A TWO-SCALE MIXING FORMULATION FOR THE ATMOSPHERIC BOUNDARY LAYER

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Abstract. This study compares different simple mixing schemes for one-dimensional models and then focuses on the two-scale mixing approach. Two-scale mixing consists of local diffusion between adjacent grid levels and nonlocal mixing over the bulk of the boundary layer (nonlocal mixing). The latter represents nonlocal mixing by the boundary-layer scale eddies. A common example of two-scale mixing is the formulation of the turbulent heat transport in terms of an eddy diffusivity to represent small-scale diffusion and a "countergradient correction" to represent boundary-layer scale transport. Most existing two-scale approaches are applied to heat and moisture transport while momentum transport is simultaneously parameterized only in terms of a local diffusivity without nonlocal mixing. This study attempts to correct this inconsistency.

The resulting model is compared with Lidar observations of spatially averaged winds which are found to be superior to radiosonde and aircraft data for determining the mean structure. The two-scale mixing correctly predicts the observed well mixed conditions for momentum while the original model based on a local diffusivity for momentum fails to produce a well mixed state. Unfortunately, the "best" value for the adjustable coefficient in the nonlocal mixing part of the two-scale approach appears to depend on baroclinity in a way which can not be completely resolved from existing data.

1. Introduction

Within the heated boundary layer, eddies on the scale of the boundary layer dominate the vertical transport of heat and other quantities. These eddies normally assume the form of thermals or sheared-thermals and they transport properties across the bulk gradient of the boundary layer. The resulting heat flux may be counter to the local gradient of potential temperature in the middle and upper part of the boundary layer. The eddy diffusivity represents local diffusion by small-scale turbulence and cannot account for the transport by the larger scale eddies. For this reason, Priestley and Swinbank (1947) modified the usual eddy diffusivity formulation to include a correction due to boundary-layer scale transport. This correction was formalized in terms of free convection similarity theory by Deardorff (1972). Troen and Mahrt (1986) further generalized this formulation to include the effect of shear-driven mixing and applied the formulation to moisture transport as well. Holtslag and Moeng (1991) also modified this approach to accommodate bottom-up, top-down diffusion and to allow the nonlocal mixing coefficient to vanish smoothly with decreasing instability. Holtslag and Boville (1993) reformulated this smoothness condition in terms of readily available model variables. They explicitly show how the use of only an eddy diffusivity without a nonlocal correction leads to serious underestimation of the depth of mixing.

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We shall refer to the above nonlocal approaches as two-scale mixing formulations consisting of small-scale diffusion by an eddy diffusivity and boundary-layer scale mixing by a nonlocal term or gradient correction. Two-scale mixing formulations are physically oversimplified; however, they seem to be enjoying increasing application. Boundary-layer formulations with more complete physics require prohibitive resolution in large-scale models or lead to erratic behavior in one or more of the numerous meteorological situations encountered in global or operational models. In addition, existing observations of the momentum flux and geostrophic wind seem inadequate for definitive testing of more complicated schemes.

The previous two-scale formulations have not included the influence of transport of momentum by boundary-layer scale eddies. This omission is a serious inconsistency since large-scale eddies must mix momentum as well as heat and moisture. Nonlocal mixing of momentum is the main subject of this study. Unfortunately, the transport of momentum is more complicated than that of heat and moisture since it is a vector and is influenced by both baroclinity and pressure fluctuations. Pressure fluctuations in the heated boundary layer apparently reduce the correlation between vertical and horizontal velocity fluctuations (Zilitinkevich, 1973; Bergström and Högström, 1989; Shaw *et al.*, 1990). For example, Mahrt (1991a) found that thermals in a sheared boundary layer are characterized by a systematic phase lag between the vertical and horizontal velocity fluctuations leading to a low correlation coefficient and inefficient momentum transport.

Partly due to this low correlation, flux sampling criteria for the momentum flux appear to require a larger record length than that required for heat and moisture (Mahrt and Gibson, 1992; Lenschow *et al.*, 1994). As an additional complication to data analysis, the horizontal velocity seems to be affected by low frequency variations referred to as inactive eddies in Högström (1990) and Mahrt and Gibson (1992). The dynamics of these inactive eddies and their influence on the momentum flux at higher levels in the boundary layer are not well understood. Near the surface, these observed motions may be simply the large eddies whose vertical motion is much reduced by the presence of the ground surface and may include roll vortices or transient mesoscale pressure disturbances.

The entrainment of momentum can be important and sometimes dominant. Recent observations (Xu and Gal-Chen, 1993; Gal-Chen *et al.*, 1992) of momentum fluxes in the heated boundary layer indicate that entrainment fluxes are significant and can exceed the stress values in the boundary-layer interior. Based on the analysis of a number of field experiments, Garratt *et al.* (1982) find that advection and entrainment of momentum seem to be more important than baroclinity in the unstable boundary layer.

Clearly the details of momentum transport are complex. However, some characteristics of momentum transport can be inferred by their influence on mean profiles. Brost *et al.* (1982) observed quasi-barotropic marine flows which were approximately well mixed in momentum with strong wind-induced mixing. In this case, the streamwise momentum flux component was quasi-linear. More recently, Lidar observations (Piironen and Eloranta, 1993) in the heated boundary layer during FIFE 1989 (First International Satellite Land Climate Program Field Experiment, Sellers *et al.*, 1988) showed well mixed momentum profiles as well. The study of Arya and Wyngaard (1975) suggests that even with baroclinity, convective mixing reduces the vertical gradient of mean momentum to values much smaller than the thermal wind shear.

When the boundary-layer mean momentum becomes well mixed, the eddy diffusivity or any purely local scheme fails in the interior of the mixed layer where the mean gradient vanishes but the flux remains significant. Purely local diffusion in the model simulation leads to a nonzero vertical gradient of the mean wind in order to produce nonzero stress everywhere within the boundary layer. In order to better compare with observations, the transport by the large boundary-layer eddies must be included in the mixing formulation for momentum as well as for heat and moisture.

Previous simple formulations for nonlocal mixing by the large eddies have been primarily confined to the case of convectively driven circulations. However numerous laboratory studies of neutrally stratified boundary layers show that both longitudinal and transverse eddies lead to significant mixing on the scale of the boundary layer. Longitudinal roll vortices are known to occur under a variety of conditions in the atmosphere (Kelly, 1984; Puhakka and Saarikivi, 1986). The case of cloud streets appears to be a small subclass of roll vortices where sufficient moisture is available and competing circulations are not present. Longitudinal vortices appear to modulate the smaller scale turbulence flux (LeMone, 1976) and lead to significant mixing of momentum on the scale of the boundary layer (e.g., Brown and Mourad, 1990; Etling and Brown, 1993) even in the near neutral and weakly stratified cases.

Nonlocal mixing may occur in the very stable case where intermittent bursting leads to destabilization of the entire boundary layer. It is not clear if such bursting consists of eddies on the scale of the boundary layer or if the mixing occurs through a sequence of smaller eddies. Unfortunately, fluxes have not been adequately sampled in the very stable boundary layer and little observational evidence is available for model construction. This study omits the stable case.

In Section 2, we show how two-scale mixing can be derived from a more general point of view. We then extend this approach to include mixing of momentum by large eddies where certain inconsistencies with the existing two-scale approaches are identified. Finally we shall directly compare the above formulation with observed atmospheric flux data and improved wind profile information which has been previously lacking (Section 3.1).

2. General Formulation

An extensive survey of nonlocal schemes is detailed in Stull (1993). The various simple formulations for the turbulent flux at a given level z are schematically

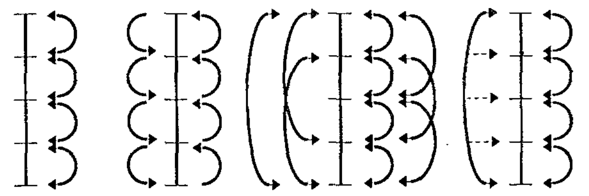

Fig. 1. Examples of mixing schemes for a 5-level model.

shown in Figure 1. Using a 5-level model, these formulations are now discussed in the order shown in Figure 1:

1. In the usual application of local diffusion, "direct" mixing occurs only between adjacent grid levels and only one height-dependent mixing coefficient is specified.
2. In the case of top-down, bottom-up diffusion (Wyngaard and Brost, 1984), two height-dependent mixing coefficients are specified.
3. In transilient theory (e.g., Stull, 1993), direct mixing is allowed to occur between all combinations of levels in the model. Altogether, $N = M(M-1)/2$ coefficients are required, where M is the number of grid points; for example, with $M = 5$ levels within the boundary layer, $N = 10$, so that 10 height-independent coefficients are required.
4. An asymmetric version of top-down bottom-up diffusion was developed by Pleim and Chang (1992). In their model, upward transport by convective plumes is modelled as a nonlocal process while downward mixing is modelled as local diffusion between adjacent grid levels.
5. In the two-scale approach (Blackadar, 1978; Brown and Mourad, 1990), the flux due to the large eddies is formulated as distinct from the small-scale mixing. In the study of Brown and Mourad, the large eddies were formulated in terms of roll vortices. With the two-scale approach, small-scale diffusion occurs between adjacent grid levels and mixing by large eddies simultaneously occurs across the bulk of the boundary layer. The version considered in this study requires two height-dependent mixing coefficients ($N = 2$).

Here we choose the two-scale mixing approach because it recognizes the limited complexity allowed in large-scale models and the fact that large eddies mix momentum. This simple approach also accommodates comparisons with the limited existing flux data in the interior of the atmospheric boundary layer. The local eddy diffusivity with the gradient correction will be considered as a special case of the two-scale approach. The gradient correction approaches of Priestley and

Swinbank (1947), Deardorff (1972), Troen and Mahrt (1986), Holtslag and Moeng (1991) and Holtslag and Boville (1993) can be expressed in the format

$$(\overline{w'f'}) = -K_f(z)\left(\frac{\partial \overline{f}}{\partial z} - \gamma_c\right), \tag{1}$$

where K_f is the eddy diffusivity and γ_c is a correction to the local gradient which parameterizes the contribution of the large-scale eddies to the total flux $(\overline{w'f'})$.

For heat flux, γ_c is referred to as the "countergradient correction" since the heat flux in the interior of the well mixed boundary layer remains upward even though the local gradient becomes weakly stable. Note that specifying γ_c is equivalent to directly specifying that the height dependence of the large eddy flux is $K_f(z)\gamma_c$. However, the height dependence of the large eddy flux is not necessarily expected to be directly proportional to the height dependence of the small-scale diffusion coefficient $K_f(z)$ since the two processes are distinctly different. This connection between the vertical structure of the large eddy flux and small-scale diffusivity cannot be formally justified. However, existing data do not allow separation of the two contributions to the total flux.

From a more general point of view, the total flux can be written as

$$(\overline{w'f'}) = -K_f(z)\frac{\partial \overline{f}}{\partial z} + (\overline{w'f'})_L, \tag{2}$$

where $(\overline{w'f'})_L$ is the large-eddy heat flux. A general formulation of the large-eddy flux $(\overline{w'f'})_L$ can be written as

$$(\overline{w'f'})_L = S_f\left(\frac{z}{L}\right)(\overline{w'f'})_s\left(\frac{z}{h}\right)^q\left(1-\frac{z}{h}\right)^p, \tag{3}$$

where $S_f(z/L)$ is a coefficient which depends on stability; for the case of momentum, this coefficient also depends on baroclinity. $(\overline{w'f'})_s$ is the surface flux and h is the boundary-layer depth.

Choosing $q = 0$ and $p = 1$ in (3) leads to a linear decrease of the large-eddy flux expected for the total heat flux in a convecting mixed boundary layer. Brost and Wyngaard (1978) use $q = 1$ and $p = 1.5$. Troen and Mahrt (1986) and Holtslag and Boville (1993) use $q = 1$ and $p = 2$ for the unstable case. At the surface, $S_f(z/L)$ becomes the fraction of the flux due to the large eddies. Since w' is associated with small-scale motions near the surface, $(\overline{w'f'})_L$ and $S_f(z/L)$ might be expected to vanish as $z \to 0$. However, as shown in Etling and Brown (1993), the cospectra for heat near the surface do not vanish at boundary-layer scales. Perhaps the flux near the surface is still modulated or organized by the large eddies. Nonetheless, the main flux shifts to smaller scale local diffusion in the surface layer (Højstrup, 1982). The decrease of the large eddy flux toward the surface is controlled by the exponent q in (3). For $q = 1$, both the local eddy diffusivity and the large eddy flux

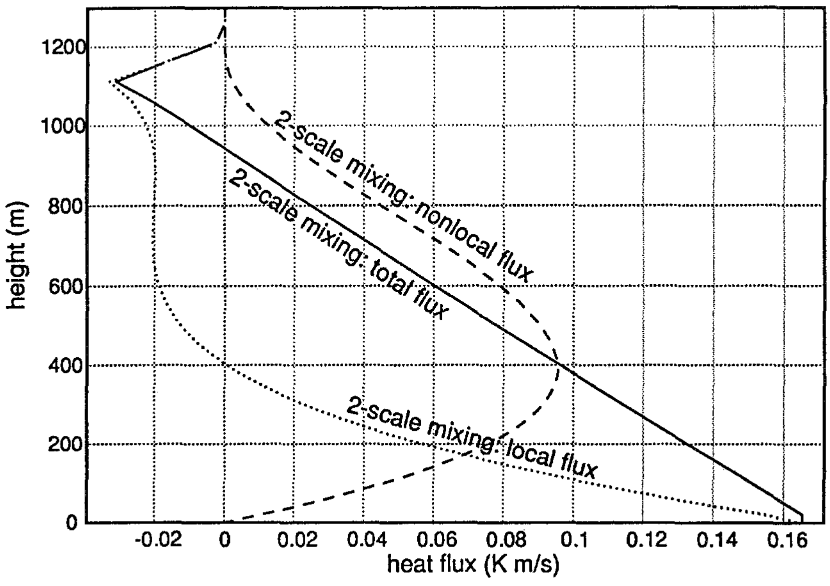

Fig. 2. Predicted heat flux profile (K m/s) using the two-scale mixing scheme for 28 July 1989 at 1200 LST during FIFE (Section 3.2). Plotted are the local (dotted) and nonlocal (dashed) components of the total heat flux (solid line).

have a similar height dependence; however, in actual model runs, the small-scale diffusion is dominant in the surface layer where local gradients are large while nonlocal mixing dominates at higher levels (Figure 2).

3. Nonlocal Mixing of Momentum

3.1. Basic Formulation

Application of (3) to the large eddy momentum transport must recognize that f becomes a vector and an additional condition is required to determine the direction of the large eddy stress. The present development assumes that the large eddy stress vector is aligned with the bulk shear vector between the top of the surface layer and the layer just below the boundary-layer top. This assumption is briefly reconsidered in Section 3.2. We calculate the bulk shear vector as the difference between the wind vector at the top of the surface layer and at the model level just below the top of the boundary layer.

We propose the following form for the large eddy stress (here the x-component)

$$(\overline{w'u'})_L = -S_m u_*(w_* + u_*)\left(\frac{z}{h}\right)\left(1-\frac{z}{h}\right)^2 \frac{u_{sh}}{|\vec{v}_{sh}|}, \tag{4}$$

where S_m is an adjustable coefficient and u_{sh} is the x-component of the bulk shear vector and q in (3) is specified to be 1. $S_m u_*(w_* + u_*)$ replaces $S_f(z/L)$

in (3). The free convection velocity scale w_* represents the generation of vertical velocity fluctuations by the buoyancy. The scaling with $u_*(w_* + u_*)$ provides the simplest stability dependence with consistent asymptotic limits. The large eddy flux scales with the surface stress u_*^2 for the neutral limit ($w_* \rightarrow 0$). In the free convection limit ($u_* \rightarrow 0$), the large eddy momentum flux becomes zero. The stability dependence in (4) attempts to simulate the observations that the correlation between the horizontal and vertical velocity perturbations u' and w' of the main eddies decreases with increasing instability (Mahrt, 1991a). The coefficient S_m is a parameter to be adjusted against data and is expected to be a function of baroclinity. The y-component assumes a form analogous to (4).

The two horizontal components of the prognostic equation for momentum at level z in the 1-D model are written as

$$\frac{\partial u}{\partial t} = f(v - v_g) - w\frac{\partial u}{\partial z} + \frac{\partial}{\partial z}\left(K_m \frac{\partial u}{\partial z}\right) - \frac{\partial}{\partial z}(\overline{w'u'})_L, \tag{5}$$

$$\frac{\partial v}{\partial t} = f(u_g - u) - w\frac{\partial v}{\partial z} + \frac{\partial}{\partial z}\left(K_m \frac{\partial v}{\partial z}\right) - \frac{\partial}{\partial z}(\overline{w'v'})_L, \tag{6}$$

where f is the Coriolis parameter, v_g and u_g are the geostrophic wind components, w is the large-scale vertical motion, K_m is the eddy diffusivity for momentum and $(\overline{w'u'})_L$ and $(\overline{w'v'})_L$ are determined as in (4).

The local eddy diffusivity for momentum is defined as (Troen and Mahrt, 1986):

$$K_m(z) = w_s k h \left(\frac{z}{h}\right)\left(1 - \frac{z}{h}\right)^2, \tag{7}$$

where

$$\begin{aligned} w_s &= \left(u_*^3 + 15\frac{z_s}{h}k w_*^3\right)^{1/3}, \\ z_s/h &= 0.1. \end{aligned} \tag{8}$$

The following subsection applies formulations (4–8) to two cases of modest baroclinity.

3.2. Data Comparison

The one-dimensional model with two-scale mixing is compared with the mean wind profiles obtained from Lidar measurements during FIFE 1989 for 28 July and 3 August 1989 (Piironen and Eloranta, 1993). We use volume-averaged Lidar wind profiles which have been averaged over one-hour periods. We also compare the predicted momentum profiles with Twin Otter aircraft data (MacPherson *et al.*, 1992; Eloranta and Forrest, 1992) and radiosonde profiles. In general, Lidar

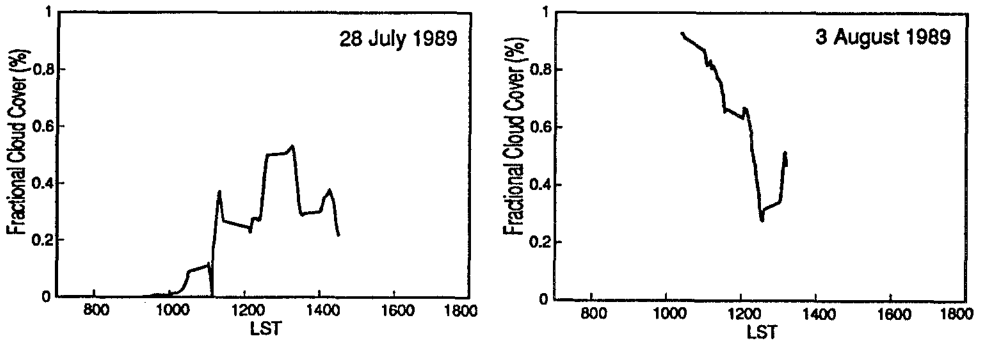

Fig. 3. Fractional cloud cover for 28 July and 3 August 1989 during FIFE obtained from the Lidar.

wind profiles provide the best sampling statistics. In contrast, radiosonde data contain virtually no averaging while aircraft data are horizontal line measurements and may contain inadequate averaging (Grossman, 1992; Lenschow *et al.*, 1994; Mahrt and Gibson, 1992). To help isolate errors due to momentum mixing in the model, we prescribe the fractional cloud cover observed by the Lidar (Figure 3). Cloud cover reduces the incoming solar radiation available for surface heating. We do not account for any modification of the turbulent mixing due to clouds. We initialize the model temperature and moisture profiles with radiosonde data. The horizontal advection terms for the boundary-layer heat and moisture budget appear to be important based on the analysis of radiosonde data for 28 July and 3 August 1989. We crudely account for advection by "updating" the initial temperature and moisture sounding above 1000 m utilizing afternoon sounding data.

Geostrophic wind profiles are estimated from the NMC upper air grid data extracted from the NOAA operational analysis system. Using gridpoints in the polar-stereographic projection over the FIFE area, we produce height maps for 1000, 850 and 700 hPa. The horizontal pressure gradients over the FIFE site are then calculated in finite difference form. We linearly interpolate the geostrophic wind with height between the observational levels. The geostrophic wind profiles are estimated for 0600 and 1800 Local Standard Time (LST) and interpolated linearly in time (Figure 4). The calculated geostrophic wind profiles for 0600 LST are used to initialize the model winds for the data comparison in order to avoid strong inertial oscillations.

3.2.1. 3 August 1989:

On 3 August 1989, winds are nearly stationary with speeds of the order 10 m/s. Parameter values for the numerical simulation are obtained from the FIFE Information System (FIS, Strebel *et al.*, 1990). The prescribed subsidence increases linearly with height from the surface, reaching a maximum value of -1 cm/s at 3000 m. The subsidence was tuned to approximate the observed boundary-layer depth in order to help isolate errors due to the mixing scheme.

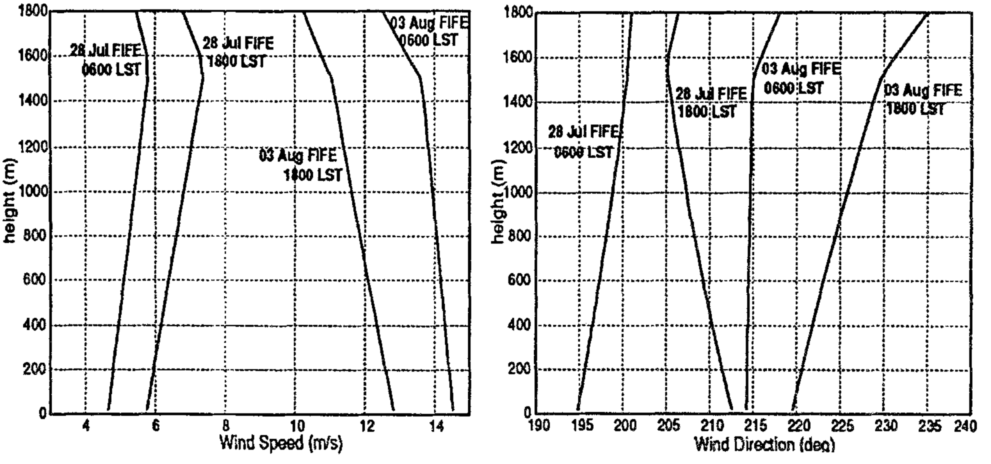

Fig. 4. Calculated geostrophic wind speed (m/s) and direction (deg) from NMC data.

The Lidar wind speed profile (Figure 5) indicates well mixed momentum up to the boundary-layer top. The model compares most closely with the Lidar wind profiles for $S_m = 1.4$. As S_m decreases below unity, the predicted wind profiles are no longer well mixed. For S_m equal to about two or greater, the mixing scheme in the present model leads to formation of an unphysical wind maximum within the boundary layer. This unrealistic behavior appears to be due to overestimation of the large eddy stress.

The wind speeds predicted by the two-scale mixing scheme and the purely local scheme are both about 1 m/s stronger than the Lidar wind speed within the boundary-layer interior, which is a small difference considering the uncertainty in the geostrophic wind speed. The local scheme predicts continuous shear throughout the boundary layer and therefore fails to predict the observed well-mixed mean momentum profile.

The wind in the surface layer predicted by the two-scale approach is stronger due to the more efficient downward transport of momentum by the parameterized large eddy transport. The two-scale approach for momentum therefore predicts a 10% larger surface stress compared to that for the local diffusion scheme. The surface stress is determined by a drag law which is quadratically proportional to the first model-level wind speed. Momentum fluxes are available only from the aircraft data which contain too much scatter for definite validation of the model.

3.2.2. 28 July 1989:

We also test the proposed formulation (4) by modeling 28 July 1989 in FIFE. The Lidar measured wind speed is well mixed up to the boundary-layer top (Figure 6). The numerical simulation with $S_m = 0.8$ for the nonlocal flux parameterization compares best with the Lidar wind profiles for 28 July 1989. The optimum S_m is less than unity because the geostrophic wind backs with height, as will be discussed

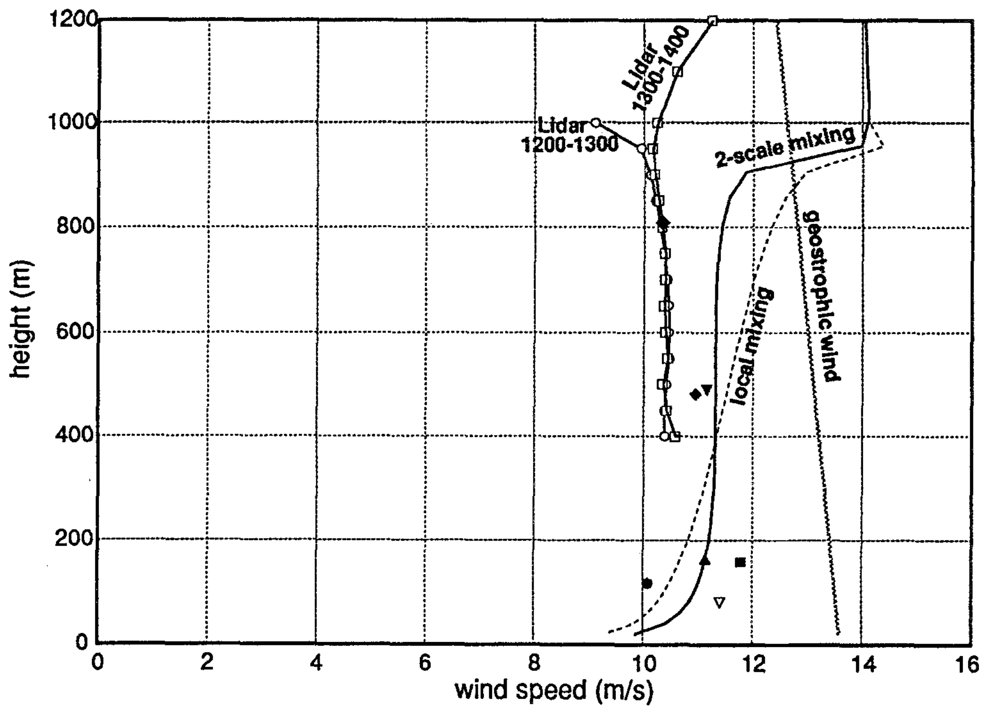

Fig. 5. Model-predicted and observed wind speed (m/s) for 3 August 1989 at 1300 LST during FIFE: individual symbols show aircraft data between 1205 and 1321 LST. Each aircraft data point represents a leg-average (Eloranta and Forrest, 1992). The winds derived from Lidar data (circles and squares) are unreliable below 400 m and are therefore omitted.

in more detail below. The local scheme shows continuous weak shear throughout the boundary layer. However, for this case the differences between the two-scale and local approach are not as large as on 3 August 1989. We find that the magnitude of the predicted wind speeds for both mixing schemes agrees well with the Lidar mean wind fields. Both the two-scale and local schemes produce wind directions which are about 20 deg clockwise from the Lidar-observed winds (not shown). The wind direction predicted by the model is sensitive to the geostrophic wind direction. The two-scale approach again predicts a 10% larger surface stress compared to the local scheme.

In summary, the above results indicate that two-scale mixing of momentum produces better results than the usual local mixing formulation. However, the results are sensitive to the coefficient S_m. Both days represent weak baroclinic conditions. To produce well-mixed profiles, a smaller value of S_m is required (less nonlocal mixing) for backing of the geostrophic wind where the geostrophic shear is acting to generate mean shear opposite to veering induced by the stress divergence (as in the case of 28 July 1989 during FIFE). A larger value of S_m is required to completely mix momentum when the geostrophic wind veers with

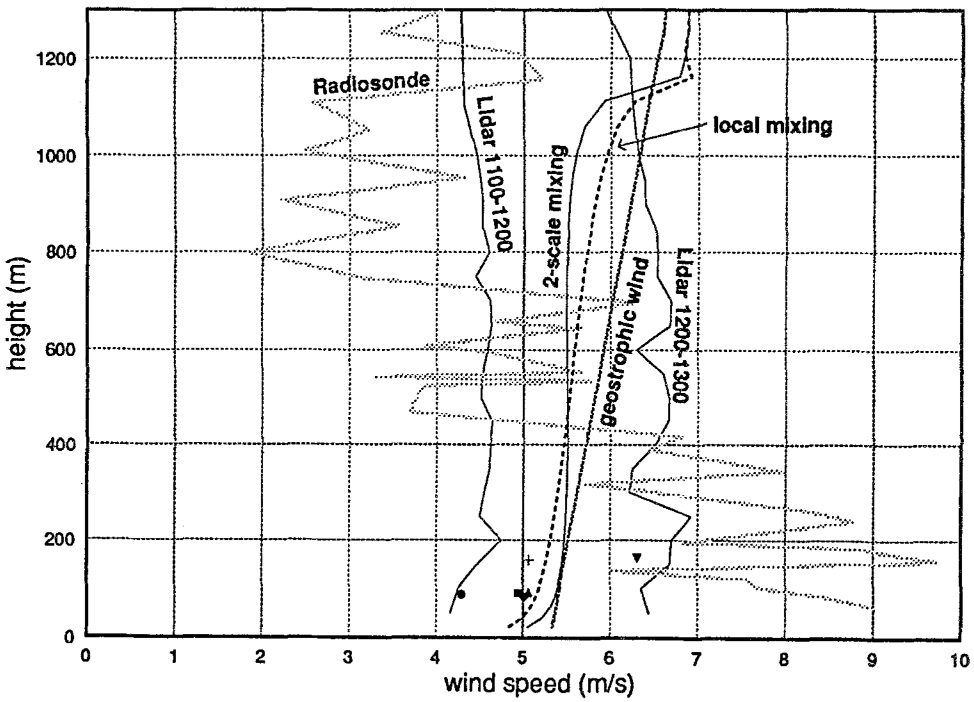

Fig. 6. Model-predicted and observed wind speed (m/s) for 28 July 1989 at 1200 LST during FIFE: individual symbols show aircraft data between 1101 and 1143 LST.

height and acts to enhance the shear induced by the stress divergence (as in the case of 3 August 1989 during FIFE). It appears that a constant value of S_m is unable to describe mixing of momentum in baroclinic conditions. As an alternative to a variable S_m, the model behavior can be improved by including a variable nonzero angle between the direction of the large eddy stress and the bulk shear vector. But again, existing data are insufficient to evaluate the dependence of this angle on baroclinity.

4. Conclusions

The simple two-scale mixing formulation for momentum was implemented to account for the momentum transport by large eddies and to be more consistent with existing formulations of heat and moisture transport. The small-scale mixing consists of local gradient diffusion and large-scale transport on the scale of the boundary layer (nonlocal mixing). The local heat diffusion and the usual "countergradient correction" for heat is a previous example of two-scale mixing. Here it is shown that for existing two-scale approaches, the height-dependence of the large eddy flux is proportional to the height dependence of the small-scale diffusivity. This proportionality cannot be formally justified; however, the inadequacy of existing data does not allow improvement upon this scheme.

The two-scale mixing scheme for momentum is compared with Lidar wind profiles. Lidar profiles are superior to aircraft and radiosonde data because they contain both time and three-dimensional spatial averaging. Two-scale mixing better predicts the observed well-mixed structure of momentum observed by Lidar. The local mixing scheme fails to predict the well-mixed wind profile. The two-scale mixing increases surface stress compared to the usual local mixing scheme due to the more effective downward transport of momentum. Significant enhancement of the surface momentum flux by large eddies has also been suggested by Brown and Foster (1994).

Although the two-scale mixing scheme for momentum is more consistent and compares better with the data than that from local mixing alone, the new scheme requires specification of a nondimensional coefficient S_m which represents the relative importance of nonlocal mixing compared to local mixing. The optimum value of this coefficient appears to depend on baroclinity in a way that cannot be adequately determined from existing data. Quantitative data comparisons are difficult because of the uncertainties in the specified geostrophic wind and the greater complexity of mixing a vector quantity compared to a scalar mixing. A larger data set with better determination of the geostrophic wind and momentum flux is required in order to refine and extend the limited physics of the two-scale mixing scheme.

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