An Approximate Analytical Model of the Vapor Deposition and Aggregation Growth of Snowflakes

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ABSTRACT

It is well known that snowflakes tend to distribute exponentially with respect to their melted diameter. This fact is used to formulate an approximate analytical model of the deposition and aggregation growth of snow in stratiform clouds. The model predicts the height evolution in a steady-state, vertically heterogeneous cloud of the slope and intercept parameters, $N(h)$ and $\lambda(h)$, of the size distribution of snowflakes which is assumed to be given by $n(D,h)=N(h)\exp[-\lambda(h)D]$, where h is the height in the cloud and D the snowflake diameter. Solutions for $N(t)$ and $\lambda(t)$ for a time-dependent spatially homogeneous cloud are also presented. Results from this technique compare well with numerical integrations for the case of perfect geometric coalescence of raindrops. This stratiform snow model predicts the existence of radar reflectivity–snowfall rate relations although, for this first-order model, there is fair agreement between theoretical and observed values. The model suggests that "equilibrium" snow size spectra owe their existence to the counteracting effects of deposition and aggregation growth.

1. Introduction

The measurement of precipitation size distributions in clouds by means of radar and aircraft sensing techniques has become almost routine in the past decade. However, the interpretation of these measurements in terms of particle growth processes has lagged behind the recent technological advances. The basic physics of precipitation growth are fairly well known, but the resulting equations are complex and usually solved by numerical integration. This procedure is expensive and the results of model calculations are often as difficult to interpret as is Nature herself. Thus it is advantageous to develop simple analytical models whenever possible.

As part of the University of Chicago's study of winter storm microphysics, snow size spectra are measured at various levels in Midwest extratropical cyclones. These storms are characterized by weak updrafts and supercooled liquid water contents which are typically at or below our level of detection. Snowflake growth is due primarily to vapor deposition and crystal aggregation. What is desired is a technique by which the rate of snowflake growth via these two mechanisms can be deduced from measurements of the precipitation size spectra in locally steady precipitation. This paper describes a simple analytical model for approximating the vapor deposition and aggregation growth of snow in stratiform precipitation.

Past measurements of snow size spectra show that they are almost always exponential in form such that $n(D_m)=N_m e^{-\lambda_m D_m}$, where $n(D_m)dD_m$ is the concentration of particles of equivalent melted diameter D_m in the size interval D_m to D_m+dD_m and N_m and λ_m are the distributional parameters (e.g., Gunn and Marshall, 1958). Numerical integrations of the collection show that when initially exponential particle size distributions evolve by gravitational, geometric collection, they tend to retain their exponential form and that initially narrow spectra obtain an exponential tail (e.g., Srivastava, 1967).

These findings suggest a parameterization of the deposition and aggregation growth of snow which is based on the exponential form of snow size spectra. By assuming at the outset that the size distribution is exponential, the distributional parameters can be found as functions of time or height by solving moment conservation equations which are derived from the stochastic collection equation. For a general discussion of the collection equation and its moments see Drake (1972).

The technique of constraining the size distribution function and solving moment equations for the collection process is not new. The first application for gravitational geometric collection was probably due to Schumann (1940), although it is not readily apparent that this is what he intended. He used the asymptotic solution to the collection equation for a constant kernel as his solution form $[n(D)\sim e^{-D^3}]$ and the total concentration and mass moments to

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estimate the effects of coalescence on fog droplet distributions. Enukashvili (1964a) utilized the method of moments to derive an approximate form for the size distribution function which he then used to investigate cloud droplet coalescence in vertically heterogeneous clouds (Enukashvili, 1964b). Both of these authors used geometric kernels employing Stokes' fall velocities since their studies were concerned with cloud droplet coalescence. They did not include the effects of condensation in their models. Srivastava (1978) has formulated a parameterization of raindrop coalescence, mass deposition and drop breakup based on the assumption that the drop size distribution is exponential with respect to the drop diameter. His resultant differential equations are based on the moment equations for the total mass and drop concentration and must be solved numerically.

The approximate analytical technique presented here was developed to model the evolution of snow size spectra in stratiform clouds. The size distribution of snowflakes is assumed to be exponential such that $n(D)=Ne^{-\lambda D}$, where D is the snowflake diameter. Snowflake growth by vapor deposition and aggregation is considered. A gravitational, geometric collection kernel is used in which the fallspeeds are specified by an arbitrary power law relation. For the case of steady stratiform precipitation in a vertically heterogeneous cloud, a system of two, nonlinear, first-order, ordinary differential equations in N and λ are derived from the moment conservation equations for the total mass and radar reflectivity. Solution of these equations yields expressions for N and λ as functions of height below a reference level. Solutions for N and λ as functions of time are also presented for the case of spatially homogeneous clouds and these are compared with results of numerical integrations of the collection equation for the special case of no condensation growth. The stratiform snow model is then used to derive an "equilibrium" relationship between the distributional parameters which is analogous to empirically derived precipitation rate–radar reflectivity relationships.

2. Theory

a. *The moment conservation equations*

The governing equation for ice crystals growing by vapor deposition and aggregation in a horizontally homogeneous cloud may be written

$$\frac{\partial}{\partial t}n(x,t,h)=-\frac{\partial}{\partial x}[\dot{x}n(x,t,h)]-\frac{\partial}{\partial h}\{[w-v_T(x)]n(x,t,h)\}$$
$$+\frac{1}{2}\int_0^x n(x-y,\,t,\,h)n(y,t,h)k(x-y,\,y)dy$$
$$-n(x,t,h)\int_0^\infty n(y,t,h)k(x,y)dy, \quad (1)$$

where $n(x,t,h)$ is the number density of particles of mass x at time t and height h, w is the vertical air velocity and $v_T(x)$ the particle fallspeed. The collection kernel for particles of mass x collecting those of mass y is $k(x,y)$ and $\dot{x}$ is the time rate of change of mass of a particle of mass x due to deposition growth. The terms on the right side of (1) represent the time rate of change of the number density due to deposition growth, advection in a vertically heterogeneous cloud, the creation of particles of mass x due to aggregation of particles of masses $x-y$ and y, and the destruction of particles of mass x due to their aggregation with any other particle.

The rate of change of the mth moment of the size distribution can be found by multiplying both sides of (1) by $x^m dx$ and integrating over all x. The deposition term can be integrated by parts very simply if it is assumed that

$$\lim_{x\to 0} x^m\dot{x}n(x,t,h)=\lim_{x\to\infty} x^m\dot{x}n(x,t,h)=0. \quad (2)$$

The moment conservation equations for the total mass ($m=1$) and the total "radar reflectivity factor" ($m=2$) are given by

$$\frac{\partial \chi}{\partial t}+\frac{\partial}{\partial h}(w\chi-\chi_f)=\int_0^\infty \dot{x}n(x,t,h)dx, \quad (3)$$

$$\frac{\partial Z}{\partial t}+\frac{\partial}{\partial h}(wZ-Z_f)=2\int_0^\infty \dot{x}xn(x,t,h)dx$$
$$+\int_0^\infty\int_0^\infty xyn(x,t,h)n(y,t,h)k(x,y)dxdy, \quad (4)$$

where

$$\chi=\int_0^\infty xn(x,t,h)dx, \quad (5)$$

$$\chi_f=\int_0^\infty v_T(x)xn(x,t,h)dx, \quad (6)$$

$$Z=\int_0^\infty x^2n(x,t,h)dx, \quad (7)$$

$$Z_f=\int_0^\infty v_T(x)x^2n(x,t,h)dx. \quad (8)$$

Here χ, Z, χ_f and Z_f are, respectively, the total mass and what shall be called the radar reflectivity (although this quantity is really proportional to the radar reflectivity factor for Rayleigh scatterers) and the downward fluxes of these quantities due to the particle fallspeeds.

The primary reason for not using the zeroth moment (the total concentration) in the formulation of the problem is that the exponential distribution may not be a good approximation to the distribution of small particles. Typically, the total concentration is strongly weighted by these small particles. The reported observations of exponential snow size spectra have been made with a view toward predicting the behavior of higher order moments such as the snowfall rate and radar reflectivity. Thus these moments were selected to formulate the problem of predicting the behavior of exponential distributions.

b. Simplifying assumptions

It is desirable to express the moment conservation equations in terms of the snowflake diameter rather than the mass. It is well known that ice crystals occur in a variety of crystal habits and degrees of elaboration depending on the environmental conditions. However, in view of the current state of knowledge about crystal aggregates, it seems acceptable to assume spherical symmetry such that

$$x=\pi\rho_i D^3/6, \tag{9}$$

where D is the actual snowflake diameter and ρ_i the bulk density of the aggregate.

The rate of particle growth by vapor deposition can be expressed as

$$\dot{x}=f(t,h)D^\delta, \tag{10}$$

where classically $\delta=1$ and $f(t,h)$ depends on the crystal type and environmental conditions. The collection kernel, in terms of snowflake diameters, is assumed to be gravitational and geometric, i.e.,

$$k(x,y)=(\pi/4)(D_1+D_2)^2\bar{E}a|D_1^b-D_2^b|, \tag{11}$$

where D_1 and D_2 are the snowflake diameters corresponding to the masses x and y, $\bar{E}$ is a mean collection efficiency assumed independent of particle size, and where the particle fallspeeds are given by an arbitrary power of law of the form

$$v_T=aD^b, \tag{12}$$

where a and b depend on the particle type.

Based on the observations and numerical work which were cited in the Introduction, it is assumed that the size distribution is given by

$$n(x,t,h)=N(t,h)\exp[-\lambda(t,h)D]\frac{dD}{dx}, \tag{13}$$

where $N(t,h)$ and $\lambda(t,h)$ are the distributional parameters for the snowflake size spectrum. Note that the exponential distribution satisfies (2) for $\delta=1$ for the total mass and reflectivity equations. To convert to the melted diameter spectrum one can use (9) and (13) to find that

$$\frac{\lambda_m}{\lambda}=\frac{N_m}{N}=\left[\frac{\rho_L}{\rho_i}\right]^{\frac{1}{3}}, \tag{14}$$

where ρ_L is the density of water.

Inserting (9), (10), (11) and (13) into (3)–(8) yields

$$\frac{\partial \chi}{\partial t}+\frac{\partial}{\partial h}(w\chi-\chi_f)=\frac{Nf(t,h)\Gamma(\nu+1)}{\lambda^{\delta+1}}, \tag{15}$$

$$\frac{\partial Z}{\partial t}+\frac{\partial}{\partial h}(wZ-Z_f)=\frac{2(\pi\rho_i/6)Nf(t,h)\Gamma(\delta+4)}{\lambda^{\delta+4}}$$
$$+(\pi\rho_i/6)^2(\pi/4)a\bar{E}N^2\int_0^\infty\int_0^\infty D_1^3D_2^3(D_1+D_2)^2$$
$$\times|D_1^b-D_2^b|\exp[-\lambda(D_1+D_2)]dD_1dD_2, \tag{16}$$

where

$$\chi=(\pi\rho_i/6)N\Gamma(4)\lambda^{-4}, \tag{17}$$

$$\chi_f=(\pi\rho_i/6)aN\Gamma(4+b)\lambda^{-4-b}, \tag{18}$$

$$Z=(\pi\rho_i/6)^2N\Gamma(7)\lambda^{-7}, \tag{19}$$

$$Z_f=(\pi\rho_i/6)^2aN\Gamma(7+b)\lambda^{-7-b}. \tag{20}$$

The double integral in (16) can be scaled by λ so that the last term becomes

$$\left(\frac{\pi\rho_i}{6}\right)^2\frac{\pi a\bar{E}I(b)N^2}{4\lambda^{10+b}}, \tag{21}$$

where

$$I(b)=\int_0^\infty\int_0^\infty x^3y^3(x+y)^2|x^b-y^b|e^{-(x+y)}dxdy. \tag{22}$$

Evaluating I yields

$$I(b)=\Gamma(p)2^{1-p}\sum_{i=1}^{3}C_i\left[\frac{F(1,p;8-i;\frac{1}{2})}{7-i}-\frac{F(1,p;4+b+i;\frac{1}{2})}{3+b+i}\right], \tag{23}$$

where F represents Gauss' hypergeometric function $p=10+b$, $C_1=C_3=1$ and $C_2=2$. The series representation for F converges quite rapidly for these arguements (10^{-6} in 20 terms). Note that for $b=0$, I is identically zero which illustrates that there can be no aggregation without dispersion in the particle fallspeeds.

For the case of stratiform snow, the moment conservation equations (15) and (16) can be further simplified in two ways. The first is to assume that the vertical fluxes of mass and reflectivity are due primarily to the particle fallspeeds. This is a good assumption when one considers that mean vertical air motions in stratiform clouds are of the order of a few centimeters per second. The second assumption which can be made for stratiform clouds is that precipitation is steady. Aside from eliminating time derivatives in the moment conservation equations, this constraint requires the convergence of the flux of water vapor due to the weak vertical air motions to be balanced by an increase in the downward flux of mass due to precipitation fallspeeds. Thus in (15) and (16), rather than specifying the crystal growth function $f(h)$, the rate of change of the total mass flux can be found from the dynamic and thermodynamic properties of the cloud, i.e.,

$$\frac{\partial}{\partial h}(\omega\rho_{vs})=\frac{\partial \chi_f}{\partial h}, \tag{24}$$

where ρ_{vs} is the water vapor density. One can consider $f(h)$ to be forced by the rate at which moisture becomes available in the weak updraft.

c. Solution of moment conservation equations for stratiform snow

Applying the weak updraft and steady state assumptions, combining (15) and (16) through elimination of $f(h)$, letting $\chi_f'(h)$ denote the height derivative of the mass flux, utilizing (18) to eliminate reference to N, and reversing the sign of the height coordinate yields, after some manipulation,

$$\frac{\partial}{\partial h}\lambda(h)=\frac{\chi_f'(h)\lambda(h)}{\chi_f(h)3}\left[1-\frac{2\Gamma(4+b)\Gamma(\delta+4)}{\Gamma(7+b)\Gamma(\delta+1)}\right]-\frac{\pi\bar{E}\chi_f(h)\lambda^{2+b}(h)}{12\Gamma(7+b)\Gamma(4+b)(\pi\rho_i/6)a}. \tag{25}$$

This equation is the governing equation for the size distribution slope parameter $\lambda(h)$, where h is the height below a reference level; $\chi_f(h)$ and $\chi_f'(h)$ are assumed to be known functions of height. The first term on the right is the contribution due to deposition growth and will be positive for $\delta=1$, the classical value. Therefore, deposition will act to increase the slope of the size distribution. The aggregation term will always act to decrease the slope of the size distribution.

Although (25) is nonlinear, it is a Bernoulli equation with solution

$$\lambda^{-(1+b)}(h)=\lambda_0^{-1-b}[\chi_{f0}/\chi_f(h)]^{K_1}+\frac{K_2}{\chi_f^{K_1}(h)}\int_0^h \chi_f^{K_1+1}(h)dh, \tag{26}$$

where

$$K_1=\left[\frac{1+b}{3}\right]\left[1-\frac{2\Gamma(\delta+4)\Gamma(4+b)}{\Gamma(\delta+1)\Gamma(7+b)}\right], \tag{27}$$

$$K_2=\left[\frac{1+b}{3}\right]\left[\frac{\pi\bar{E}I(b)}{4a\Gamma(7+b)\Gamma(4+b)(\pi\rho_i/6)}\right]. \tag{28}$$

Here χ_{f0} and λ_0 are the initial conditions at height $h=0$; $N(h)$ can be found from (26) and (18) for a known $\chi_f(h)$.

The analogous solution to (15) and (16) for a spatially homogeneous cloud is found in the identical manner. Assuming that $\chi(t)$ and its time derivative are known functions of time yields the solution

$$\lambda^{b-1}(t)=\lambda_0^{b-1}[\chi_0/\chi(t)]^{c_1}+\frac{c_2}{\chi^{c_1}(t)}\int_0^t \chi_{(t)}^{c_1+1}dt, \tag{29}$$

where

$$c_1=\left[\frac{1-b}{3}\right]\left[1-\frac{2\Gamma(4)\Gamma(\delta+4)}{\Gamma(7)\Gamma(\delta+1)}\right], \tag{30}$$

$$c_2=\left[\frac{1-b}{3}\right]\left[\frac{\pi a\bar{E}I(b)}{4\Gamma(7)\Gamma(4)(\pi\rho_i/6)}\right]. \tag{31}$$

The initial conditions in (29) refer to $t=0$. $N(t)$ is found from (29) and (17) for a known $\chi(t)$.

Thus for either the weak updraft, steady cloud or the homogeneous cloud, the size distribution parameters N and λ can be found as functions of height or time, respectively. To do this of course requires knowledge of the updraft and temperature structure of the cloud [i.e., $\chi_f(h)$ or $\chi(t)$ must be known functions]. The bulk density of the aggregates and the collection efficiency must also be known along with the fallspeed relation constants.

3. Comparison with numerical integration

To assess the accuracy of this approximate analytical technique, the solution for the homogeneous cloud can be compared to results from Srivastava's (1971) numerical integration for the case of perfect, geometric coalescence of raindrops in a constant liquid water content cloud ($\chi=1$ g m^{-3}). Under these constraints Eq. (29) becomes

$$\lambda^{b-1}(t)=\lambda_0^{b-1}+\left[\frac{1-b}{3}\right]\frac{\pi aI(b)\chi t}{4\Gamma(7)\Gamma(4)(\pi\rho_L/6)}, \tag{32}$$

where

$$N(t)=\lambda^4(t)\chi/(\pi\rho_L). \tag{33}$$

The mean collection efficiency was taken as unity for perfect, geometric coalescence, and ρ_L is the density of water. For raindrops, $a=1420$ and $b=0.5$ (Spil-

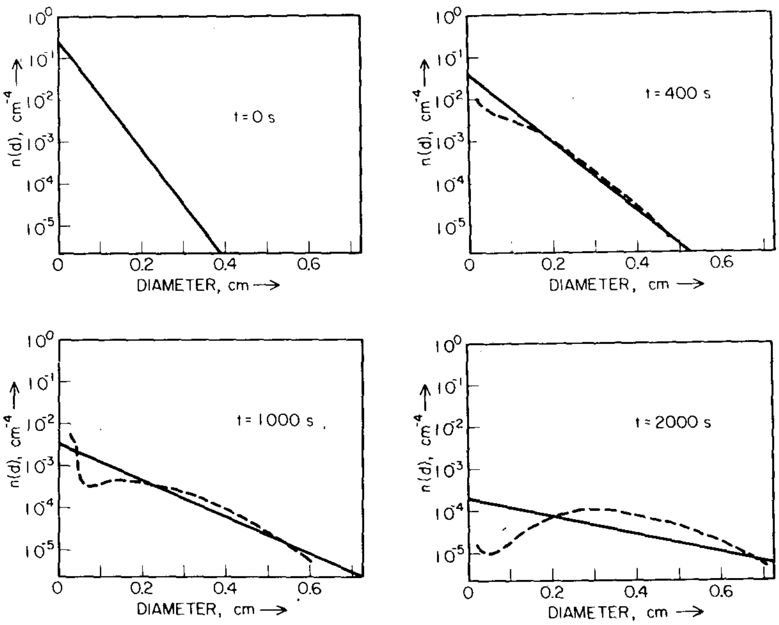

FIG. 1. Comparison between the approximate analytical technique (solid) and Srivastava's numerical integration (dashed) for perfect geometric raindrop coalescence.

haus, 1948), and from (23), $I=1610$ so that (32) becomes

$$\lambda^{-\frac{1}{2}}(t)=\lambda_0^{-\frac{1}{2}}+1.32\times10^{-4}t \text{ (cgs units)}. \quad (34)$$

The slope tends to zero asymptotically as time goes to infinity.

Raindrop size distributions calculated from (33) and (34) are shown in Fig. 1 along with those calculated by Srivastava by numerical integration for the same initial condition. At 400 s the comparison is quite good, but at 1000 s a significant mode starts to develop in the numerically modeled spectrum. However, the agreement is still fair for drops >0.2 cm. At 2000 s the mode becomes very pronounced and the agreement between the two techniques is rather poor.

The exponential distribution probably would be a better approximation if either nucleation or particle breakup were acting to renew the number of small particles and thus reduce the mode. However, nucleation rates in stratiform snow and the extent of crystal breakup are not known. For the case of rain, breakup has been shown to be important in maintaining the exponential form of drop size distributions (Srivastava, 1971; Young, 1975; List and Gillespie, 1976).

It is important to note that the total mass and reflectivity moments were used to find $N(t)$ and $\lambda(t)$ because past measurements of exponential size distributions were made with a view toward predicting the behavior of these higher order moments. If, instead, the total concentration and total mass were used, Eq. (34) would have the same form and the constant would differ by only 2%. Thus for no deposition growth, the choice of moments (concentration-mass or reflectivity-mass) is not critical for the homogeneous cloud case.[1]

4. "Equilibrium" snow size spectra

The solutions for the steady-state, vertically heterogeneous cloud can be evaluated by assuming something about the dynamic and thermodynamic properties of the atmosphere. Under the constraint that the cloud is moist adiabatic, the vapor density will decrease logarithmically with height above cloud base so that for a constant updraft in a steady, balanced cloud Eq. (24) can be integrated to yield

$$\chi_f(h)=\chi_{f_0}e^{Ah}, \quad (35)$$

where A is a constant. The balance condition requires that the upward flux of vapor be equal to the downward flux of precipitation mass. Using (35) to evaluate the integral in the slope equation, Eq. (25) yields

$$\lambda^{-(1+b)}=\lambda_0^{-(1+b)}\exp(-AK_1h)+\frac{K_2\chi_f}{A(K_1+1)} \times\{1-\exp[-A(K_1+1)h]\}. \quad (36)$$

For snow growth by vapor deposition ($\delta=1$), $K_1>0$ so that in the lower regions of the cloud, the expo-

[1] See Passarelli (thesis in preparation) for a more detailed discussion.

nential terms in (36) become small so that

$$\lambda^{-(1+b)} \sim \frac{K_2 \chi_f}{A(K_1+1)} \tag{37}$$

or from (28) and (18)

$$N \sim \left[\frac{12A(K_1+1)\Gamma(7+b)}{\pi(1+b)I(b)\bar{E}}\right]\lambda^3. \tag{38}$$

This relation may be thought of as an equilibrium relation between the slope and intercept parameters. It depends on the fallspeed exponent b, the collection efficiency $\bar{E}$, the deposition growth parameter δ, and the thermodynamic properties of the cloud A. It is independent of the fallspeed parameter a and the bulk density ρ_i. The equilibrium relation does not depend on the initial conditions.

The equilibrium relation exists because of the balance between deposition and aggregation growth. For a given snowfall rate, those spectra which contain a large number of particles relative to equilibrium evolve primarily by aggregation since the aggregation rate goes approximately as the square of the particle concentration. Those with relatively few particles evolve primarily by deposition since the same amount of mass is distributed among fewer particles. Thus a balanced growth occurs when the effects of deposition and aggregation counteract one another.

This concept of an equilibrium relation for snow is quite different from that of raindrop equilibrium size distributions. For rain, coalescence and breakup are thought to balance such that if a size spectrum is perturbed from its equilibrium state, it will return to its initial state (Srivastava, 1971; Young, 1975; List and Gillespie, 1976). For the case of snow, this analysis shows that the spectrum does not return to its precise initial condition, but instead, to the equilibrium line determined by (38). The net result of either equilibrium process is that for a given precipitation rate one expects to observe a particular spectrum.

This equilibrium growth can help explain the observations made by many investigators that on the average the snowfall rate is related to the radar reflectivity factor by a power law relation (e.g., Gunn and Marshall, 1958; Sekhon and Srivastava, 1970). These relations are usually derived from ground measurements of snow size spectra taken from a number of different storms and, therefore, probably represent some kind of climatological mean of snow storm dynamics and microphysics.

Sekhon and Srivastava (1970; hereafter SS) used snow size spectra data taken from studies made by a number of different investigators to derive empirical relationships between N and χ_f, λ and χ_f, and Z and χ_f. However, all of these relations were based

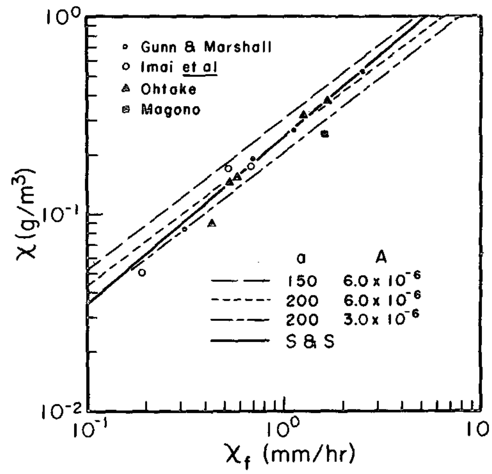

FIG. 2. Theoretical χ,χ_f relations from (39) for various values of the fallspeed coefficient a and the vapor density gradient parameter A, along with Sekhon and Srivastava's empirical χ,χ_f relation. The data points correspond to the measurements of the various investigators who were cited in the Sekhon and Srivastava study.

on a power law fit to their plot of the χ vs χ_f data (their Fig. 8). A theoretical equilibrium χ,χ_f relationship can be derived from (17), (18) and (38), i.e.,

$$\chi = \Gamma(4)\left\{\frac{\left[\dfrac{2\rho_i A(K_1+1)\Gamma(7+b)}{(1+b)I\bar{E}}\right]^{b}}{a\Gamma(4+b)}\right\}^{[1/(1+b)]} \chi_f^{[1/(1+b)]}. \tag{39}$$

Sekon and Srivastava's χ,χ_f plot is reproduced in Fig. 2 along with χ,χ_f lines computed from (39) for various values of the vapor density gradient parameter A and the fallspeed coefficient a. The values for the other parameters in (39) were assumed to be $\rho_i = 0.05$ g cm^{-3}, $\delta = 1$ ($K_1 = 0.292$), $b = 0.31$, $I(b) = 751$ and $\bar{E} = 1$. For a moist adiabatic atmosphere with a surface temperature between -20 and 0°C, $A \approx 6 \times 10^{-6}$ cm^{-1}. Note that A does not have to correspond to moist adiabatic conditions since steady stratiform snow can form in a rising saturated stable layer. To reflect this, one of the lines in Fig. 2 was computed assuming $A = 3\times 10^{-6}$ cm^{-1} which is half the moist adiabatic value. The values $b = 0.31$ and $a = 150$ were used in the SS study.

The observations are fairly well bracketed by these theoretical lines. The middle line fits the data almost as well as the SS line for precipitation rates >0.5 mm h^{-1}. Within the scatter of the data the theoretical equilibrium can explain the observations fairly well. The scatter of the data can be explained by storm-to-storm variations in cloud dynamics and microphysics which would lead to variations in the physical parameters in (39).

5. Summary and conclusions

This type of approximate analytical technique could be used to parameterize precipitation growth processes in the context of a more complex model of snowstorm dynamics and microphysics, and to analyze snow size spectra data from vertical incidence Doppler radar or aircraft measurements. An application to the latter is described by Passarelli (1977) where this analytical technique is used to estimate $\bar{E}$ from measurements of snow size spectra which were taken at different levels in a Midwest extratropical cyclone.

The technique suggests the existence of an equilibrium relation between N and λ which is due to the counteracting effects of deposition and aggregation growth. It is possible that observed correlations between the snowfall rate and radar reflectivity are due to this type of equilibrium rather than the coalescence breakup mechanism which is thought to occur for heavy rain.

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