) s(^{-1})</td>
</tr>
<tr>
<td>(a_s)</td>
<td>Fall speed constant for snow</td>
<td>1094.3</td>
<td>cm(^{(1-b_s)}) s(^{-1})</td>
</tr>
<tr>
<td>(a_x)</td>
<td>Fall speed constant for an particle species (x)</td>
<td></td>
<td>cm(^{(1-b_x)}) s(^{-1})</td>
</tr>
<tr>
<td>(a_z)</td>
<td>Fall speed constant for an particle species (z)</td>
<td></td>
<td>cm(^{(1-b_z)}) s(^{-1})</td>
</tr>
<tr>
<td>(A')</td>
<td>Exponent in Bigg freezing of cloud droplets</td>
<td>-.66</td>
<td>K(^{-1})</td>
</tr>
<tr>
<td>(A_{B_1})</td>
<td>Thermodynamic term in deposition onto ice</td>
<td></td>
<td>cm(^2) s(^{-1})</td>
</tr>
<tr>
<td>(A_{B_{w}})</td>
<td>Thermodynamic term in evaporation of rain and wet precipitation ice</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(A_x)</td>
<td>First constant for ventilation of precipitation</td>
<td>.31</td>
<td>cm(^{-3}) s(^{-1})</td>
</tr>
<tr>
<td>(b_g)</td>
<td>First fall speed exponent for graupel</td>
<td>.37</td>
<td></td>
</tr>
<tr>
<td>(b_h)</td>
<td>First fall speed exponent for frozen drops/hail</td>
<td>.6384</td>
<td></td>
</tr>
<tr>
<td>(b_i)</td>
<td>First fall speed exponent for ice crystals</td>
<td>1.0</td>
<td></td>
</tr>
<tr>
<td>(b_r)</td>
<td>First fall speed exponent for raindrops</td>
<td>1.0</td>
<td></td>
</tr>
<tr>
<td>(b_s)</td>
<td>First fall speed exponent for snow</td>
<td>1.0</td>
<td></td>
</tr>
<tr>
<td>(b_x)</td>
<td>First fall speed exponent for species (x)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(b_z)</td>
<td>First fall speed exponent for species (z)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(B')</td>
<td>Constant in Bigg freezing of cloud droplets</td>
<td>10(^{-4})</td>
<td>cm(^{-3}) s(^{-1})</td>
</tr>
<tr>
<td>(B_x)</td>
<td>Second constant for ventilation of precipitation</td>
<td>.78</td>
<td></td>
</tr>
<tr>
<td>(c_g)</td>
<td>Mass constant for graupel</td>
<td>(\pi/6\rho_g)</td>
<td>g cm(^{-3})</td>
</tr>
<tr>
<td>(c_h)</td>
<td>Mass constant for frozen drops/hail</td>
<td>(\pi/6\rho_h)</td>
<td>g cm(^{-3})</td>
</tr>
<tr>
<td>(c_i)</td>
<td>Mass constant for ice crystals</td>
<td>.044</td>
<td>g cm(^{-3})</td>
</tr>
<tr>
<td>(c_r)</td>
<td>Mass constant for raindrops</td>
<td>(\pi/6)</td>
<td>g cm(^{-3})</td>
</tr>
<tr>
<td>(c_s)</td>
<td>Mass constant for snow</td>
<td>(\pi/6\rho_s)</td>
<td>g cm(^{-3})</td>
</tr>
<tr>
<td>(c_x)</td>
<td>Mass constant for a species (x)</td>
<td></td>
<td>g cm(^{-3})</td>
</tr>
<tr>
<td>(c_w)</td>
<td>Specific heat of water</td>
<td>4187</td>
<td>J kg(^{-1}) K(^{-1})</td>
</tr>
<tr>
<td>(c_z)</td>
<td>Mass exponent for a species (z)</td>
<td></td>
<td>g cm(^{-3})</td>
</tr>
<tr>
<td>(C_p)</td>
<td>Specific heat of air at constant pressure</td>
<td>1005</td>
<td>J kg(^{-1}) K(^{-1})</td>
</tr>
<tr>
<td>(C_x)</td>
<td>Constant in radar reflectivity of wet ice species (x)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(C_x')</td>
<td>Constant in radar reflectivity of dry ice species (x)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CND</td>
<td>Change in water vapor mixing ratio by net condensation</td>
<td></td>
<td>g (^{-1})</td>
</tr>
<tr>
<td>(d_g)</td>
<td>Mass exponent for graupel</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>(d_h)</td>
<td>Mass exponent for frozen drops/hail</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>(d_i)</td>
<td>Mass exponent for ice crystals</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>(d_r)</td>
<td>Mass exponent for raindrops</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>(d_s)</td>
<td>Mass exponent for snow</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>(d_x)</td>
<td>Mass exponent for a species (x)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(d_z)</td>
<td>Mass exponent for a species (z)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(D_1)</td>
<td>Lower threshold diameter in rain collection (QXACRY, QRACXY)</td>
<td></td>
<td>cm</td>
</tr>
<tr>
<td>(D_{1gh})</td>
<td>Minimum diameter of graupel converted by riming to frozen drops/hail</td>
<td></td>
<td>cm</td>
</tr>
<tr>
<td>(D_{1kg})</td>
<td>Minimum diameter of frozen drops/hail converted by riming to graupel</td>
<td></td>
<td>cm</td>
</tr>
<tr>
<td>(D_{1lg})</td>
<td>Minimum diameter of snow converted by riming to graupel</td>
<td></td>
<td>cm</td>
</tr>
<tr>
<td>(D_{1xy})</td>
<td>Minimum diameter of species (x) converted to species (y) by riming</td>
<td></td>
<td>cm</td>
</tr>
<tr>
<td>(D_2)</td>
<td>Larger threshold diameter in rain collection (QXACRY, QRACXY)</td>
<td></td>
<td>cm</td>
</tr>
<tr>
<td>(D_{2xy})</td>
<td>Maximum diameter of species (x) converted to species (y) by riming</td>
<td></td>
<td>cm</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
<td>Value</td>
<td>Units</td>
</tr>
<tr>
<td>----------</td>
<td>------------------------------------------------------------------------------</td>
<td>-------</td>
<td>-------</td>
</tr>
<tr>
<td>$D_{by}$</td>
<td>Final diameter of species $x$ that is converted by riming to species $y$</td>
<td></td>
<td>cm</td>
</tr>
<tr>
<td>$D_g$</td>
<td>Graupel particle diameter</td>
<td></td>
<td>cm</td>
</tr>
<tr>
<td>$D_h$</td>
<td>Frozen drop/hail particle diameter</td>
<td></td>
<td>cm</td>
</tr>
<tr>
<td>$D_l$</td>
<td>Ice crystal diameter</td>
<td></td>
<td>cm</td>
</tr>
<tr>
<td>$D_{df}$</td>
<td>Ice crystal diameter where $x_1 = 1$</td>
<td></td>
<td>cm</td>
</tr>
<tr>
<td>$D_{IHR}$</td>
<td>Diameter of ice crystals initiated by ice enhancement</td>
<td>.01</td>
<td>cm</td>
</tr>
<tr>
<td>$D_{lm}$</td>
<td>Diameter of ice splinters</td>
<td>.01</td>
<td>cm</td>
</tr>
<tr>
<td>$D_{imax}$</td>
<td>Maximum size of ice crystals, converted to snow</td>
<td>.05</td>
<td>cm</td>
</tr>
<tr>
<td>$D_{0r}$</td>
<td>Median raindrop diameter</td>
<td></td>
<td>cm</td>
</tr>
<tr>
<td>$D_r$</td>
<td>Raindrop diameter</td>
<td></td>
<td>cm</td>
</tr>
<tr>
<td>$D_{ni}$</td>
<td>Minimum diameter of drops collecting cloud ice</td>
<td>40</td>
<td>$\mu$m</td>
</tr>
<tr>
<td>$D_s$</td>
<td>Snow particle diameter</td>
<td></td>
<td>cm</td>
</tr>
<tr>
<td>$D_{si}$</td>
<td>Minimum diameter of snow collecting cloud ice</td>
<td>40</td>
<td>$\mu$m</td>
</tr>
<tr>
<td>$D_t$</td>
<td>Threshold cloud droplet diameter for ice enhancement</td>
<td></td>
<td>cm</td>
</tr>
<tr>
<td>$D_w$</td>
<td>Cloud droplet diameter</td>
<td></td>
<td>cm</td>
</tr>
<tr>
<td>$D_{wa}$</td>
<td>Mean cloud droplet diameter for autoconversion</td>
<td>20</td>
<td>$\mu$m</td>
</tr>
<tr>
<td>$D_{wmin}$</td>
<td>Minimum mean cloud droplet diameter</td>
<td>5</td>
<td>$\mu$m</td>
</tr>
<tr>
<td>$D_x$</td>
<td>Diameter of particle species $x$</td>
<td></td>
<td>cm</td>
</tr>
<tr>
<td>$D_{x1}$, $D_{x2}$</td>
<td>Diameters of colliding ice crystals and snowflakes of different sizes in aggregation process</td>
<td></td>
<td>cm</td>
</tr>
<tr>
<td>$D_{st}$</td>
<td>Minimum diameter that graupel and frozen drops collect cloud ice</td>
<td>40</td>
<td>$\mu$m</td>
</tr>
<tr>
<td>$D_{xm}$</td>
<td>Equivalent melted diameter for ice species $x$</td>
<td></td>
<td>cm</td>
</tr>
<tr>
<td>$D_{sw}$</td>
<td>Minimum diameter that an ice particle of species $x$ becomes wet</td>
<td></td>
<td>cm</td>
</tr>
<tr>
<td>$D_z$</td>
<td>Diameter of particle species $z$</td>
<td></td>
<td>cm</td>
</tr>
<tr>
<td>DEP</td>
<td>Change in water vapor mixing ratio due to net deposition</td>
<td></td>
<td>g $g^{-1}$</td>
</tr>
<tr>
<td>$E_{ri}$</td>
<td>Efficiency of rain collecting ice crystals</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E_{st1}$</td>
<td>Constant for efficiency of dry snow collecting ice crystals</td>
<td>.25</td>
<td></td>
</tr>
<tr>
<td>$E_{st2}$</td>
<td>Exponent for efficiency of dry snow collecting ice crystals</td>
<td>.05</td>
<td>$K^{-1}$</td>
</tr>
<tr>
<td>$E_{xi}$</td>
<td>Efficiency of species $x$ collecting ice crystals</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E_{xr}$</td>
<td>Efficiency of species $x$ collecting rain</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$(E_{xs})_{dry}$</td>
<td>Efficiency of snow collected by dry graupel, frozen drops</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>$(E_{xs})_{wet}$</td>
<td>Efficiency of snow collected by wet graupel, frozen drops</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>$E_{xw}$</td>
<td>Efficiency of species $x$ collecting cloud droplets</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E_{xx}$</td>
<td>Aggregation efficiencies of ice crystals and snow</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E_{x1}$</td>
<td>Constant for efficiency of dry graupel and frozen drops collecting ice crystals and snow</td>
<td>.1</td>
<td></td>
</tr>
<tr>
<td>$E_{x2}$</td>
<td>Exponent for efficiency of dry graupel and frozen drops collecting ice crystals and snow</td>
<td>.1</td>
<td>$K^{-1}$</td>
</tr>
<tr>
<td>$(E_{xw})_{wet}$</td>
<td>Efficiency of wet graupel and frozen drops collecting ice crystals and snow</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>$E_{xz}$</td>
<td>Efficiency of species $x$ collecting species $z$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f_x$</td>
<td>Second fall speed exponent for graupel</td>
<td>0</td>
<td>cm$^{-1}$</td>
</tr>
<tr>
<td>$f_h$</td>
<td>Second fall speed exponent for frozen drops/hail</td>
<td>0</td>
<td>cm$^{-1}$</td>
</tr>
<tr>
<td>$f_i$</td>
<td>Second fall speed exponent for ice crystals</td>
<td>0</td>
<td>cm$^{-1}$</td>
</tr>
<tr>
<td>$f_r$</td>
<td>Second fall speed exponent for raindrops</td>
<td>1.95</td>
<td>cm$^{-1}$</td>
</tr>
<tr>
<td>$f_s$</td>
<td>Second fall speed exponent for snow</td>
<td>0</td>
<td>cm$^{-1}$</td>
</tr>
<tr>
<td>$f_z$</td>
<td>Second fall speed exponent for species $x$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$F$</td>
<td>Notation for Gauss' hypergeometric function</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$F_{gw}$</td>
<td>Liquid water mass fraction on graupel</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$F_{hw}$</td>
<td>Liquid water mass fraction on frozen drops/hail</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$F_{sw}$</td>
<td>Liquid water mass fraction on snow</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### APPENDIX E—Continued

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_{sw}$</td>
<td>Liquid water mass fraction on precipitation ice species $x$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$F_{wm}$</td>
<td>Maximum mass fraction of liquid water on wet precipitation ice species $x$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$FN_{xt}$</td>
<td>Fractional collection of snow number concentration by wet ice species $x$</td>
<td>.5</td>
<td></td>
</tr>
<tr>
<td>$FQ_{sw}$</td>
<td>Fractional collection of snow mass by wet graupel</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$FQ_{sh}$</td>
<td>Fractional collection of snow mass by wet frozen drops</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$FQ_{sx}$</td>
<td>Fractional collection of snow mass by wet ice species $x$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$HM_{T}$</td>
<td>Temperature-dependent factor for Hallet-Mossop rime splintering</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$HM_{w}$</td>
<td>Dependence of cloud droplet spectra upon rime splintering</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$HM_{x}$</td>
<td>Rimer fall speed dependence upon rime splintering</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$HM_{x1}$</td>
<td>First factor used to calculate $HM_{w}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$HM_{x2}$</td>
<td>Second factor used to calculate $HM_{w}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$HM_{w}$</td>
<td>Rimer fall speed effects upon rime splintering integrated over particle spectra (species $x$)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$K_{a}$</td>
<td>Thermal conductivity of air</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$</td>
<td>K</td>
<td>_{1}$</td>
<td>Dielectric factor for pure ice</td>
</tr>
<tr>
<td>$</td>
<td>K</td>
<td>_{2}$</td>
<td>Dielectric factor for water</td>
</tr>
<tr>
<td>$</td>
<td>K</td>
<td>_{x}$</td>
<td>Dielectric factor for ice species $x$</td>
</tr>
<tr>
<td>$L_{f}$</td>
<td>Latent heat of fusion</td>
<td>$3.336 \times 10^{5}$</td>
<td>J kg$^{-1}$</td>
</tr>
<tr>
<td>$L_{s}$</td>
<td>Latent heat of sublimation</td>
<td>$2.833 \times 10^{6}$</td>
<td>J kg$^{-1}$</td>
</tr>
<tr>
<td>$L_{v}$</td>
<td>Latent heat of vaporization</td>
<td>$2.5 \times 10^{6}$</td>
<td>J kg$^{-1}$</td>
</tr>
<tr>
<td>$m_{0}$</td>
<td>Initial mass of a nucleated ice crystal</td>
<td>$6.88 \times 10^{-10}$</td>
<td>g</td>
</tr>
<tr>
<td>$m_{r}$</td>
<td>Mass of raindrop of diameter $D_{r}$</td>
<td></td>
<td>g</td>
</tr>
<tr>
<td>$m_{s}$</td>
<td>Mass of snow of diameter $D_{s}$</td>
<td></td>
<td>g</td>
</tr>
<tr>
<td>$m_{x}$</td>
<td>Mass of species $x$ particle of diameter $D_{x}$</td>
<td></td>
<td>g</td>
</tr>
<tr>
<td>$n_{g}$</td>
<td>Graupel number concentration</td>
<td></td>
<td>cm$^{-3}$</td>
</tr>
<tr>
<td>$n_{h}$</td>
<td>Frozen drop/hail number concentration</td>
<td></td>
<td>cm$^{-3}$</td>
</tr>
<tr>
<td>$n_{i}$</td>
<td>Ice crystal number concentration</td>
<td></td>
<td>cm$^{-3}$</td>
</tr>
<tr>
<td>$n_{i_{max}}$</td>
<td>Max ice crystal number concentration for ice enhancement</td>
<td></td>
<td>cm$^{-3}$</td>
</tr>
<tr>
<td>$n_{in}$</td>
<td>Number concentration of nucleated ice crystals</td>
<td></td>
<td>cm$^{-3}$</td>
</tr>
<tr>
<td>$n_{in1}$</td>
<td>Number concentration of nucleated ice crystals at $T_{c} \approx -5^\circ$C</td>
<td></td>
<td>cm$^{-3}$</td>
</tr>
<tr>
<td>$n_{in2}$</td>
<td>Number concentration of nucleated ice crystals at $T_{c} &lt; -5^\circ$C</td>
<td></td>
<td>cm$^{-3}$</td>
</tr>
<tr>
<td>$n_{io1}$</td>
<td>Constant in ice crystal nucleation at $T_{c} \approx -5^\circ$C</td>
<td></td>
<td>cm$^{-3}$</td>
</tr>
<tr>
<td>$n_{io2}$</td>
<td>Constant in ice crystal nucleation at $T_{c} &lt; -5^\circ$C</td>
<td></td>
<td>cm$^{-3}$</td>
</tr>
<tr>
<td>$n_{og}$</td>
<td>Intercept of graupel size distribution</td>
<td></td>
<td>cm$^{-(0.5-0.6)}$</td>
</tr>
<tr>
<td>$n_{oh}$</td>
<td>Intercept of frozen drop/hail size distribution</td>
<td></td>
<td>cm$^{-(0.5-0.6)}$</td>
</tr>
<tr>
<td>$n_{oi}$</td>
<td>Intercept of ice crystal size distribution</td>
<td></td>
<td>cm$^{-(0.5-0.6)}$</td>
</tr>
<tr>
<td>$n_{or}$</td>
<td>Intercept of rain size distribution</td>
<td></td>
<td>cm$^{-(0.5-0.6)}$</td>
</tr>
<tr>
<td>$n_{os}$</td>
<td>Intercept of snow size distribution</td>
<td></td>
<td>cm$^{-(0.5-0.6)}$</td>
</tr>
<tr>
<td>$n_{ox}$</td>
<td>Intercept of size distribution for species $x$</td>
<td></td>
<td>cm$^{-(0.5-0.6)}$</td>
</tr>
<tr>
<td>$n_{oxm}$</td>
<td>Intercept for melted ice distribution for species $x$</td>
<td></td>
<td>cm$^{-(0.5-0.6)}$</td>
</tr>
<tr>
<td>$n_{oz}$</td>
<td>Intercept of size distribution for species $z$</td>
<td></td>
<td>cm$^{-(0.5-0.6)}$</td>
</tr>
<tr>
<td>$n_{ri}$</td>
<td>Number of ice crystals collected by drop of diameter $D_{r}$</td>
<td></td>
<td>cm$^{-3}$</td>
</tr>
<tr>
<td>$n_{rx}$</td>
<td>Number concentration of a redistributed raindrop spectrum</td>
<td></td>
<td>cm$^{-3}$</td>
</tr>
<tr>
<td>$n_{s}$</td>
<td>Snow number concentration</td>
<td></td>
<td>cm$^{-3}$</td>
</tr>
<tr>
<td>$n_{w}$</td>
<td>Cloud droplet number concentration</td>
<td></td>
<td>cm$^{-3}$</td>
</tr>
<tr>
<td>$n_{wl}$</td>
<td>Number concentration of large droplets ($\geq 25 \mu$m)</td>
<td></td>
<td>cm$^{-3}$</td>
</tr>
<tr>
<td>$n_{w}$</td>
<td>Number concentration of large droplets rimed onto ice</td>
<td></td>
<td>cm$^{-3}$</td>
</tr>
<tr>
<td>$n_{ws}$</td>
<td>Number concentration of small droplets ($\leq 12 \mu$m)</td>
<td></td>
<td>cm$^{-3}$</td>
</tr>
<tr>
<td>$n_{ws}$</td>
<td>Number concentration of small droplets rimed onto ice</td>
<td></td>
<td>cm$^{-3}$</td>
</tr>
<tr>
<td>$n_{x}$</td>
<td>Number concentration of species $x$</td>
<td></td>
<td>cm$^{-3}$</td>
</tr>
</tbody>
</table>
### APPENDIX E—Continued

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta T_{\text{mit}}$</td>
<td>Air temperature after calculating latent heating at $T_e \geq 0^\circ$C; prevent unrealistic melting rates</td>
<td></td>
<td>K</td>
</tr>
<tr>
<td>$\Delta V_n$</td>
<td>Scaled fall speed difference in number concentration collection kernel, function only of $\lambda_1$ and $\lambda_2$</td>
<td></td>
<td>cm s$^{-1}$</td>
</tr>
<tr>
<td>$\Delta V_q$</td>
<td>Scaled fall speed difference in mass collection kernel, function only of $\lambda_1$ and $\lambda_2$</td>
<td></td>
<td>cm s$^{-1}$</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Dispersion of drop distribution</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Air resistance effects on particle fall speed, $(\rho_0/\rho)^{0.5}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma^*(x, y)$</td>
<td>Incomplete gamma function</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Gamma$</td>
<td>Gamma function</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\kappa$</td>
<td>Exponent in Exner function</td>
<td>0.286</td>
<td>cm$^{-1}$</td>
</tr>
<tr>
<td>$\lambda_g$</td>
<td>Slope of graupel size distribution</td>
<td></td>
<td>cm$^{-1}$</td>
</tr>
<tr>
<td>$\lambda_h$</td>
<td>Slope of frozen drop/hail size distribution</td>
<td></td>
<td>cm$^{-1}$</td>
</tr>
<tr>
<td>$\lambda_i$</td>
<td>Slope of ice crystal size distribution</td>
<td></td>
<td>cm$^{-1}$</td>
</tr>
<tr>
<td>$\lambda_0$</td>
<td>Minimum slope of ice crystals, converted to snow</td>
<td>50</td>
<td>cm$^{-1}$</td>
</tr>
<tr>
<td>$\lambda_r$</td>
<td>Slope of rain size distribution</td>
<td></td>
<td>cm$^{-1}$</td>
</tr>
<tr>
<td>$\lambda_s$</td>
<td>Slope of snow size distribution</td>
<td></td>
<td>cm$^{-1}$</td>
</tr>
<tr>
<td>$\lambda_{00}$</td>
<td>Minimum slope of snow size distribution</td>
<td>10</td>
<td>cm$^{-1}$</td>
</tr>
<tr>
<td>$\lambda_x$</td>
<td>Slope of size distribution for species $x$</td>
<td></td>
<td>cm$^{-1}$</td>
</tr>
<tr>
<td>$\lambda_{ii}$</td>
<td>Slope of size distribution for dry ice species $x$</td>
<td></td>
<td>cm$^{-1}$</td>
</tr>
<tr>
<td>$\lambda_{mm}$</td>
<td>Slope of melted ice distribution for species $x$</td>
<td></td>
<td>cm$^{-1}$</td>
</tr>
<tr>
<td>$\lambda_z$</td>
<td>Slope of size distribution for species $z$</td>
<td></td>
<td>cm$^{-1}$</td>
</tr>
<tr>
<td>$\Lambda_n$</td>
<td>Scale parameter in number concentration collection kernel</td>
<td></td>
<td>cm$^{-1}$</td>
</tr>
<tr>
<td>$\Lambda_q$</td>
<td>Scaled parameter in mass collection kernel</td>
<td></td>
<td>cm$^{-1}$</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Kinematic viscosity</td>
<td></td>
<td>cm$^2$ s$^{-1}$</td>
</tr>
<tr>
<td>$\Pi$</td>
<td>Exner function, $(\rho_0/\rho)^*$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\theta$</td>
<td>Potential temperature</td>
<td></td>
<td>K</td>
</tr>
<tr>
<td>$\rho_{\text{a}}$</td>
<td>Air density</td>
<td></td>
<td>g cm$^{-3}$</td>
</tr>
<tr>
<td>$\rho_g$</td>
<td>Density of graupel</td>
<td>0.4</td>
<td>g cm$^{-3}$</td>
</tr>
<tr>
<td>$\rho_s$</td>
<td>Density of dry graupel</td>
<td>0.9</td>
<td>g cm$^{-3}$</td>
</tr>
<tr>
<td>$\rho_{\text{dl}}$</td>
<td>Density of dry frozen drops/hail</td>
<td>1.0</td>
<td>g cm$^{-3}$</td>
</tr>
<tr>
<td>$\rho_{\text{dih}}$</td>
<td>Density of dry frozen drops/hail</td>
<td>1.0</td>
<td>g cm$^{-3}$</td>
</tr>
<tr>
<td>$\rho_{\text{ls}}$</td>
<td>Density of liquid water</td>
<td></td>
<td>g cm$^{-3}$</td>
</tr>
<tr>
<td>$\rho_0$</td>
<td>Surface air density</td>
<td></td>
<td>g cm$^{-3}$</td>
</tr>
<tr>
<td>$\rho_s$</td>
<td>Density of snow</td>
<td></td>
<td>g cm$^{-3}$</td>
</tr>
<tr>
<td>$\rho_{\text{si}}$</td>
<td>Density of dry snow</td>
<td></td>
<td>g cm$^{-3}$</td>
</tr>
<tr>
<td>$\rho_{\text{r}}$</td>
<td>Density of (wet or dry) ice species $x$</td>
<td></td>
<td>g cm$^{-3}$</td>
</tr>
<tr>
<td>$\rho_{\text{ri}}$</td>
<td>Density of a dry ice species $x$</td>
<td></td>
<td>g cm$^{-3}$</td>
</tr>
<tr>
<td>$\rho_{\text{yi}}$</td>
<td>Density of ice drop mixture due to collisions between raindrops and precipitation ice</td>
<td></td>
<td>g cm$^{-3}$</td>
</tr>
<tr>
<td>$\rho_{\text{z}}$</td>
<td>Density of (wet or dry) ice species $z$</td>
<td></td>
<td>g cm$^{-3}$</td>
</tr>
<tr>
<td>$\tau_{\text{xy}}$</td>
<td>Factor used to calculate $D_{\text{xy}}$</td>
<td></td>
<td>g$^{-1}$ cm$^{1-\alpha}\text{ s}^{-1}$</td>
</tr>
<tr>
<td>$\psi$</td>
<td>Diffusivity of water vapor in air</td>
<td></td>
<td>cm$^2$ s$^{-1}$</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>Constant used to adjust deposition rates</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### REFERENCES


