Equivalent Radar Reflectivity Factors for Snow and Ice Particles

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From time to time, questions arise concerning how to compare radar observations of snow or ice particles with corresponding particle size data or how to estimate snowfall from radar observations. Weather radar systems customarily measure the equivalent radar reflectivity factor Z_e, so any calculations involving particle sizes must ultimately be expressed in terms of Z_e. Persons skilled in radar meteorology can (and normally do) work out correct procedures, but standard works on radar meteorology (e.g., Battan, 1973) do not discuss the matter explicitly. The purpose of this note is to clarify the situation for the benefit of those who may encounter difficulty with such problems.

1. Relationships between Z and Z_e

Both physical factors and accepted conventions complicate the subject, so it is best to work from the basic definitions. The radar reflectivity factor Z for rain can be expressed in terms of raindrop sizes as

$$Z = \sum D^6/V_c = Z_e. \tag{1}$$

Here D represents the drop diameter and the summation must be carried out over all the drops in the radar contributing region of volume V_c. The last part of (1) indicates that, for spherical drops with diameters small compared to the radar wavelength, the equivalent radar reflectivity factor Z_e is equal to Z.

For snowflakes or other ice particles, the fact that the particle shapes are generally far from spherical complicates matters. That difficulty can be handled, according to Marshall and Gunn (1952), by noting that for particles small enough to fall in the Rayleigh scattering region (which requirement is reasonably well fulfilled at the usual weather radar wavelengths by snowflakes and small ice particles), the radar cross section of an irregular particle composed of a weak dielectric like ice is the same as that of a sphere of the same mass. In other words, the exact shape of the particle is immaterial. The general absence of substantial depolarization (apart from that due to propagation effects) in radar echoes from snow or ice particles (e.g., Hendry and Antar, 1981) corroborates this idea.

One can therefore write the radar cross section of such an ice particle as

$$\sigma = \pi^5 |K|_i^2 D^6/\lambda^4. \tag{2}$$

(This and the other basic expressions used here can be found in Chapter 4 of Battan, 1973). Here $|K|^2$ is the dielectric factor $|(\epsilon_r - 1)/(\epsilon_r + 2)|^2$, ϵ_r being the relative permittivity. (It is related to the index of refraction n by $\epsilon_r = n^2$.) The subscript i indicates that a value appropriate for ice should be used. The diameter implied in (2) is that of a sphere having the same mass as the particle in question.

The radar reflectivity η (or radar cross section per unit volume) of the array of particles in the radar contributing region of volume V_c is

$$\eta = \sum_{V_c} \frac{\sigma}{V_c} = \frac{\pi^5 |K|_i^2}{\lambda^4} \sum_{V_c} \frac{D^6}{V_c}. \tag{3}$$

The equivalent radar reflectivity factor Z_e is defined as

$$Z_e = \frac{\lambda^4 \eta}{\pi^5 |K|_w^2}, \tag{4}$$

where the subscript w indicates that the value appropriate for water (approximately 0.93 for the usual meteorological radar wavelengths) is used by convention. That convention is adopted because when radar measurements are made, one is often not certain whether the particles are water or ice. (Moreover, particles of each type frequently occur in different parts of the scanned volume.) Substituting the value of η from (3) into (4),

For ice particles:

$$Z_e = \frac{|K|_i^2}{|K|_w^2} \sum_{V_c} \frac{D^6}{V_c}. \tag{5}$$

When particle size data are analyzed to determine radar variables, the quantity usually calculated is the radar reflectivity factor Z and not the equivalent radar reflectivity factor Z_e. The analysis yields values of

$$\sum_{V_s} \frac{D^6}{V_s} \approx \sum_{V_c} \frac{D^6}{V_c} = Z, \tag{6}$$

where V_s is some sampling volume, much smaller than V_c. Comparing (5) and (6), one can see that

For ice particles:

$$Z_e = \frac{|K|_i^2}{|K|_w^2} Z. \tag{7}$$

2. Values for the dielectric factor

The next point concerns the appropriate value for $|K|_i^2$ in (7). To determine that requires recognition of an artifice, first introduced by Marshall and Gunn (1952), which has become a generally used convention in analyzing ice particle size data. They determined the size of each snowflake by melting it and measuring the diameter of the resulting water drop. This diameter is smaller than that of the ice sphere of mass equivalent to the original particle, by a factor $0.92^{1/3}$ (0.92 being the specific gravity of solid ice). If the melted diameter were used in (2) with the actual dielectric factor for ice, which has the value 0.176, the calculated radar cross section of the particle would be too small by the factor $(0.92)^2 = 0.846$. The Marshall and Gunn artifice consists of multiplying the true dielectric factor by the quantity $(\frac{1}{0.846}) = 1.18$, resulting in the value 0.208.[1] Then the melted drop diameter can be used with that value for $|K|_i^2$ in (2) to obtain the correct radar cross section for the particle. Of course, the same result could be obtained by using 0.176 for the dielectric factor and adjusting the melted drop diameter by the factor $0.92^{-1/3} = 1.028$.

Consequently, there are two possible "correct" values of $|K|_i^2$ in the foregoing equations, depending upon how the particle sizes are determined. If, in calculating Z the particle sizes used are melted drop diameters, as in the work of Gunn and Marshall (1958), Sekhon and Srivastava (1970), and others, the appropriate value for $|K|_i^2$ is 0.208 and

$$Z_e = 0.224Z. \tag{8}$$

In logarithmic form, this becomes

$$Z_e \text{ (in dBz)} = Z \text{ (in dBz)} - 6.5 \text{ dB}. \tag{9}$$

If, on the other hand, the particle sizes are expressed as equivalent ice sphere diameters, the appropriate value for $|K|_i^2$ is 0.176 and

$$Z_e = 0.189Z. \tag{10}$$

In logarithmic form (10) becomes

$$Z_e \text{ (in dBz)} = Z \text{ (in dBz)} - 7.2 \text{ dB}. \tag{11}$$

To determine the dielectric factor $|K|_i^2$, Marshall

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[1] The incorrect value 0.197 appeared in the original Gunn and East (1954) paper. A corrigendum was published in Vol. 81 of *Quart. J. Roy. Meteor. Soc.* (p. 653), but the erroneous value has been perpetuated by Battan (1973) and other authors.

and Gunn (1952) used a theory originated by Debye to calculate the relative permittivity of the ice–air mixture making up a particle. The subject of the dielectric properties of mixtures has recently received considerable attention (cf. Evans, 1965; Bohren and Battan, 1980, 1982) and questions have been raised about the applicability of the Debye theory to snow. In fact, it may not be possible to calculate exact values for the permittivity of heterogeneous mixtures (de Loor, 1983), although limiting boundaries can be specified. Nevertheless, the expression given as Eq. (10) in Bohren and Battan (1980) agrees better with experimental data than the Debye function, and the two can be compared to illustrate the magnitude of possible differences.

The former expression gives values for the relative permittivity of ice–air mixtures which are, at most, about 5% higher than the values obtained from the Debye function. The difference varies with the composition of the mixture, being zero when the "mixture" is either 100% air or 100% ice and reaching the maximum at an ice fraction of around 60%. The corresponding differences in the dielectric factor $|K|_i^2$ are larger, reaching a maximum of 18% at an ice fraction of about 45%. This implies that the equivalent radar reflectivity factors could be as much as 0.7 dB higher than those indicated above. Because of uncertainties in the theory and the fact that the difference varies with the composition of the mixture (which is seldom known for individual particles and probably varies over the array of particles within the radar contributing region), no more exact value can be given.

3. Z_e–R relationships for snow

As noted in the opening paragraph, weather radar systems are customarily calibrated to measure the "water equivalent" Z_e defined by (4) with $|K|_w^2 = 0.93$. The dielectric factor is incorporated into a radar calibration constant, and that constant is not altered when the precipitation form changes from liquid to solid. This means that some care must be used in employing published snow Z–R relationships derived from particle size observations.

To illustrate this, consider the snow Z–R relationship obtained by Sekhon and Srivastava (1970):

$$Z = 1780R^{2.21} \tag{12}$$

with Z in $\text{mm}^6 \text{ m}^{-3}$ and R in mm h^{-1}. In logarithmic form,

$$Z \text{ (in dBz)} = 32.5 + 22.1 \log R. \tag{13}$$

The snowflake size data used by Sekhon and Srivastava were melted diameters, so (8) or (9) is the appropriate relationship between Z_e and Z. Thus, the Sekhon and Srivastava result corresponds to

$$Z_e \text{ (in dBz)} = 26 + 22.1 \log R. \tag{14}$$

This would be an expression appropriate for estimating snowfall rates from radar measurements.

Table 1 compares equivalent radar reflectivity factors calculated for precipitation rates of 1 and 10 mm h^{-1} for rain, using the Marshall-Palmer relationship

$$Z_e = 200R^{1.6}, \tag{15}$$

and for snow, using (14). One should remember that here the precipitation rates for snow have to be expressed in terms of melted water equivalents. At $R = 1$ mm h^{-1}, the Z_e value for snow is 3 dB higher that that for rain. Two main factors contribute to the difference, in opposite senses. One is that ice is a weaker dielectric than water, which tends to reduce the reflectivities for snow. But the fall speeds of snowflakes are lower than those of raindrops, so the other factor is that larger sizes or greater concentrations of snowflakes are needed to achieve the same precipitation rate; that tends to increase the reflectivities. At 1 mm h^{-1}, the latter factor is evidently dominant.

The more rapid increase of Z_e for snow between 1 and 10 mm h^{-1} reflects the tendency for increased precipitation rates in snow to be associated with aggregation into larger flakes. In fact, the snowflake number concentration tends to decrease as R increases, whereas the opposite is true for the raindrop concentration. The larger aggregates have correspondingly greater radar cross sections because of the D^6 factor in (2). That leads to an exponent higher in (12) than in (15), so that Z_e increases more rapidly with R for snow than for rain.

Statements are frequently made to the effect that radar echoes from snow are weaker than those from rain. Sometimes the difference is attributed to the weaker dielectric properties of ice. The foregoing discussion shows that the extent to which such statements are true must reflect a tendency for the precipitation rates to be generally lower in snow more than any factor related to the scattering properties of the individual hydrometeors.

4. Concluding remarks

This note is intended to aid in comparing radar and particle-size observations of snow or ice particles, or in using radar to measure snowfall. The main objective has been to clarify the differences between

TABLE 1. Example values of R and Z_e for rain and snow.

	Precipitation rate R (mm h^{-1})	
	1	10
Z_e (rain) — dBz	23	39
Z_e (snow) — dBz	26	48

Z and Z_e for those situations, so that the calculations based on particle-size data can be correctly performed. The results apply for dry snowflakes or small ice particles only; for wet particles or sizable hailstones, the Marshall and Gunn argument mentioned in Section 1 is not valid. The treatment of those situations is more complicated.

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