The Integration of a Low Order Spectral Form of the Primitive Meteorological Equations

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Abstract

The interest for spectral forms of the meteorological equations has grown considerably over the past several years. Integrations in terms of spherical harmonics provide us with an interesting alternative to the grid point method. The results of an extension of the method to the complete meteorological equations will be presented here.

A model based on five levels and 15 coefficients is integrated for 200 days starting from an atmosphere at rest and at a uniform temperature. The integration is then continued for another 22 days with 45 coefficients. Cross-sections show a jet stream in each hemisphere and low level easterlies along the equatorial belt. The amplitudes and the phase speeds of the planetary waves in the model compare favourably with their atmospheric equivalents. The results of this integration indicate that spherical harmonics could be used profitably in general circulation models or in the preparation of extended range forecasts.

1. Introduction

The meteorologist cannot contemplate as a possibility, any direct experimentation with the large scale processes of the atmosphere. This is unfortunate in a sense, because it deprives us of the method most commonly used in physics. It deprives us of a powerful tool that could have raised meteorology to the level of an exact science.

An insight into the mechanisms that generate and sustain the large scale flow patterns, can result also from the study of adequate analogues. This alternative method contains serious weaknesses, the conclusions drawn from studies of the analogue do not necessarily apply to the atmosphere, but it can still provide us with a considerable amount of knowledge and useful information. At present, it appears to be the best possible alternative to the direct experiment.

Analogues of the atmosphere fall into two classes: the scale model and the numerical model. Models of the first type attempt to simulate the atmospheric circulations in the laboratory, the rotating dishpan experiments belong to this category. The numerical model uses the electronic computer, it represents a relatively new field of research activity.

The first numerical integrations of the meteorological equations using electronic equipment were performed about 15 years ago. The grid point method was used for these experiments and other methods of representation were not seriously considered until recently. The grid point method requires finite difference approximations and these by necessity, generate appreciable truncation errors. This undesirable feature of the grid point method can seriously corrupt the numerical integrations. The study of various finite difference operators and their properties, still represents a large fraction of the time devoted to the simulation problem in meteorology.

Spectral forms of the meteorological equations do not use any approximations for the evaluation of space derivatives. They do not eliminate entirely the necessity for truncation, but they permit a rigid control over the resulting truncation errors. The integration of spectral equations may require an excessive amount of computing power. This would definitely be true at very high resolution. This problem will always remain, it might be serious today but it will likely be trivial in another generation.

The results of the first successful integration of the barotropic vorticity equation in terms of spherical harmonics were published by Baer (1964). The application of this method to baroclinic models does not present any additional problems. An experiment carried out by Li Peng (1965) shows that spectral baroclinic models can produce useful results even at very low resolution. An extension to the primitive equations appears more problematic and failure to overcome this difficulty would make the method useless. It would not be possible to find any merit in a spectral form that cannot be extended to the complete meteorological equations.

Fortunately, the work of Kubota (1959) and a few others indicates that the complete meteorological equations may be treated in terms of spherical harmonics. He uses the differentiated form (vorticity and divergence equations) of the two equations for horizontal motion. The divergence equation in particular contains a large number of difficult terms to handle. The method appears as extremely laborious and it can discourage any attempts towards implementation. A different approach will be used in the present experiment because of its greater simplicity.

In order to avoid using the differentiated form of the meteorological equations it has been found that the horizontal components of the wind must be represented by functions other than spherical harmonics. The functions that must be used do not differ much in nature from the truncated series of spherical harmonics used to represent temperature and vertical motion. Once that the problem of representing the horizontal components of the wind has been solved, the differentiated form no longer has to be used and this eliminates the most serious barrier that prevented us from integrating the primitive meteorological equations.

2. The method

Spherical harmonics are associated Legendre polynomials of the first kind and the meteorological equations may be treated in terms of the elements that constitute these polynomials instead of using the polynomials themselves. This simplifies the multiplication process significantly and eliminates some of the difficulties present in the method of multiplication described by Silberman (1954). The basic elements required to generate spherical harmonics are simple analytic functions.

$$G_n{}^m(\lambda,\varphi)=e^{im\lambda}\cos^M\varphi\sin^n\varphi \qquad (1)$$

where λ and φ represent the longitude and the latitude respectively, M simply represents the absolute value of m. Both M and n will be either positive integers or zero. The number M gives the number of waves along any given latitude circle while n is only an exponent and cannot be given any simple physical interpretation at this stage.

These functions will be used later to represent the meteorological variables. They are continuous and all their physically significant derivatives are continuous, this will become obvious after a discussion of the properties of these functions. The evaluation of the two partial derivatives is straightforward.

$$\frac{\partial G_n{}^m}{\partial\lambda}=imG_n{}^m \qquad (2)$$

$$\cos\varphi\frac{\partial G_n{}^m}{\partial\varphi}=nG_{n-1}{}^m-(M+n)G_{n+1}{}^m \qquad (3)$$

The evaluation of non-linear terms in the equations of motion involves the product of functions. For a description of the multiplication we will use two functions with m and k assigned as upperscripts. The upperscript with the largest absolute value will be represented by m so that $M\geqslant K$. The product of two functions is then given by:

$$G_n{}^mG_l{}^k=G_{n+l}{}^{m+k} \qquad \text{if}: mk\geqslant 0 \quad (4)$$

$$G_n{}^mG_l{}^k=\sum_{j=0}^{K}\frac{(-1)^jK!}{j!(K-j)!}G_{n+l+2j}{}^{m+k} \quad \text{if}: mk<0 \quad (5)$$

In both cases the result takes a simple form in comparison with the multiplication of true spherical harmonics. With the multiplication and the two derivatives we have the basic operations needed for the integration of the meteorological equations. A few additional operations involving the horizontal "del" operator ∇_H will be discussed in order to clarify the application to physical processes.

$$\nabla_H{}^2G_n{}^m=\frac{1}{a^2}[n(n-1)G_{n-2}{}^m-(M+n)(M+n+1)G_n{}^m] \qquad (6)$$

Here a is the radius of the earth, also:

$$\frac{G_n{}^m G_l{}^k}{\cos^2\varphi}=\sum_{j=0}^{K-1}\frac{(-1)^j(K-1)!}{j!(K-1-j)!}G_{n+l+2j}{}^{m+k} \quad \text{if: } mk<0 \tag{7}$$

The above identity is valid only when $M\geqslant K$ as in (4) and (5) and it will be used in the two following operations:

$$\nabla_H G_n{}^m \cdot \nabla_H G_l{}^k=\frac{1}{a^2}[nlG_{n-1}{}^m G_{l-1}{}^k-(M+n)\cdot (K+l)G_n{}^m G_l{}^k]+\frac{(MK-mk)}{a^2\cos^2\varphi}G_n{}^m G_l{}^k \tag{8}$$

The last term in (8) vanishes when $mk\geqslant 0$. When $mk<0$ the division by $\cos^2\varphi$ may be carried out in accordance with (7). The next operation also has the same property

$$\nabla_H G_n{}^m \times \nabla_H G_l{}^k \cdot \underset{\sim}{K}=\frac{i}{a^2}(ml-kn)G_n{}^m G_{l-1}{}^k +\frac{i(MK-mk)}{a^2\cos^2\varphi}G_n{}^m G_{l+1}{}^k \tag{9}$$

The last term in (9) vanishes again when $mk\geqslant 0$. When $mk<0$, the division by $\cos^2\varphi$ may be carried out in accordance with (7).

The operator ∇_H when applied twice as in (6), (8) and (9) necessitates a division by $\cos^2\varphi$. In all cases the numerator is exactly divisible by $\cos^2\varphi$ and the results are expressible in terms of the functions selected in (1).

In order to integrate the meteorological equations, each field will be expressed in terms of the functions defined in (1)

$$P(\lambda,\varphi)=\sum_m\sum_n A_n{}^m G_n{}^m(\lambda,\varphi) \tag{10}$$

The vector representing the horizontal gradient of this field is defined by its two components:

$$P_x=\frac{\partial P}{\partial x}=\frac{1}{\cos\varphi}\sum_m\sum_n\frac{im}{a}A_n{}^m G_n{}^m \tag{11}$$

$$P_y=\frac{\partial P}{\partial y}=\frac{1}{\cos\varphi}\sum_m\sum_n\frac{1}{a}[(n+1)A_{n+1}{}^m -(M+n-1)A_{n-1}{}^m]G_n{}^m \tag{12}$$

Both P_x and P_y take the same form as P except for an additional division by $\cos\varphi$. This division presents no problem because the corresponding numerator is always exactly divisible by $\cos\varphi$. The division is not carried out because it would not be possible then to transform the results back to the form used in (10). The distinction being made here is emphasized by calling (10) a "true scaler" and the expression used in both (11) and (12) is called a "horizontal vector component". The only difference between the two is a division by $\cos\varphi$, but this distinction is very important. It forms the basis of the method described here.

When $\cos\varphi$ appears more than once in the denominator, the division is carried out with the formula:

$$\frac{P}{\cos^2\varphi}=\sum_m\sum_n\sum_{i=0}^{\infty}A_{n-2j}{}^m G_n{}^m \tag{13}$$

When such a division is required in the meteorological equations as in (6), (8) and (9), it truns out that the polynomial appearing in the numerator is always exactly divisible by $\cos^2\varphi$.

For a complete description of the multiplication we will use the fields: Q with coefficients $B_n{}^m$, R with coefficients $C_n{}^m$ and S with coefficients $D_n{}^m$.

$$PQ=R+S \tag{14}$$

$$R=\sum_M\sum_K\sum_n\sum_l[A_n{}^M B_l{}^K G_n{}^M G_l{}^K +A_n{}^{-M}B_n{}^{-K}G_l{}^{-M}G_l{}^{-K}] \tag{15}$$

$$S=\sum_M\sum_K\sum_n\sum_l[A_n{}^M B_l{}^{-K}G_n{}^M G_l{}^{-K}] +A_n{}^{-M}B_l{}^K G_n{}^{-M}G_l{}^K] \tag{16}$$

When the upperscript is preceded by a minus sign, the corresponding term is deleted when the upperscript is made zero. The coefficients of the results will be determined from:

$$C_n{}^M=\sum_{K=0}^{M}\sum_{l=0}^{n}A_{n-l}{}^{M-K}B_l{}^K \tag{17}$$

$$C_n{}^{-M}=\sum_{K=1}^{M-1}\sum_{l=0}^{n}A_{n-l}{}^{K-M}B_l{}^{-K} \tag{18}$$

$$D_n{}^M=\sum_{K=1}^{\infty}\sum_{l=0}^{\infty}\sum_{j=0}^{K}\frac{(-1)^jK!}{j!(K-j)!} [A_{n-l-2j}{}^{K+M}B_l{}^{-K}+A_{n-l-2j}{}^{-K}B_l{}^{K+M}] \tag{19}$$

$$D_n{}^{-M}=\sum_{K=1}^{\infty}\sum_{l=0}^{\infty}\sum_{j=0}^{K}\frac{(-1)^jK!}{j!(K-j)!} [A_{n-l-2j}{}^K B_l{}^{-K-M}+A_{n-l-2j}{}^{-K-M}B_l{}^K] \tag{20}$$

Equations (18) and (20) are not used for $M=0$. The upper limits of the first two summation in (19) and (20) have been left undefined. The summation ends when it generates terms which are not part of the representation.

In the meteorological equations, the various operations are performed without any truncation. The calculations proceed until they generate the partial time derivative of a true scalar. At this point, the array of coefficients is truncated by making the truncation errors orthogonal with each of the terms retained for further calculations. This process uses the functions

$$H_n{}^m=\sum_{j=0}^{\infty}\frac{(-1)^j n!\left(M+n-j-\frac{1}{2}\right)!}{4^j\left(M+n-\frac{1}{2}\right)!\,j!\,(n-2j)!}G_{n-2j}{}^m \quad (21)$$

These functions have the following property:

$$\overline{H_n{}^m H_l{}^k}=0 \quad \text{if } m\neq k \text{ or } n\neq l$$
$$\overline{H_n{}^m H_n{}^m}\neq 0 \quad (22)$$

where the "bar" represents the integral over the entire surface of the earth.

$$\overline{F}=\int_{-\pi/2}^{+\pi/2}\int_0^{2\pi}F\cos\varphi\,\delta\lambda\,\delta\varphi \quad (23)$$

The elimination of a term $G_n{}^m$ is achieved by replacing this term by $G_n{}^m-H_n{}^m$. The highest order term in this last expression is $G_{n-2}{}^m$ as may be seen from (21). This technique is always applied to the highest value of n which is then reduced by two units. It is repeated until all the undesired terms have been eliminated. It should be noted that the functions $H_n{}^m$ are the spherical harmonics presented in a different form.

The various operations described in this section are required for the integration of the meteorological equations. Each variable is represented by a truncated double series in accordance with (10). The derivatives of a meteorological variable are calculated by using (11) and (12) and the product of two variables requires the application of (17), (18), (19) and (20). Any particular term in the meteorological equations may be calculated by using the spectral operations described in this section and the result represented by a double series of the functions $G_n{}^m$. These functions do not form an orthogonal system but this is unimportant provided that the orthogonal functions $H_n{}^m$ are used to truncate any series that contains too many terms.

Since each variable is represented by a double series of the functions $G_n{}^m$ and since each term in the meteorological equations is calculated separately and represented by a double series, this implies that the calculation of each time derivative requires a long succession of spectral operations. An advantage of the successive spectral operations resides in the fact that they eliminate the need for an explicit statement of the primitive equations in their spectral form. Also, this method permits the calculation of the truncation errors and this facilitates the assessment of their relative importance.

3. The modeling equations

The complete meteorological equations must form the basis from which all studies of atmospheric phenomena should start. The physicists discovered these laws and tested them over a much wider range of applications than we can expect to encounter in the atmosphere. It is true that we cannot incorporate into these equations all the features of the atmosphere and its boundaries, but this has not been the main handicap over the past few years. Most difficulties stemmed from the numerical procedures used to integrate the meteorological equations. The development of improved finite difference approximations contributed considerably to solve the simulation problem.

Functional representations of the meteorological variables require different integration procedures which generate errors of a different nature. The present experiment will simply show that the method discussed in the preceding section works successfully and gives adequate results. Comparisons with the grid point method are not possible at this stage and will have to be delayed to a later date. The spectral method is applied to the primitive meteorological equations with the vertical structure specified by five levels. In order to avoid the initial value problem, the model is applied to a rather simple general circulation experiment. The model uses the following set of equations expressed in pressure coordinates.

$$\frac{du}{dt}-2\Omega v\sin\varphi-\frac{uv}{a}\tan\varphi=-\frac{\partial\phi}{\partial x}+\alpha\frac{\partial}{\partial p}\left(p^2\frac{\partial u}{\partial p}\right) \tag{24}$$

$$\frac{dv}{dt}+2\Omega u\sin\varphi+\frac{u^2}{a}\tan\varphi=-\frac{\partial\phi}{\partial y}+\alpha\frac{\partial}{\partial p}\left(p^2\frac{\partial v}{\partial p}\right) \tag{25}$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}-\frac{v}{a}\tan\varphi+\frac{\partial\omega}{\partial p}=0 \tag{26}$$

$$\frac{dT}{dt}-\frac{RT\omega}{pC_p}=\gamma(T_E-T) \tag{27}$$

$$\frac{\partial\phi}{\partial p}=-\frac{RT}{p} \tag{28}$$

where:

u and v are the horizontal components of the wind.

$\omega=\frac{dp}{dt}$

T is temperature.

p is pressure.

ϕ is the geopotential.

a is the radius of the earth.

R is the gaz constant for dry air.

C_p is the heat capacity at constant pressure.

α is an eddy viscosity coefficient.

γ is a net radiative cooling coefficient.

T_E is a radiative equilibrium temperature.

The following boundary conditions are used

$$\omega=0 \quad \text{at} \quad p=0$$

$$w=\frac{dz}{dt}=0 \quad \text{at} \quad z=0 \tag{29}$$

The experiments performed by Mintz (1964) indicate that the absence of topography will not affect significantly some aspects of the results. Topography is not used in the present model in order to insure the presence of transient planetary modes exclusively. The lower boundary condition adds another predictive equation to the system.

$$\frac{\partial\phi_S}{\partial t}+u_S\frac{\partial\phi_S}{\partial x}+v_S\frac{\partial\phi_S}{\partial y}-\frac{RT_S w_S}{p_S}=0 \tag{30}$$

The subscript S designates the lower boundary assumed to be at 1000 mb in the above equation. Additional boundary conditions are required for the frictional stresses.

$$\frac{\partial u}{\partial p}=\frac{\partial v}{\partial p}=0 \quad \text{at} \quad p=0$$

$$\left(\frac{\partial u}{\partial p}\right)_S=-\frac{\varepsilon u_S}{p_S} \qquad \left(\frac{\partial v}{\partial p}\right)_S=-\frac{\varepsilon v_S}{p_S} \tag{31}$$

where ε is a surface drag coefficient. The following values were assigned to the two constants related to the eddy viscous stresses.

$$\begin{array}{ll} \alpha=0.0002\ \text{hour}^{-1} & \varepsilon=40 \\ u_S=0.7\,u_9 & v_S=0.7\,v_9 \\ \omega_S=\omega_{10} & T_S=T_{10} \end{array} \tag{32}$$

These values would give a friction layer with a depth of 800 meters and a drag coefficient representative of a flat land area.

The vertical grid is given in Fig. 1, equations (24), (25) and (26) are applied at all the odd levels, equations (27) and (28) are applied at all even levels except the top of the atmosphere. Centered finite difference approximations are used to evaluate the vertical derivatives whenever possible. If a variable is required at an intermediate level, it is obtained by simply averaging the level immediately above with the level immediately below.

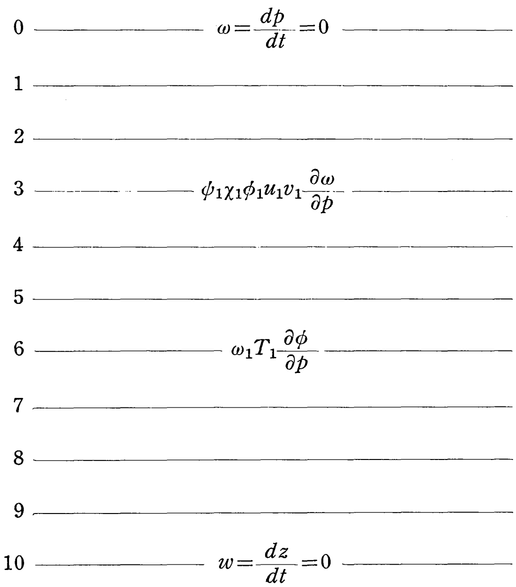

Fig. 1. The vertical grid used for the representation of the meteorological variables. The evenly spaced levels are 100 millibars apart.

The calculation of the cooling rates is based on the following parameters

$$\gamma=0.002\,\text{hour}^{-1}$$
$$(T_E)_2=203-\sin\varphi-7\sin^2\varphi$$
$$(T_E)_4=251-7\sin\varphi-51\sin^2\varphi$$
$$(T_E)_6=279-18\sin\varphi-67\sin^2\varphi$$
$$(T_E)_8=295-26\sin\varphi-73\sin^2\varphi$$
$$(T_E)_{10}=315-32\sin\varphi-86\sin^2\varphi \tag{33}$$

Because of the presence of dissipative terms the ordinary integration using centered time steps was unstable. A slight modification of the procedure was used in this case.

$$F^*(t+\Delta t)=F(t-\Delta t)+2\Delta t\left(\frac{\partial F}{\partial t}\right)^* \tag{34}$$

$$F(t)=F^*(t)+0.01[F^*(t+\Delta t)+F(t-\Delta t)-2F^*(t)] \tag{35}$$

The star represents a preliminary value and the absence of a star represents a final value. As indicated by (34) all time steps use centered finite differences except the first one where forward differences are used. The application of (35) starts after the second time step. The scheme described above acts as a weak filter and it was the only filter used in the model. The properties of (34) and (35) are discussed in detail in a report by the author (1965). A time step of 20 minutes was used for the integration.

At the beginning of a time step, only true scalars are available, namely the stream function ψ, the potential function χ, and then ω, T and ϕ. The horizontal components of the wind are then calculated. For the integration of (24) and (25) the following relations indicate that there will be no difficulty with divisions by $\cos^2\varphi$

$$\frac{du}{dt}-\frac{uv}{a}\tan\varphi=\frac{1}{\cos\varphi}\frac{d}{dt}(u\cos\varphi) \tag{36}$$

$$\frac{dv}{dt}+\frac{u^2}{a}\tan\varphi=\frac{1}{\cos\varphi}\frac{d}{dt}(v\cos\varphi)+\frac{(u^2+v^2)}{a}\tan\varphi \tag{37}$$

In these equations, $u\cos\varphi$ and $v\cos\varphi$ are true scalars so that (8) and (9) apply to the non-linear terms of the total derivatives of the right hand side and the last term of (37) may be written as

$$u^2+v^2=\nabla_H\chi\cdot\nabla_H\chi+\nabla_H\psi\cdot\nabla_H\psi+2\nabla_H\psi\times\nabla_H\chi\cdot\boldsymbol{K} \tag{38}$$

and here also (8) and (9) apply.

The predictive equations are used to evaluate the local time derivatives. The local time derivatives of vorticity and divergence are obtained from the local time derivatives of u and v. All local time derivatives of true scalars are then truncated to their original size by using the spherical harmonics defined in (21). Forecasts of vorticity and divergence are then obtained and finally converted into forecasts of ψ and χ. This method has the advantage that it uses the same truncation process for all the variables. Horizontal vector components appear at intermediate stages of the calculations. The attempt made by Kubota to avoid this occurence produces very cumbersome equations. Allowing horizontal vector components to appear at intermediate stages clearly simplifies the numerical procedure.

The model described above is sufficiently elaborate to test the applicability of the spectral method, but it does not contain, at present, all the features required for a high quality general circulatic experiment.

4. Results

The integration was started from an atmosphere at rest and at a uniform temperature of 280°K. All the coefficients with indices in the following range were used

$$-2\leqslant m\leqslant 2$$
$$0\leqslant n\leqslant 2 \tag{39}$$

giving a set of 15 permissible coefficients for each variable. The integration was carried over a period of 200 days and then at this point the resolution was increased to

$$-4\leqslant m\leqslant 4$$
$$0\leqslant n\leqslant 4 \tag{40}$$

giving a set of 45 permissible coefficients for each variable. The integration was then continued for another 22 days. The north south cross-section of the mean zonal flow obtained at the end of this period is presented in Fig. 2. The zonal flow has been averaged around latitude circles only. The

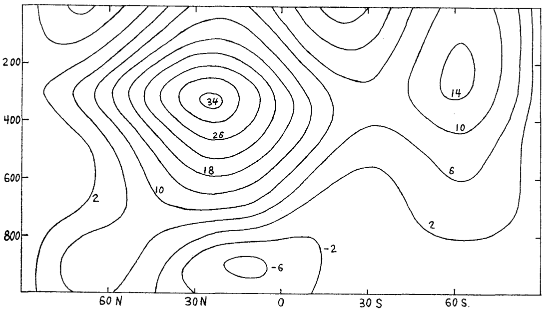

Fig. 2. The zonal wind in m. sec.$^{-1}$ averaged around latitude circles as a function of latitude in degrees and pressure in millibars at the end of the 222 day integration.

two jet streams and the equatorial easterlies present reasonably realistic characteristics. The parameters used in the model represent the month of January, this is evidenced in the cross-section with the stronger stream appearing in the northern hemisphere and at low latitudes. The mean zonal flow did not show very significant variations during the last 10 days of the integration indicating that the model atmosphere reached a form of dynamic equilibrium at this stage.

The integration of the meteorological equations in terms of spherical harmonics is particularly well adapted for studies of the behaviour of planetary waves. In the atmosphere, these waves are either stationary

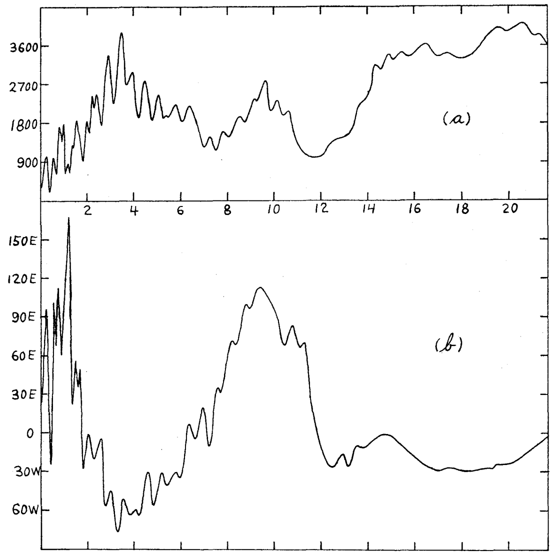

Fig. 3. The coefficients of the terms with $M=2$ and $n=2$ for the geopotential at 500 mb as a function of time in days. (a) amplitude in m^2 sec^{-2} (b) phase angle in degrees longitude.

or move very slowly. The work of Eliasen and Machenhauer (1965) establishes the presence in the atmosphere of planetary waves with relatively high phase speeds, but their amplitudes are not significant. Barotropic and baroclinic models require empirical corrections to slow down the retrogression of the longest waves, but this should no longer

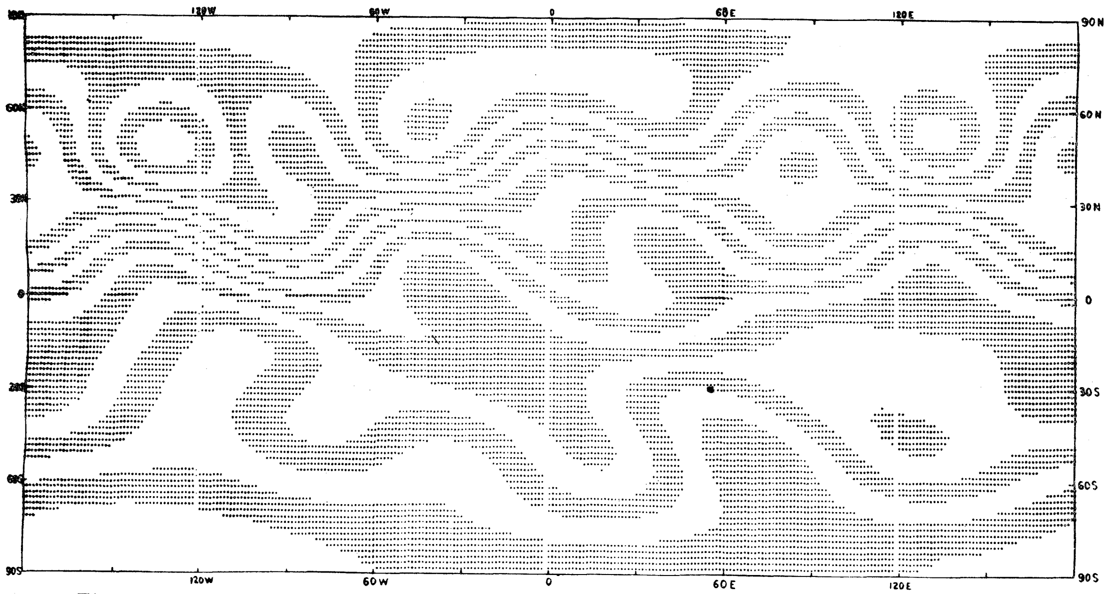

Fig. 4. The 500 mb stream function at the end of the 222 day integration. The abscissa is longitude and the ordinate is latitude.

be necessary in primitive equations models. The time series of Fig. 3 indicate that both amplitude and phase fluctuate sensibly, but the wave shows no net tendency to move either westward or eastward over the period of 22 days even in the absence of topographical effects. Other planetary waves gave similar results. The fluctuations in amplitude and phase decrease considerably towards the end of the integration indicating that the fluctuations were probably caused by the increase in resolution.

The resolution used in the model is not high enough to give realistic mean meridional circulation cells. Charts of the geopotential or the stream function (Fig. 4) lack in detail and because of the absence of topography, they have no characteristics that could be attributed to particular regions of the earth. These shortcomings did not prevent the simulation from being realistic on the planetary scale and within the limitations mentioned above this experiment has been a successful test of the spectral method.

5. Conclusions

Spherical harmonics or the elements that constitute these functions may be used successfully to integrate the complete meteorological equations. There are no serious difficulties associated with this method. An extension to a three-dimensional functional representation should logically follow. A functional representation in the vertical coordinate would have to account for the presence of boundaries, but this does not appear to be a fundamental difficulty. A completely three-dimensional spectral representation would require finite difference approximations in the local time derivatives only and the truncation errors in this case would be more easily controlled.

Spectral forms of the meteorological equations may never compete successfully with the grid point method for the operational production of weather forecasts, but there are a number of problems that could use functional representations advantageously. General circulation experiments and extended forecasts do not require a very high resolution and for this reason might be treated more effectively with spherical harmonics.

In the immediate future, comparisons between the spectral method and the grid point method will provide a considerable amount of information about the relative merits of each method. It is only after a series of comparisons that it will be possible to decide which method should be used for any particular group of experiments.

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Low order spectral 形式による primitive 方程式の積分について

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カナダ気象局

この論文では，大気の運動方程式・熱力学の式の球函数展開による数値実験についての議論が行なわれる。球函数による時間積分は格子点による時間積分にかわる興味ある方法である。

静止・等温の初期条件から出発する5層モデルは，球函数の 15 成分をとって200日間時間積分され，さらにそれにつづく 22 日間については 45 成分を使用して積分が行なわれた。

数値実験の結果，両半球の jet stream および赤道に沿う下層の偏東風が示された。数値実験で得られた planetary wave の振幅・位相速度は，実際の大気のそれらとよく対応していた。この数値実験の結果は，大気大循環の数値実験に球函数の使用が有利である事を示している。