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On the critical Richardson number in stably stratified turbulence

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Abstract

The critical Richardson number, Ri_c, is used in studies of stably stratified turbulence as a measure of flow laminarization. The accepted range of Ri_c is between 0.2 and 1. A growing body of experimental and observational data indicates, however, that turbulence survives for $Ri \gg 1$. This result is supported by a new spectral theory of turbulence that accounts for strong anisotropy and waves. The anisotropization results in the enhanced horizontal mixing of both momentum and scalar. Internal wave contribution preserves vertical momentum mixing above its molecular level. In the absence of laminarization, Ri_c becomes devoid of its conventional meaning. Copyright © 2007 Royal Meteorological Society

Keywords: turbulence; stable stratification; critical Richardson number

The gradient Richardson number, Ri, defined as

$$Ri = \frac{N^2}{S^2} \tag{1}$$

is a measure of the relative strength of the density gradient in stably stratified shear flows. Here, $N^2 = -g(\partial\rho/\partial z)/\rho_0$ is the square of the buoyancy, or Brunt–Väisälä frequency, g is the gravity acceleration, ρ is the fluid density, ρ_0 is the reference density, S is the vertical shear of the horizontal velocity, and z is the vertical coordinate. The critical Richardson number, Ri_c, appears in the classical papers by Miles (1961) and Howard (1961) as a linear stability threshold of steady, two-dimensional, stably-stratified, horizontal shear flows of an ideal Boussinesq fluid. Namely, these flows are stable if $Ri > Ri_c = 1/4$ everywhere in the fluid. Abarbanel *et al*. (1984) have considered three-dimensional non-linear deformations of a planar shear flow. They established that a formal stability of this flow requires the local Richardson number defined by variations across surfaces of constant density, i.e., isopycnals, to be greater than unity. Results by Miles (1961), Howard (1961) and Abarbanel *et al*. (1984) have highlighted the importance of the range $Ri \in [1/4, 1]$ for stability of stably stratified flows and called for further theoretical and experimental investigations of this range.

When the basic flow is time-dependent, its stability analysis becomes more complicated. Drazin (1977) has shown that a propagating unidirectional internal gravity wave is unstable for large local values of Ri. Majda and Shefter (1998) have constructed a family of explicit, elementary, stably-stratified, time-dependent and non-parallel flows that are unstable for all arbitrarily large values of Ri. The effect of viscosity in simple shear flows was considered by Gage (1971).

The increase of the Reynolds number causes non-stationarity of the basic field, bifurcations and transition to turbulence, and the analysis of the effect of stable stratification in this case needs to take into account the increasing complexity. Laval *et al*. (2003) considered the effect of the Reynolds number, $Re = VL_h/\nu$, on forced, stably-stratified shear layers (here, V and L_h are the horizontal velocity and length scales, respectively, and ν is the kinematic viscosity). In order to be able to deal with shearless, freely decaying flows, Laval *et al*. (2003) also used a Froude number defined as $Fr = V/NL_v$, where L_v is a vertical length scale. Using the Kolmogorov scaling for the rate of viscous dissipation ϵ, $\epsilon = V^3/L_v$, one can re-define the Froude number as $Fr = \epsilon/NK$, where $K = V^2/2$ is the turbulence kinetic energy. An increasing strength of stable stratification corresponds to decreasing Fr. With increasing Re, a simulated flow underwent a series of successive transitions to states with increasing anisotropy dominated by horizontal pancake-like vortices. These vortices, however, could become unstable with increasing Re. The study concluded that "for any Froude number, no matter how small, there are Reynolds numbers large enough so that a sequence of transitions to nonpancake motions will always occur and, conversely, for any Reynolds number, no matter how large, there are Froude numbers small enough so that these transitions are suppressed."

From these results, one can surmise that the adaptation of the critical Richardson number to turbulent flows is not straightforward. Since turbulence is an unstable, stochastic phenomenon, it lacks a well-defined basic state whose stability can be analyzed. However, Ri_c can be introduced phenomenologically, based upon a loose analogy with laminar flows. The Glossary of Meteorology of the American Meteorological Society defines the critical Richardson number as "the value of the gradient Richardson number below which air becomes dynamically unstable and turbulent. This value is usually taken as $Ri_c = 0.25$, although suggestions in the literature range from 0.2 to 1.0. There is also some suggestion of hysteresis, where laminar air flow must drop below $Ri = 0.25$ to become turbulent, but turbulent flow can exist up to $Ri = 1.0$ before becoming laminar." Similarly, Andreas (2002) suggests that "in stable conditions, turbulence is presumed to cease and the flow becomes laminar when the Richardson number exceeds a critical value Ri_c" and Canuto (2002a) proposes a definition of Ri_c as a value of Ri "above which there is no longer turbulent mixing" (see also Canuto (1998)). He argued that Ri_c may be larger than unity if the effects of the radiative losses and gravity waves are taken into account. Applying these results in the case of a stellar convection, he argued that the use of $Ri_c \sim 1$ would lead to substantially enhanced estimates of mixing in the convection zone and radical change of the conclusions reached in studies by Garcia-Lopez and Spruit (1991) and Schatzman *et al.* (2000). Majda and Shefter (1998) discuss examples of a similarly restrictive usage of the linear stability criterion in meteorological and oceanographic applications. One of them is the Lilly-Smagorinsky model of the atmospheric/oceanic circulation (Smagorinsky, 1963; Lilly, 1967) which assumes that the eddy diffusivity is completely turned off for $Ri \geq Ri_c \geq 1/4$, $Ri_c = O(1)$. In another study, Martin (1985) demonstrated that simulations of the stably stratified oceanic mixed layer can only replicate observations when $Ri_c \sim 1$. To attain higher values of Ri_c in the framework of the Reynolds stress (RS) turbulence closure modeling, Canuto suggested and implemented a formalism that accounts for the combined effect of turbulence and internal waves and resolves the dichotomy of shear- and gravity wave-dominated scenarios (Canuto, 2002a,b). Studies by Monin and Yaglom (1975), Yamamoto (1975) and Lettau (1979) go even further and suggest that the critical Richardson number does not exist at all.

A strong impetus for utilization of the critical Richardson number criterion in models of atmospheric/oceanic/planetary circulation emanates from the use of RS models for parameterization of small-scale turbulent mixing. The RS models make an analytical prediction that at some value of the gradient Richardson number (it can be different for different models but usually lies in the range between 0.1 and 1), the eddy viscosity and eddy diffusivity fall to zero thus signaling the termination of turbulent mixing (see e.g. Mellor (1973); Mellor and Yamada (1982) and Canuto *et al.* (2001)). The constancy of Ri_c is a result of the rigidity of the RS models' coefficients dictated by the principles of the invariant modeling (Lewellen, 1977). In a more flexible RS model such as, for instance, that by Ristorcelli (1997), the coefficients are functions of the Reynolds stress tensor invariants and Ri_c would also depend on these invariants. Clearly, the dependence of Ri_c on the choice of a model undermines the utility of this parameter.

The anisotropization of stably stratified turbulence (see e.g. Kimura and Herring (1996) and Laval *et al.* (2003)) further complicates the use of the critical Richardson number. The horizontal pancake-like vortex structure of the flow can be expected to enhance the horizontal mixing of momentum and scalars even when the vertical mixing is largely suppressed. Indeed, a strong isopycnal (i.e., along constant density surfaces) mixing has been observed in oceanic flows with strong stable stratification (Polzin and Ferrari, 2004). Since Ri involves vertical gradients only, it cannot reflect anisotropization and enhanced horizontal mixing.

The discussion so far pertained to the values of Ri calculated in terms of the mean fields. The Reynolds averaging implied in this approach lumps together processes on different scales and provides no information on the spectral aspects of the effect of stable stratification on turbulence. This information is useful for further elucidation of the physical meaning of Ri_c and its role in characterization of such processes as the interaction between turbulence and internal gravity waves. A spectral theory of turbulent flows with stable stratification was developed by Sukoriansky *et al.* (2005, 2006). This theory is based upon a quasi-normal mapping of the velocity and temperature fields using the Langevin equations for every Fourier mode. These equations reflect strong nonlinear interactions between the modes which cause stirring and damping of every mode. The quasi-normality of the forcing is assumed to be a result of random interactions between a large number of independently fluctuating modes. The Langevin equations are instrumental in the development of a self-consistent procedure of successive elimination of small-scale modes (i.e., ensemble averaging over these modes; hence the abbreviation of the theory – QNSE – quasi-normal scale elimination) that yields scale-dependent, horizontal and vertical eddy viscosities and eddy diffusivities. Note that the assumption of quasi-normality is the basic (closure) hypothesis of the QNSE theory; all other results can be derived analytically from primitive equations of motion in Boussinesq approximation. Among these results are the Kolmogorov spectrum for neutrally stratified flows including the numerical value of the Kolmogorov constant. Numerical experiments indicate that for stably stratified turbulence, the hypothesis of quasi-normality holds even better than for neutral flows (Métais and Herring, 1989; Kimura and Herring, 1996).

The QNSE scale elimination process starts at the smallest scales of the system around the Kolmogorov scale for viscous dissipation, $k_d = (\epsilon/\nu_0^3)^{1/4}$, where ν_0 is the molecular viscosity. Elimination of a small

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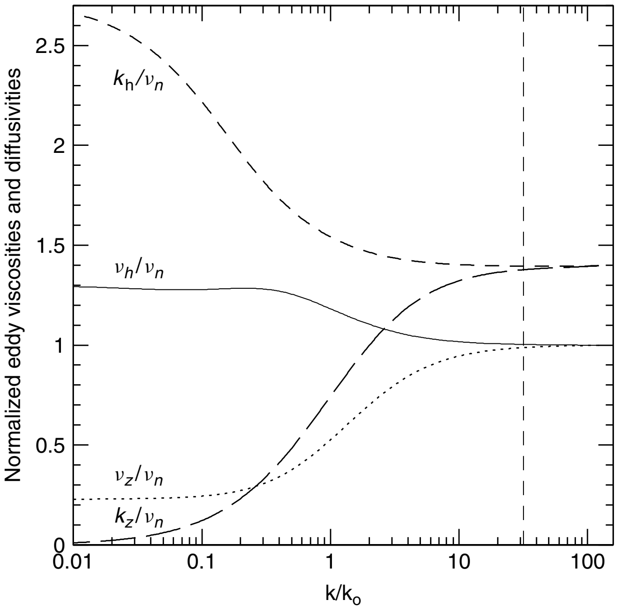

Figure 1. Horizontal and vertical eddy viscosities and eddy diffusivities normalized with the corresponding value of the eddy viscosity in the neutral case, $\nu_n \simeq 0.46\epsilon^{1/3}k^{-4/3}$, as functions of k/k_O. The dashed vertical line indicates the threshold of internal wave generation in the presence of turbulence (Sukoriansky *et al.*, 2005)

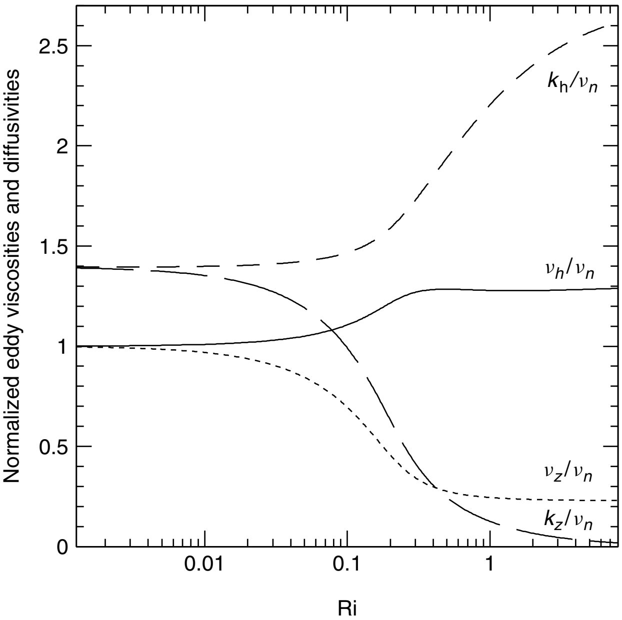

Figure 2. Horizontal and vertical eddy viscosities and eddy diffusivities normalized with ν_n as functions of *Ri*

shell of fast modes results in coarsening of the system's domain of definition and generation of small corrections to the viscosity and diffusivity. The scale elimination is repeated for successive shells of fast modes until an arbitrarily predetermined wave number k (a dynamic dissipation cutoff) is attained. Within this procedure, internal waves and turbulence naturally appear as one entity rather than an ad hoc dichotomy. The successive scale elimination process yields a system of four coupled ODEs for horizontal and vertical eddy viscosities, ν_h, ν_z, and diffusivities, κ_h and κ_z, respectively, as functions of k/k_O, $k_O = (N^3/\epsilon)^{1/2}$ being the Ozmidov wave number; its solution is shown in Figure 1. At the smallest scales, $k/k_O > 30$, eddy viscosities and eddy diffusivities are isotropic and behave similarly to the neutral case. At $k/k_O \sim 30$, two effects become noticeable: the appearance of internal waves and the beginning of turbulence anisotropization. As evident from Figure 1, ν_h, ν_z, κ_h and κ_z exhibit different tendencies with increasing stratification. While the vertical viscosity and diffusivity are reduced compared to their values in the neutral case, their horizontal counterparts are enhanced. There is also a significant difference between ν_z and κ_z: while the vertical eddy diffusivity becomes vanishingly small and approaches its molecular value, the vertical eddy viscosity remains finite even on the largest scales strongly dominated by stratification. The behavior of ν_h, ν_z and κ_h shows no turbulent-laminar transition on any scale, in contradiction with the conventional definition of the critical Richardson number in turbulence.

If the process of scale elimination is extended to the turbulence macroscale, one obtains an equivalent of a Reynolds-averaged model (Sukoriansky *et al.* 2005). In this case, expressions for ν_h, ν_z, κ_h and κ_z become functions of either the Richardson number, *Ri*, or the Froude number, $Fr = \epsilon/NK$. The former dependence is shown in Figure 2. One can notice an increase of the horizontal mixing (as compared to the mixing in neutral flows with the same ϵ and K) with increasing *Ri*, particularly for the scalar. Similar flow anisotropization has been observed in stratified momentum wakes (Spedding 1997, 2001) where the vertical spreading of the wake decreases while the horizontal spreading remains almost unaffected by stratification. Assuming the scales of the horizontal and vertical spreading of the wake proportional to the respective horizontal and vertical eddy viscosities (Meunier *et al.*, 2006), one can trace this phenomenon directly to the behavior of ν_h and ν_z shown in Figure 2.

With regard to the vertical mixing, both ν_z and κ_z decrease for $Ri > 0.1$. The diffusivity decreases faster than the viscosity, and for $Ri > 0.4$, κ_z becomes smaller than ν_z. Although at $Ri \sim 1$, κ_z falls to about 10% of its value under neutral stratification, it still remains well above the level of the molecular diffusivity, in contradiction to the Lilly-Smagorinsky model (Smagorinsky, 1963; Lilly, 1967). The faster decrease of the eddy diffusivity is well known from laboratory experiments and observations (Webster, 1964; Itsweire and Helland, 1989; Strang and Fernando, 2001; Mahrt and Vickers, 2006; Mauritsen and Svensson, 2007) and has been attributed to the tendency of internal waves to mix the momentum more effectively than a scalar (Monin and Yaglom, 1975; Mahrt, 1998; Yagüe *et al.*, 2006). Faster decrease of κ_z results in the increase of the turbulent Prandtl number, $Pr_t = \nu_z/\kappa_z$, with *Ri*. Figure 3 compares the analytical dependence $Pr_t^{-1}(Ri)$ calculated from the QNSE theory with various data. One is alluded not only to a good agreement between the theoretical and observational results but also to the presence of turbulent mixing for values of *Ri* far exceeding unity. This figure is in good agreement with the dependence $Pr_t \propto Ri$ that can be derived from the turbulence energy balance equation, $\nu_z S^2 - \kappa_z N^2 = \epsilon$, and the expression $\kappa_z \propto \epsilon N^{-2}$ obtained in stably stratified atmospheric (Weinstock, 1978) and oceanic (Osborn, 1980; Gregg *et al.*, 2003) flows for relatively large *Ri*. The linear dependency, $Pr_t \propto Ri$ for $Ri > 0.2$, has been used in some ocean circulation models to facilitate realistic results (Blanke and Delecluse, 1993; Meier, 2001). Note in this regard that almost all atmospheric and oceanic circulation models employ 'background' mixing coefficients which kick in when stratification is strong and eddy viscosities and eddy diffusivities become too small (see, e.g., Zhang and Steele (2007)). Usually, these coefficients are assumed constant but the simulation results are known to depend upon their values. Adopting *Ri*-dependent eddy viscosities and eddy diffusivities shown in Figure 2 may be a viable, physically sound alternative to the use of constant background mixing coefficients.

Concluding, we re-iterate that the extensive body of experimental, observational and theoretical results points to the fact that a single-valued critical Richardson number at which turbulence is totally suppressed and laminarized simply does not exist; in fact, turbulence can survive in flows with *Ri* far exceeding unity. Models that use Ri_c as a threshold of turbulence extinction often suffer from insufficient mixing. Rather than being quenched at high values of *Ri*, turbulence becomes strongly affected by internal waves and acquires structure of anisotropic pancakes with enhanced horizontal mixing. This mixing may be important in terrestrial, planetary and astrophysical systems, either on its own merit (Polzin and Ferrari, 2004; Rafkin *et al.*, 2001; Izakov, 2003) or in combination with the effect of differential rotation (Maeder and Meynet, 1996; Ligniéres *et al.*, 1999; Mathis *et al.*, 2004). Although conjectures about the absence of Ri_c have been made in the past (see e.g. Monin and Yaglom (1975); Yamamoto (1975); Lettau (1979)), only recent observational and theoretical studies have clarified that the effects of the nonstationarity, internal waves and strong anisotropization preclude laminarization of turbulence and thus make Ri_c devoid of its conventional meaning. It appears therefore that the use of Ri_c as a criterion of turbulence extinction is ill advised and should be avoided.

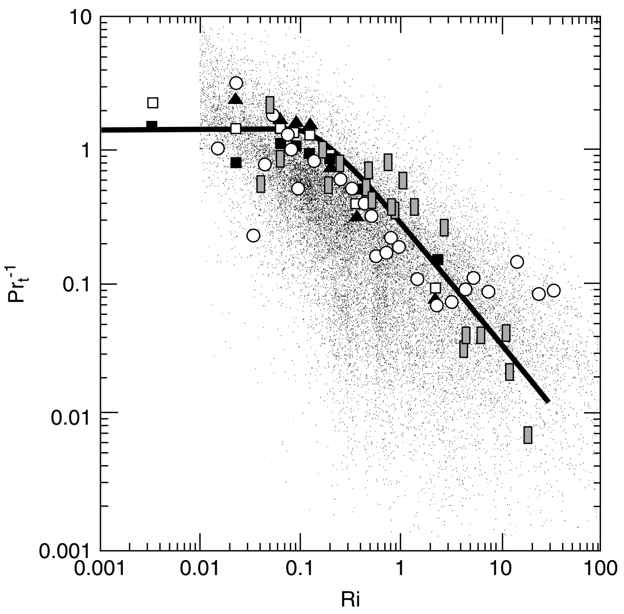

Figure 3. Inverse Prandtl number, $Pr_t^{-1} = \kappa_z/\nu_z$, as a function of *Ri*. Solid line shows theoretical prediction by QNSE theory with the correction described in Sukoriansky *et al.* (2006). Black and white squares and black tringles are data from Halley Base, Antarctica collected in 1986 (Yagüe *et al.*, 2001). White circles show data by Monti *et al.* (2002) and grey rectangles are data by Strang and Fernando (2001). Small dots show data from Halley Base collected by the British Antarctic Survey in 2003–2004

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