(1) Show that the quasi-geostrophic version of the Sawyer-Eliassen PDE is equivalent to the Q vector form of the omega equation for adiabatic flow, assuming that the flow does not depend on the x direction (i.e., $\frac{\partial}{\partial x} = 0$).

(2) Derive the necessary condition for the Sawyer-Eliassen equation to result in an elliptical PDE. What is the physical significance of this condition (hint: it involves Ertel PV on pressure surfaces)? How restrictive is this condition?

(3) Derive an equation for the three-dimensional frontogenesis $\left(\frac{D}{Dt}|\nabla_3 \theta|\right)$. What physical processes will increase the vertical temperature gradient? How does your answer change if the winds are geostrophic?

(4) Given the following figure showing 850 hPa and 250 hPa geopotential height (solid) and temperature (dashed), indicate regions of frontogenesis and frontolysis using the dynamic frontogenesis equation (hint: consider all physical processes). Indicate regions of vertical motion related to these fronts.
250 hPa Height (solid) and temperature (dashed)

850 hPa Height (solid) and temperature (dashed)