Theoretical and Observational Study of Snow-Size Spectra and Snowflake Aggregation Efficiencies

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(Manuscript received 17 January 1977, in final form 31 January 1978)

ABSTRACT

Mean snowflake collection efficiencies can be estimated by studying the height evolution of snow-size spectra in natural precipitation. This was done using coordinated aircraft and Doppler radar measurements of snow-size spectra and analytical and numerical techniques to model the stochastic collection process. For aggregating stellar and dendritic crystals in the temperature range −15 to −12°C the estimated mean collection efficiency was 1.4±0.6.

1. Introduction

The growth of snowflakes by aggregation is a fundamental aspect of cloud physics which has received relatively little attention compared to the analogous process of raindrop coalescence. This is primarily due to the greater complexity of crystal-crystal and crystal-aggregate interactions and the difficulties in simulating these interactions in the laboratory and studying them directly in natural clouds.

One of the main difficulties in modeling snowflake aggregation is that of determining the efficiency at which crystals of one size and type collect other crystals of various sizes and types. This is complicated by the nonuniform vertical and horizontal motions of ice crystals (Sasyo, 1971; List and Schemenauer, 1971), the aerodynamics of mutually interacting crystals (Jayaweera and Mason, 1965), and the dependency of sticking on such things as crystal habit, relative humidity and air temperature (Hosler and Hallgren, 1960; Hobbs *et al.*, 1974; Rogers, 1974). The effects of these processes can be combined by expressing them as a collection efficiency E which is the ratio of the actual number of collected ice crystals to the number of crystals within a vertically swept geometrical volume. Thus the rate at which a snowflake of mass x collects snowflakes of mass y is simply

$$E(x,y)\tfrac{1}{4}\pi[D(x)+D(y)]^2|v(x)-v(y)|c(y),$$

where $D(y)$, $V(y)$ and $c(y)$ are, respectively, the snowflake diameter, fall velocity and concentration.

There exists no formulation for $E(x,y)$ which accounts for all of the abovementioned effects. However, it is possible to estimate an average collection efficiency which is independent of particle size by studying the height evolution due to aggregation of actual snow size distributions in steady stratiform precipitation. This paper presents a technique for doing this and then discusses an application of this technique to snow size distributions which were measured by the University of Chicago's Snow Shower Cloud Physics Project.

2. Theory

Passarelli (1978) formulated an approximate analytical technique to model the deposition and aggregation growth of snow. This technique is based on the fact that snow-size spectra tend to be exponential with respect to the equivalent melted diameter such that

$$n(D_m)=N_m\exp(-\lambda_m D_m), \tag{1}$$

where D_m is the equivalent melted diameter, $n(D_m)dD_m$ is the concentration of particles in the size interval $D_m\pm dD_m/2$, and λ_m and N_m are the distributional parameters. Very little data exist for the actual size distribution of aggregate snowflakes. Those presented in this paper and those of Rogers (1974) show that the same basic form given by (1) is valid such that

$$n(D)=Ne^{-\lambda D}, \tag{2}$$

where D is the actual snowflake diameter and N and λ are the distributional parameters for the spectrum of actual snowflake sizes. That the observations suggest the validity of both (1) and (2) is consistent with spherical aggregates which have a constant bulk density so that

$$\rho_L D_m^3=\rho_i D^3, \tag{3}$$

where ρ_L is the density of liquid water and ρ_i the bulk density of the aggregate. This is a reasonable first approximation to the geometry of aggregate snowflakes.

0022-4928/78/0882-0889$05.00

Using (2) and (3), the moment conservation equations for the total mass and reflectivity factor fluxes for aggregating snowflakes can be found from the stochastic collection equation in terms of N and λ. For the case of steady, constant mass flux snow, these equations can be solved to yield N and λ as functions of height (see Passarelli, 1978), i.e.,

$$\lambda^{-(1+b)}=\lambda_0^{-(1+b)}+K_2\chi_f h, \tag{4}$$

$$N=\frac{\chi_f\lambda^{4+b}}{(\pi\rho_i/6)a\Gamma(4+b)}, \tag{5}$$

where h is the height below a reference level ($h=0$), λ_0 the slope of the size distribution at $h=0$, and χ_f the precipitation mass flux given by

$$\chi_f=(\pi\rho_i/6)a\Gamma(4+b)N\lambda^{-(4+b)}. \tag{6}$$

Here Γ represents the gamma function and the constants a and b are from an assumed power law fallspeed relation

$$v=aD^b, \tag{7}$$

where v is the snowflake terminal velocity. The aggregation constant K_2 is given by

$$K_2=\frac{(1+b)\pi\bar{E}I(b)}{12a(\pi\rho_i/6)\Gamma(4+b)\Gamma(7+b)}. \tag{8}$$

The parameter I is a function of b only and is given by the double integral

$$I(b)=\int_0^\infty\int_0^\infty x^3y^3(x+y)^2|x^b-y^b|e^{-(x+y)}dxdy \tag{9}$$

which arises from the moment conservation equation for the total reflectivity factor for an exponential size distribution function. Substituting (8) for K_2 and (6) for χ_f into (4), and rearranging, yields an explicit expression for $\bar{E}$, i.e.,

$$\bar{E}=(\lambda^{-1-b}-\lambda_0^{-1-b})\frac{12\Gamma(7+b)\lambda^{4+b}}{(1+b)\pi I(b)Nh}. \tag{10}$$

This expression for $\bar{E}$ is independent of the bulk density ρ_i and the fallspeed coefficient a. Thus $\bar{E}$ can be estimated within a layer of thickness h in steady snow provided that the size distribution parameters N and λ are known at the layer boundaries and that the mass flux is constant within the layer ($N/\lambda^{4+b}=\text{constant}$). The fallspeed exponent b must also be known within the layer.

Passarelli (1978) found that results from the approximate analytical technique compared well with those from a full numerical integration for the case of perfect geometric coalescence of raindrops. The raindrop collection kernel which was used had a form identical to that used to arrive at (10).

To evaluate the parameters in (10) which are required to estimate $\bar{E}$ requires rather specialized instrumentation and meteorological conditions. The snow-size spectra discussed here were measured by means of coordinated Doppler radar and aircraft techniques.

3. Data collection and instrumentation

The snow-size spectra discussed here were taken over Champaign, Ill., on 26 November 1975 during an intense snowfall. A large area of precipitation, in association with a Louisiana low, covered much of the central United States. The most intense region of snow developed northeast (downwind with respect to the winds above 850 mb) of a convergence line which extended from Kansas City, Mo., to Memphis, Tenn., at 2000 GMT. This region of moderate to heavy snow moved over Champaign during the time of the experiment. Snow spectra were observed by means of simultaneous vertical incidence Doppler radar and aircraft measurements.

The aircraft used in our experiments is a Lockheed Lodestar leased by the University of Chicago Cloud Physics Laboratory. It is equipped to measure a variety of microphysical parameters. An optical array precipitation spectrometer (PROPA) (Knollenberg, 1970) was used to count and size precipitation particles into 15 equally spaced, 300 μm size categories ranging from 300 to 4500±150 μm. Precipitation particles were photographed *in situ* by a precipitation particle camera (Carrera, 1976) which is a copy of the one designed and built by Cannon (1970). The primary temperature sensor was a platinum resistance element mounted in a reverse-flow housing. A number of other sensors were also carried, but these are not of primary importance in this study.

The radar used is dual-wavelength pulsed radar, 10 cm and 3 cm, equipped with Doppler processing for the 10 cm band. Known as CHILL, it is owned jointly by the Illinois State Water Survey and the University of Chicago, and is operated by the former. For these experiments, the 10 cm Doppler spectra were taken over 512 pulses (512 ms) which gives 10 cm s^{-1} velocity resolution. The thirty-two, 150 m range gates were stacked 150 m apart for maximum range resolution. Only the 10 cm vertical incidence data are discussed in this study.

The aircraft flew a series of 10 km passes spaced 600 m apart at 4.35, 3.75, 3.15 and 2.55 km AGL over the Champaign VORTAC which was located about 0.7 km from the CHILL radar at Willard Airport, while the radar collected vertical incidence data. The passes were spaced 10 min apart, the approximate time required for snow to fall 600 m. After we had completed these passes, the surface weather deteriorated to below landing minimums at Champaign and we were forced to divert to another airport.

At 4.35 and 3.75 km AGL (−20 and −17°C) the particle camera showed that the particles were very

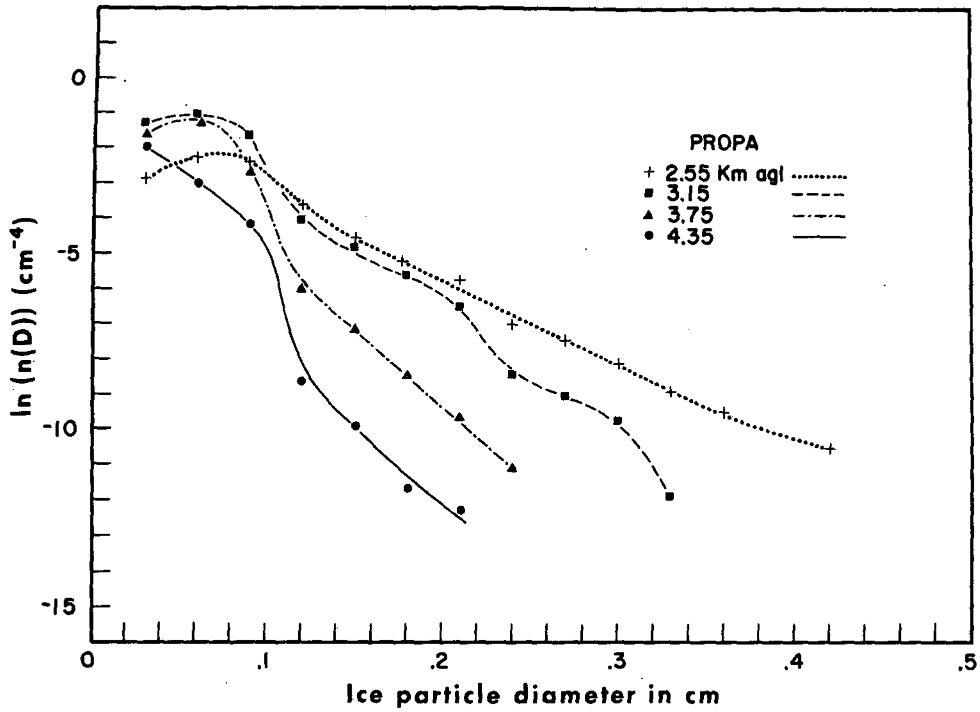

FIG. 1. Pass-averaged PROPA spectra at various flight altitudes.

similar, consisting predominantly of side plane and columnar-type crystals and aggregates. It is important to note that aggregates were common even at −20°C. At 3.25 and 2.55 km (−15 and −12°C) the snow was made up of tsuzumis capped with stellars, spatial dendrites and aggregates of these forms.

Fig. 1 shows the pass-averaged PROPA particle spectra for all size categories in which more than ten particles were counted. The spectra show increasing concentrations of large particles with decreasing altitude. This is in agreement with the camera data which show that the sizes of individual crystals and the frequency of aggregates were greater at the lower altitudes.

The PROPA snow spectra are essentially exponential. Table 1 gives the slope and intercept parameters computed by the method of least squares for the average spectra at the four different altitudes. The first size category was not included in the calculations since the size interval can be ambiguous.

The interpretation of PROPA particle spectra is complicated by the asymmetry of the particles and the diversity of particle types. It is apparent that PROPA will measure some sort of *mean particle dimension*, but how that dimension relates to the particle mass is not easily determined. An important aspect of the PROPA size distributions is that the measured particle diameter is almost a direct measure of the mean horizontal projection of ice crystals and aggregates, since the probe is mounted on the aircraft to give the particle dimension in the horizontal plane. This means that the particle diameters measured by PROPA are well suited for computing the swept volume in a geometric kernel.

a. Snowflake bulk density

While the snowflake density as defined by (3) is not specifically required to estimate $\bar{E}$, it is a useful parameter for subsequent discussions of the vertical incidence Doppler spectra. The bulk density can be found by normalizing the PROPA spectra to coincident measurements of the radar reflectivity factor which can be

TABLE 1. Slope and intercept parameters from 26 November 1975 measurements.

Height (km)	λ (cm^{-1})	N (cm^{-4})	a (cgs)	b	χ_f (mmh^{-1})*	dbZ$_e$ (mm^{6}m^{-3}) PROPA*	Radar
2.55	24.4	0.51	155	0.24	1.44	27.6	29.9
3.15	38.5	3.43	160	0.24	1.44	22.0	22.3
3.75	55.0	5.46	230	0.42	0.45	13.2	13.0
4.35	65.0	1.64	240	0.42	0.07	2.9	2.7

* Based on $\rho_i=0.09$.

expressed as

$$Z=\int_0^{\infty} D^6(\rho_i/\rho_L)^2 n(D)dD. \qquad (11)$$

The bulk density can be varied until the reflectivity factor computed from the PROPA spectra match those measured by the radar (Passarelli *et al.*, 1976). Table 1 gives a comparison between PROPA reflectivity factors assuming a particle density of 0.09 g cm^{-3}. The comparison represents pass-averaged PROPA spectra versus 10 s averages of the radar reflectivity factor taken at the time when the aircraft was at its closest approach to the radar. At all altitudes the particle density of 0.09 g cm^{-3} gives Z values which agree well with the radar measurements. The fact that the radar and aircraft dbZ values show such a close correspondence at all heights indicates that the differences between the PROPA snow-size spectra at the various altitudes are real, and not due to the statistics of particle sampling.

b. Fallspeed relationships

Locatelli and Hobbs (1974) measured fallspeeds and masses for some of the types of crystals which were observed. For the sideplane type crystals at 4.35 and 3.75 km AGL, the respective fallspeed-mass relations are $v=370M^{0.14}$ and $v=354M^{0.14}$ (cgs), after applying the Foote and du Toit (1969) density correction factor [$v=v_0(\rho_0/\rho)^{0.4}$, where v_0 refers to the fallspeed measured at an air density of ρ_0]. In terms of the aggregate diameter these relations (for $\rho_i=0.09$) are $v=240D^{0.42}$ and $v=230D^{0.42}$, respectively. Similarly for the dendritic type aggregates at 3.15 and 2.55 km the respective fallspeed-mass relations are $v=206M^{0.08}$ and $v=201M^{0.08}$ (cgs) or $v=160D^{0.24}$ and $v=155D^{0.24}$. These results are also summarized in Table 1.

c. Radar measurements of snow size spectra

The vertical incidence Doppler (VID) spectra can be converted to particle size distributions to check the accuracy of the PROPA spectra, the bulk density and the fallspeed relations. For Rayleigh scatterers

$$z(v)dv=D^6(\rho_i/\rho_L)^2 n(D)dD, \qquad (12)$$

where $z(v)$ is the radar reflectivity factor per unit velocity interval in the velocity range v to $v+dv$. Since $n(D_m)dD_m=n(D)dD$, $v=aD^b$ and $\rho_L D_m^3=\rho_i D^3$, the size distribution of snowflakes can be found from the vertical incidence reflectivity factor spectrum via

$$n(D)=ab(v/a)^{1-7/b}(\rho_L/\rho_i)^2 z(v). \qquad (13)$$

This conversion is based on the assumption that the vertical air motions are weak as compared to the snowflake fall velocities.

The conversion was done for at least 15 VID spectra which were taken when the aircraft was sampling at its closest approach to the radar for each of the four passes. The results are shown in Fig. 2 along with the pass-averaged PROPA spectra which were measured at the same altitude. Size distributions computed from Sekhon and Srivastava's (1970, hereafter referred to as SS) parameterization (adjusted for spherical aggregates of density 0.09) are also shown.

The agreement between the two techniques is excellent at 4.35 and 3.75 km. However, at lower altitudes the radar tends to underestimate the particle concentration probably due to turbulence broadening. The Doppler spectra at the lower altitudes were significantly broader than those aloft and were often multimodal, indicating the presence of subpulse volume vertical air motions. The excellent agreement between the particle spectra derived from the VID measurements and those measured by PROPA at the higher altitudes suggest that undersizing by PROPA was not a serious problem for the types of particles which were present on this day (Knollenberg, 1975).

The SS parameterized spectra are not as steep as those measured by PROPA. This could be due to the fact that the SS parameterization is based on ground measurements which are probably representative of more highly aggregated snow spectra.

On the basis of the data which were used to formulate their parameterization, SS truncated their spectra. The observed PROPA spectra extend to larger sizes than the SS spectra. In fact, it is impossible to say what the largest aggregate size is due to the limitations of sampling volume. A careful review of their manuscript indicates that the maximum size used by SS is probably too small for the same reason.

d. Precipitation rate

Fig. 3 shows the precipitation rate as a function of height as computed from the least-squares lines for the measured size distributions assuming spherical symmetry, $\rho_i=0.09$ and the fallspeed relationships given above. Also plotted are the values corresponding to the SS Z–X_f relationship.

The snowfall rate computed from the measured particle spectra increases rapidly down to 3.15 km and is essentially constant down to 2.55 km. The snowfall rates computed from the Z–χ_f relationship are substantially less than those from the measured particle spectra. Again, this may be due to the fact that the snow is not yet highly aggregated which causes low Z values and consequently low χ_f values (Passarelli *et al.*, 1976).

The spectra at 2.55 and 3.15 km are well suited for computing $\bar{E}$ since the precipitation rate is essentially the same for these levels. Time-height cross sections of radar reflectivity factor show that precipitation was very steady during the time of the observations at those two levels.

e. Calculation of $\bar{E}$

The collection efficiency for the layer between 3.15 and 2.55 km can be computed directly from (10)

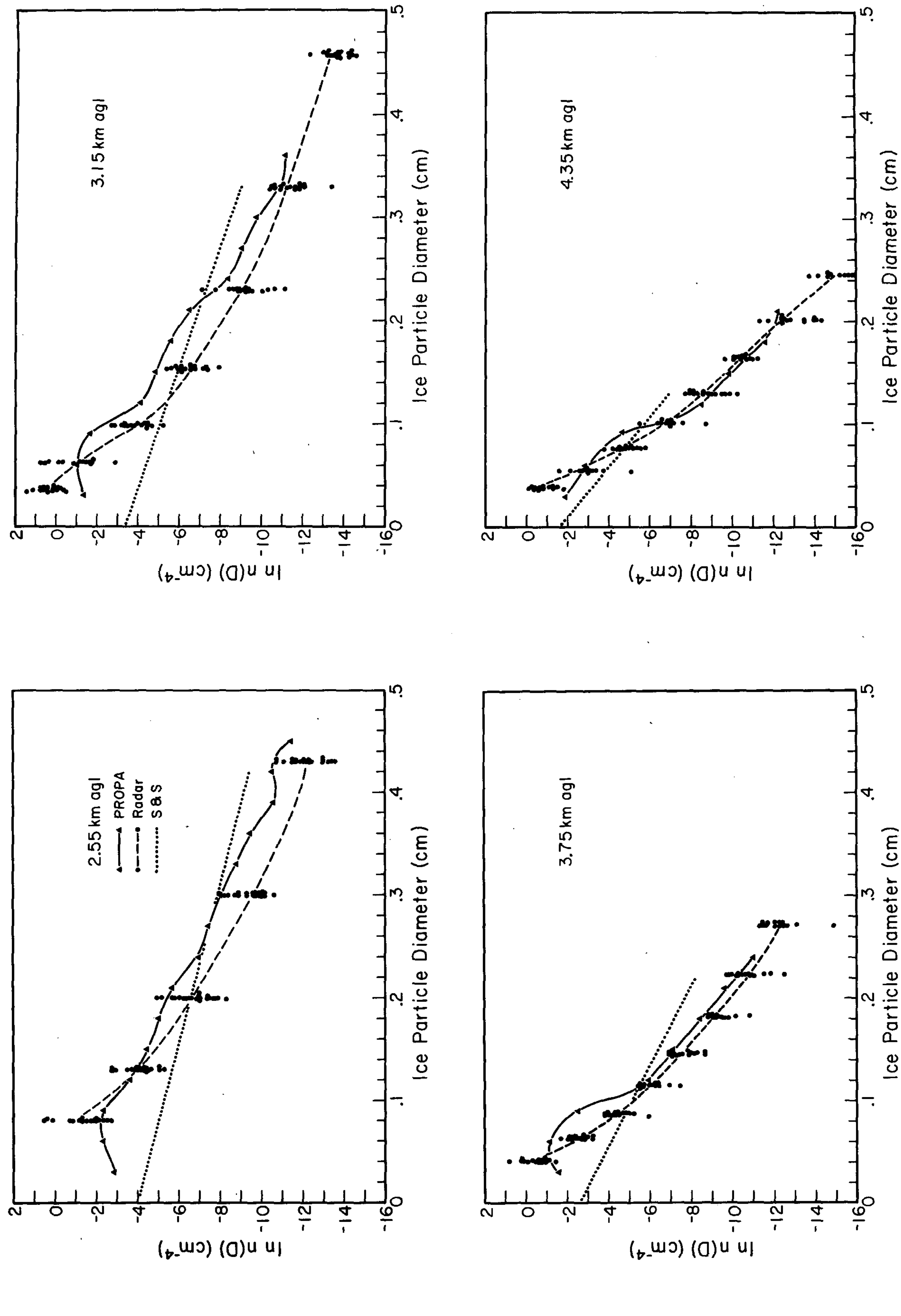

FIG. 2. Comparison between the PROPA, radar and Sekhon and Srivastava snow-size spectra at various flight altitudes.

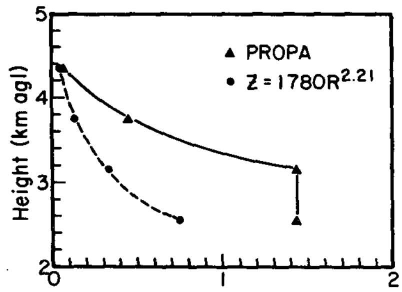

FIG. 3. Precipitation rate as a function of height computed from the log-linear fits to the PROPA data and Sekhon and Srivastava's Z–R relationship.

yielding $\bar{E}=1.4$. Note that $I(0.24)=524$. Fig. 4 shows spectra computed from the parameterization for various values of $\bar{E}$ along with the PROPA data points at the two levels to illustrate the sensitivity of the spectral evolution to the value of the collection efficiency. From this figure it is apparent that the collection efficiency is bounded between 1.0 and 2.0. Fig. 5 shows similar results except that the spectral evolution was computed by numerical integration of the stochastic collection equation utilizing the log-linear fit to the 3.15 km spectrum for the initial state.

The primary source of error in this analysis is the PROPA sizing which has an accuracy of $\pm 150\,\mu$m. Shifting the PROPA size categories by $\pm 150\,\mu$m results in a change of about ± 0.6 in the collection efficiency so that $\bar{E}=1.4\pm 0.6$. Another possible source of error is the fallspeed exponent b which governs the fallspeed dispersion of the particle spectrum. Doubling b would reduce the collection efficiency by a factor of $\sim$0.5. According to Locatelli and Hobbs (1974) and Jiusto and Bosworth (1971), it seems unlikely that this is the case for the types of crystals which were observed.

4. Discussion

The estimated collection efficiency of 1.4 ± 0.6 is considerably larger than past laboratory measurements have indicated. Hosler and Hallgren (1960) measured $\bar{E}$ in the laboratory by counting the number of ice crystals which stuck to an ice sphere which was placed in a stream of ice crystals. This was not a good simulation for measuring an $\bar{E}$, such as the one discussed here, since their ice crystals were small plates and columns, and the hydrodynamics of their problem were much different. However, for a 360 μm sphere collecting 18 μm diameter plates, they measured an $\bar{E}$ of 0.2 at -11°C. In a similar experiment, Latham and Saunders (1970) passed a stream of 5 μm ice crystals past a 0.2 cm ice collector sphere at a relative velocity of $\sim$3 m s^{-1} and found $\bar{E}\approx 0.3$. Again, the crystals which were used were unelaborated plates and columns.

In a field study of snowflake aggregation efficiencies, Rogers (1974) found collection efficiencies ranging from 0.1 to 0.6 based on his observations and a nondepleting model of ice crystal aggregation. This type of model will, of course, underestimate collection efficiencies.

The difference between the laboratory estimates of $\bar{E}$ and the one calculated here may be due to the more elaborate crystal forms and larger crystals which were present in the natural case. Nakaya (1954) pointed out that loose mechanical interlocking of spatial and planar dendrites is common. The laboratory experiments could not simulate this important aspect of

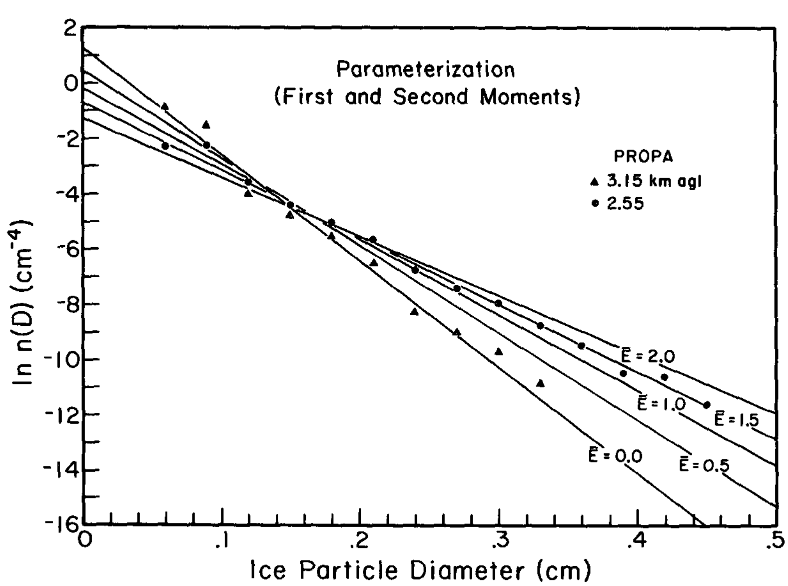

FIG. 4. The 600 m evolution of log-linear snow size spectra for various values of $\bar{E}$ as computed from the first and second moment parameterizations along with the measured size spectra at 3.15 and 2.55 km AGL.

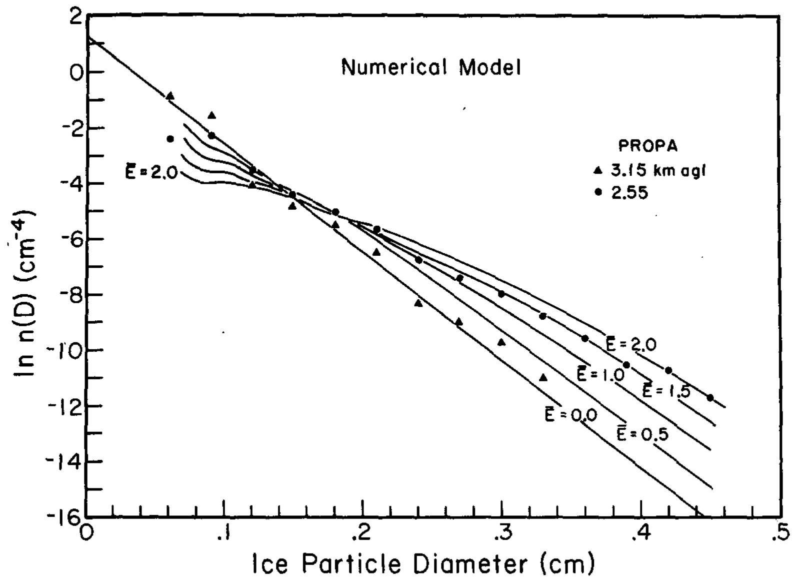

FIG. 5. As in Fig. 4 except that the spectral evolution is computed by numerical integration.

aggregation. Indeed, Hobbs *et al.* (1974) observed that the size and frequency of occurrence of aggregates tend to be greater in the dendritic growth region (-12 to -17°C).

That the estimate of $\bar{E}$ is greater than unity indicates that wake capture may be important. The strong influence of wakes has been observed in laboratory studies of falling cylinders (Jayaweera and Mason, 1965). Two equal-size columns which are initially separated in the vertical will converge due to the greater fall velocity of the upper column. This author knows of no similar studies which have been performed for more complex snowflake shapes.

Another possible explanation for the magnitude of the estimated efficiency is that the horizontal motions of the snowflakes were not accounted for in the formulation of the collection kernel. These would tend to enhance aggregation by effectively increasing the swept-volume. Rogers (1974) attributed the rapid aggregation of dendrites to horizontal motions. He estimated that horizontal motions increased the effective swept-volume by a factor of 5 for his case. While the effect of horizontal motions may not be as large in this study, it could explain the magnitude of the estimated collection efficiency.

Another possibility is that the simple "one-to-one" concept relating the fallspeed to the particle size may be inadequate to describe the aggregation of snowflakes. Differences in shape between aggregates of the same mass could lead to significant fall velocity differences (e.g., Jiusto and Bosworth, 1971). This would introduce a greater fallspeed dispersion and thus enhance the aggregation rate. This effect would be enhanced in situations where the snowfall is heterogeneous with respect to crystal type.

5. Summary and conclusions

Snow-size spectra measured by the aircraft and radar are significantly different from those computed from the Sekhon and Srivastava parameterization in that they are steeper and extend to larger particle sizes. The greater size distribution slope is consistent with the theoretical findings of Passarelli (1978) concerning equilibrium snow-size spectra.

The collection efficiency of dendritic crystals and aggregates as estimated from a simple geometric kernel is greater than unity for this particular case study. It is not clear whether the magnitude of the collection efficiency is due to an effect such as wake capture or whether the kernel which is used is inappropriate due to effects such as horizontal motions, nonspherical aggregate shape or the inapplicability of the "one-to-one" type fallspeed-size relationships. It is clear that dendritic crystals aggregate very rapidly even at temperatures between -12 and -15°C.

The coordinated radar and aircraft approach is well suited to measuring the parameters required to estimate $\bar{E}$. We anticipate making future studies of this type under a greater variety of meteorological conditions.

Acknowledgments. The author thanks Professors R. R. Braham, Jr. and R. C. Srivastava for their guidance in this research; T. R. Morris and N. J. Carrera for aircraft data collection; E. Mueller, E. Silha and the Illinois State Water Survey for radar data collection; J. Jacobs for typing, and E. Von der Porten for data reduction and drafting; D. Rogers for his helpful review of this manuscript; and the NCAR Flight Facility for ARIS tape processing and the NCAR Computer Facility for resources granted for data reduction. NCAR is funded by the National Science Foundation.

This research was supported by the National Science Foundation, Atmospheric Sciences Division, under Grant DES75-14225, and RANN program under Grant ENV72-03399-A03. Any opinions, findings and conclusions or recommendations expressed in this publication are those of the author and do not necessarily reflect the views of the National Science Foundation.

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