1. Derive the QGPV equation starting from the QG geopotential height tendency equation.

2. Compute the stationary wavelength for bartropic Rossby waves, assuming the following conditions: 45°N, and a mean zonal wind of 20 m sec\(^{-1}\)? If the entire mid-latitude flow was characterized by these type of waves, how many ridges/troughs would exist? How would your answer change if the mean zonal wind is increased to 80 m sec\(^{-1}\)? What does this answer imply about stationary ridges/troughs in strong westerly flow?

3. Determine the structure of the geopotential height perturbation and wave dispersion relationship for the case of a semi-infinite atmosphere (i.e., fixed surface with an atmosphere that goes to infinity) assuming constant static stability and a mean zonal wind that increases linearly with height and is in thermal wind balance with the mean temperature gradient. What is the necessary condition to obtain an unstable solution? Hint: Follow the steps used to derive the Eady solution, but with an appropriate boundary conditions at \(z \to \infty\).

4. Derive an expression for the time rate of change of geostrophic kinetic energy \((K = \frac{1}{2}(u_g^2 + v_g^2))\) with respect to time starting from the quasi-geostrophic equations. Physically describe the processes that lead to an increase in kinetic energy following the flow.