

Four-stream isosector approximation for canopy radiative transfer

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[1] This study develops an analytical four-stream isosector approximation for solar radiative transfer in a homogeneous canopy, based on the approximation of four spherical sectors of isotropic intensities (constant values for light intensities). Compared to results from a multilayer radiative transfer model, the four-stream isosector approximation substantially improves the accuracy in calculation of albedo, transmittance, and absorptance with respect to the corresponding two-stream approximation. For direct incident radiation, it has errors mostly under 5% for leaf area index less than 5, even when sun angles are very low, while the two-stream method has errors of about 10% or higher; more improvement is achieved in albedo and transmittance in the visible band, and transmittance and absorptance in the near-infrared (NIR) band. For diffuse incident radiation, both the two-stream and four-stream approximations always have a higher accuracy in the NIR band than in the visible band, but the improvement of the four-stream approximation is larger for the visible band than for the NIR band. In addition, they have a higher accuracy in describing canopy albedo, transmittance and absorption for direct incident radiation than for diffuse incident radiation. However, the improvement of the four-stream is higher for diffuse incident radiation than for direct incident radiation. The inclusion of soil albedo as the low boundary does not degrade the performance of the four-stream approximation. As an analytical model, the four-stream approximation can be easily applied as an efficient approach to improving the parameterization of land surface radiation in climate models.

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1. Introduction

[2] The physical process of radiative transfer is described by a differential-integral equation, whose solution in a scattering and absorbing media is made complex by the directionality of the problem [*Li and Ramaswamy*, 1996]. In cases where the angular distribution of the radiation field is of less interest, the transport equation can be integrated over angle to derive the appropriate equations for radiation fluxes [*Myneni et al.*, 1989]. Two-stream approximation methods are based on such rule. They reduce the problem of directionality into two coupled angle-integrated equations to describe fluxes (streams) in the upward and downward hemispheres. By doing so, the upward and downward fluxes of the media are obtained analytically with little computational expense and moderate accuracy [*Liou et al.*, 1988]. Two-stream approximations for canopy radiative transfer have been widely used to obtain canopy reflectance, transmittance and absorptance [*Dickinson*, 1983; *Sellers*, 1985] in climate models [*Bonan*, 1996; *Oleson et al.*, 2004]. They

are earlier used in the field of atmospheric physics [e.g., *Coakley and Chylek*, 1975; *Meador and Weaver*, 1980]. However, *King and Harshvardhan* [1986] had pointed that various two-stream approximations for atmosphere models can produce 15~20% relative errors for a number of optical depths, solar zenith angles, and single-scattering albedos. Similar errors are expected for two-stream approximations for vegetation canopies. In atmospheric sciences, four-stream discrete ordinate [*Liou et al.*, 1988], spherical harmonic expansion [*Li and Ramaswamy*, 1996] and isosector [*Li and Dobbie*, 1998] approximation have been developed to consider two radiative streams in each of the upper and lower hemispheres. A four-stream approximation for canopies can be developed as a valuable approximation method to improve the accuracy of the description of canopy radiative transfer while maintaining the computational efficiency of the two-stream approach.

[3] Recently, *Pinty et al.* [2006] elaborated the analytical two-stream solutions for the canopy that have the same analytical forms as those proposed by *Meador and Weaver* [1980]. Their analysis facilitates the communication between atmosphere and land community. With the same objective, we adopt the four-stream isosector approximation of *Li and Dobbie* [1998] for canopy radiative transfer. In the two-stream model of *Coakley and Chylek* [1975], the radiative intensities are taken to be isotropic (independent of local

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zenith angle) in the upward and downward hemispheres. The four-stream model of *Li and Dobbie* [1998] further divided each isotropic hemisphere into two portions (sectors) with different isotropic diffuse intensities within each sector. The assumption of isotropy within each sector (a form of numerical approximation) allows a simple integration of the multiple scattering source term.

[4] The basic equations of the four-stream approximation developed in this paper are patterned after equations (3) and (4) of *Li and Dobbie* [1998], but for applications to a plant canopy, we add reflection from underlying soil and consider diffuse incident radiation and factors treating the orientation of leaves relative to the solar beam and their reflection. We solve the complex equations analytically using the software “Mathematica” [*Abell and Braselton*, 2004; *Ruskeepaa*, 2004] instead of providing equations. The symbolic solutions derived from the software can be easily applied to canopy radiation modeling given required input parameters. Section 2 describes the basic equations and section 3 analyzes the calculation results, followed by discussion and conclusions in sections 4 and 5.

2. Four-Stream Isosector Approximation Scheme

[5] Considering a horizontally homogeneous leaf canopy illuminated by a monodirectional beam, the basic azimuthally independent radiative transfer equation in general form is

$$-\mu \frac{dI(L, \mu)}{dL} = -G(\mu)I(L, \mu) + \frac{\omega}{2} \int_{-1}^1 G(\mu')I(L, \mu') \cdot P(\mu, \mu')d\mu' + \frac{\omega}{4\pi} G(\mu_0)\pi I_0 \cdot P(\mu, -\mu_0)e^{-G(\mu_0)L/\mu_0}. \quad (1)$$

Where I is the diffuse intensity, $\mu = \cos \theta$, θ is the local zenith angle, $\mu_0 = \cos \theta_0$, θ_0 is the solar zenith angle, πI_0 is the incident solar flux at the top of the canopy, $G(\mu)$ is the geometry factor defined as the relative projected area of leaf elements in the direction $\cos^{-1}\mu$, ω is the single-scattering albedo, and $P(\mu, \mu')$ is the normalized azimuthally independent phase function, $\frac{1}{2} \int_{-1}^1 P(\mu, \mu')d\mu = 1$, $P(-\mu, \mu') = P(\mu, -\mu')$. The term L is the cumulative leaf area index, i.e., the total area of one side of the leaves per unit ground area above a horizontal level [*Myneni et al.*, 1989]. Because the vertical ordinate is directed downward, a canopy is confined between depth zero ($L = 0$) at the top and $L =$ total leaf area at the bottom. Hence L is a measure of depth in the canopy.

[6] In equation (1), the first term describes the divergence (net rate of photos streaming out of the canopy along the depth of the canopy) of the diffuse intensity, which is in the direction μ . The second term describes the reduction of the diffuse intensity due to events of outward scattering and absorption by leaves. The third term describes the contribution to the diffuse intensity by multiple inward scattering, arising from the scattering of a ray of radiation with solid angle $d\mu'$ in the direction of μ' [*Liou*, 1980]. The last term shows the generation of diffuse intensity in the direction μ due to single scattering of the direct solar radiation from

$-\mu_0$ (the minus sign denotes that the direct solar radiation is always downward penetrating to the specified depth L in the canopy).

[7] In the four-stream approximation, the zenith space μ is separated into four regions as $[-1, -\mu_s]$, $[-\mu_s, 0]$, $[0, \mu_s]$, $[\mu_s, 1]$, where $\mu_s = \cos \theta_s$, θ_s is the angle separating a hemisphere into two portions or sectors [*Li and Dobbie*, 1998]. The corresponding intensities in the four regions are $I_1^\uparrow(L, -\mu)$, $I_1^\uparrow(L, \mu)$ when $0 < \mu < \mu_s$, and $I_2^\uparrow(L, -\mu)$, $I_2^\uparrow(L, \mu)$ when $\mu_s < \mu < 1$. Following the derivation of *Li and Dobbie* [1998], the radiative transfer equation is formulated into a group of equations with the downward and upward diffuse intensities for each sector as

$$-\mu \frac{dI_2^\uparrow}{dL} = G(-\mu)I_2^\uparrow - S(-\mu, \mu_0), \text{ for } \mu_s < \mu < 1, \quad (2a)$$

$$-\mu \frac{dI_1^\uparrow}{dL} = G(-\mu)I_1^\uparrow - S(-\mu, \mu_0), \text{ for } 0 < \mu < \mu_s, \quad (2b)$$

$$\mu \frac{dI_1^\uparrow}{dL} = G(\mu)I_1^\uparrow - S(\mu, \mu_0), \text{ for } 0 < \mu < \mu_s, \quad (2c)$$

$$\mu \frac{dI_2^\uparrow}{dL} = G(\mu)I_2^\uparrow - S(\mu, \mu_0), \text{ for } \mu_s < \mu < 1. \quad (2d)$$

[8] Where the source term $S(\pm\mu, \mu_0)$ in each of the equations is defined as,

$$S(-\mu, \mu_0) = \frac{\omega}{2} \int_{-1}^{-\mu_s} G(\mu')I_2^\uparrow P(-\mu, \mu')d\mu' + \frac{\omega}{2} \int_{-\mu_s}^0 G(\mu')I_1^\uparrow P(-\mu, \mu')d\mu' + \frac{\omega}{2} \int_0^{\mu_s} G(\mu')I_1^\uparrow P(-\mu, \mu)d\mu' + \frac{\omega}{2} \int_{\mu_s}^1 G(\mu')I_2^\uparrow P(-\mu, \mu')d\mu' + \frac{\omega}{4} G(\mu_0)I_0 P(-\mu, -\mu_0)e^{-G(\mu_0)L/\mu_0}, \quad (2e)$$

$$S(\mu, \mu_0) = \frac{\omega}{2} \int_{-1}^{-\mu_s} G(\mu')I_2^\uparrow P(\mu, \mu')d\mu' + \frac{\omega}{2} \int_{-\mu_s}^0 G(\mu')I_1^\uparrow P(\mu, \mu')d\mu' + \frac{\omega}{2} \int_0^{\mu_s} G(\mu')I_1^\uparrow P(\mu, \mu)d\mu' + \frac{\omega}{2} \int_{\mu_s}^1 G(\mu')I_2^\uparrow P(\mu, \mu')d\mu' + \frac{\omega}{4} G(\mu_0)I_0 P(\mu, -\mu_0)e^{-G(\mu_0)L/\mu_0}. \quad (2f)$$

[9] Assuming the intensity within each of regions is independent of μ , we approximate $I_{1/2}^\uparrow(L, \pm\mu)$ with $I_{1/2}^\uparrow(L)$

by integrating over μ for the interval of each equation in equation (2),

$$\frac{dI_2^\downarrow}{dL} = \frac{1}{\mu_2} \left[(\alpha^+ - \kappa_{-2})I_2^\downarrow + \beta^+I_1^\downarrow + \beta^-I_1^\uparrow + \alpha^-I_2^\uparrow \right] + \left[\frac{G(\mu_0)}{\mu_2} \varepsilon_{-2} e^{-G(\mu_0)L/\mu_0} \right], \quad (3a)$$

$$\frac{dI_1^\downarrow}{dL} = \frac{1}{\mu_1} \left[\beta^+I_2^\downarrow + (\gamma^+ - \kappa_{-1})I_1^\downarrow + \gamma^-I_1^\uparrow + \beta^-I_2^\uparrow \right] + \left[\frac{G(\mu_0)}{\mu_1} \varepsilon_{-1} e^{-G(\mu_0)L/\mu_0} \right], \quad (3b)$$

$$\frac{dI_1^\uparrow}{dL} = \frac{1}{\mu_1} \left[-\beta^-I_2^\downarrow - \gamma^-I_1^\downarrow - (\gamma^+ - \kappa_1)\gamma^-I_1^\uparrow - \beta^+I_2^\uparrow \right] - \left[\frac{G(\mu_0)}{\mu_1} \varepsilon_1 e^{-G(\mu_0)L/\mu_0} \right], \quad (3c)$$

$$\frac{dI_2^\uparrow}{dL} = \frac{1}{\mu_2} \left[-\alpha^-I_2^\downarrow - \beta^-I_1^\downarrow - \beta^+I_1^\uparrow - (\alpha^+ - \kappa_2)I_2^\uparrow \right] - \left[\frac{G(\mu_0)}{\mu_2} \varepsilon_2 e^{-G(\mu_0)L/\mu_0} \right], \quad (3d)$$

where $\mu_1 = \int_0^{\mu_s} \mu d\mu$, $\mu_2 = \int_{\mu_s}^1 \mu d\mu$, and

$$\alpha^\pm = \frac{\omega}{2} \int_{\mu_s}^1 \int_{\mu_s}^1 G(\mu') P(\pm\mu, \mu') d\mu' d\mu, \quad (4a)$$

$$\beta^\pm = \frac{\omega}{2} \int_{\mu_s}^1 \int_0^{\mu_s} G(\mu') P(\pm\mu, \mu') d\mu' d\mu, \quad (4b)$$

$$\gamma^\pm = \frac{\omega}{2} \int_0^{\mu_s} \int_0^{\mu_s} G(\mu') P(\pm\mu, \mu') d\mu' d\mu, \quad (4c)$$

$$\varepsilon_{\pm 1} = \frac{\omega}{4} I_0 \int_0^{\mu_s} P(\pm\mu, -\mu_0) d\mu, \quad (4d)$$

$$\varepsilon_{\pm 2} = \frac{\omega}{4} I_0 \int_{\mu_s}^1 P(\pm\mu, -\mu_0) d\mu, \quad (4e)$$

$$\kappa_{\pm 1} = \int_0^{\mu_s} G(\pm\mu) d\mu, \quad (4f)$$

$$\kappa_{\pm 2} = \int_{\mu_s}^1 G(\pm\mu) d\mu. \quad (4g)$$

[10] Note that the above derivation is adopted from *Li and Dobbie* [1998] but modified to consider specific features for vegetation canopies. We include the G function in

all equations and use two more coefficients $\kappa_{\pm 1}$ and $\kappa_{\pm 2}$ to consider the directionality of G . The optical depth τ is replaced with L , so that the intensity I is a function of L instead of τ . Equations (3a)–(3d) are a system of four first-order differential nonhomogeneous equations for our four-stream approximation calculation.

3. Solutions and Results

[11] Because of the complexity of equations, direct derivation of the solution for equation (3) is algebraically complicated. Here we solve the equations symbolically using the software “Mathematica”, which is made available by Wolfram Research, Inc. and is powerful in symbolic programming, computation and solution [*Ruskeepaa*, 2004]. The solution procedure consists of three steps. The first step is to find the eigenvalues and eigenvectors for the corresponding homogeneous equation to form the general solution. The second step is to find a particular solution to the nonhomogeneous system. The third step is to solve the initial value problem (see more discussions later) to get a complete solution. Although the results appear complicated, the overall manipulations are done by the software. In particular, this software transfers the complicated analytical solution directly to Fortran codes. Note that the procedure to solve the system of differential equations refers to equation (3) with the variables of α , β , γ , ε , κ as required input parameters. Therefore different choices of G , ω , μ_0 , μ_s , and P do not require modification of the Fortran code (the Fortran code generated by Mathematica is available upon request to the author).

[12] To evaluate the solutions’ accuracy, we assume that leaves with all orientations are distributed with an equal probability ($G = 0.5$) and that the individual leaves are treated as isotropic scatters, i.e., $P = 1$, $r = t = 0.5 \omega$, where r is leaf reflectance and t is leaf transmittance. We consider both incident radiation from a direct beam (section 3.1) and diffuse radiation (section 3.2), and also assess the effects of soil reflectance on canopy radiation fields. Leaf reflectance and transmittance are set as $r = t = 0.05$ for the visible band and $r = t = 0.45$ for the NIR band, soil reflectance is 0 or 0.1 (or 0.2) for the visible (NIR) band.

[13] The accuracy of our four-stream approximation is examined by comparing the approximate results with the “exact” values computed from the RT model, a multilayer canopy radiative transfer model developed by *Myneni et al.* [1987]. This model solves the photon transport problem in slab geometry for a spatially uniform leaf canopy whose leaf-normals are distributed as (1) planophile, (2) erectophile, (3) plagiophile, (4) extremophile, or (5) uniformly (isotropically). The canopy may be illuminated by both direct (monodirectional) solar radiation as well as by an isotropic diffuse component. Leaf scattering may be described by either (1) a Henyey-Greenstein scattering function or (2) a bi-Lambertian scattering model. The soil is assumed to be a gray isotropic reflector. The successive orders of scattering approximation (SOSA) method is used to solve the transport equation, which considers 50 vegetation sublayers, 220 angular directions in zenith and azimuth directions. In this study, we assume that the leaf-normal is uniformly distributed and individual leaves are bi-Lambertian scattering elements.

[14] The relative error in percentage is calculated as $RN_{4\text{-stream}} = (N_{4\text{-stream}} - N_{RT})/N_{RT} * 100$, where $N_{4\text{-stream}}$ (N_{RT}) represents canopy albedo, transmittance, or absorptance calculated from the four-stream (RT) method. To quantify the improvement of our four-stream approximation relative to the corresponding two-stream approximation of *Sellers* [1985], we also calculate the relative error of the two-stream ($G = 0.5$, $r = t = 0.5\omega$).

3.1. Direct Beam Radiation

3.1.1. Black Soil (Soil Reflectance $\rho = 0$)

[15] The boundary condition for the black soil is,

$$I_1^\downarrow(0) = I_2^\downarrow(0) = 0, \quad (5a)$$

$$I_1^\uparrow(LAI) = I_2^\uparrow(LAI) = 0, \quad (5b)$$

where $I_{1/2}^\uparrow(LAI)$ refers to upward intensity below the canopy, at the soil surface, and LAI is the total leaf area index (the integral of L over the full canopy depth). The solution of equation (3) with boundary conditions equation (5) yields the complete solution. Therefore the upward and downward fluxes are:

$$F^\uparrow = 2\pi(\mu_1 I_1^\uparrow + \mu_2 I_2^\uparrow), \quad (6a)$$

$$F^\downarrow = 2\pi(\mu_1 I_1^\downarrow + \mu_2 I_2^\downarrow). \quad (6b)$$

[16] The albedo, transmittance and absorptance for the canopy are,

$$r_S = F^\uparrow(0)/(\mu_0 \pi I_0), \quad (7a)$$

$$t_S = F^\downarrow(LAI)/(\mu_0 \pi I_0) + \exp\left(-\frac{G \cdot LAI}{\mu_0}\right), \quad (7b)$$

$$a_S = 1 - r_S - t_S(1 - \rho). \quad (7c)$$

[17] Equation (7b) shows that canopy transmittance is determined by two parts: (1) the direct radiation contribution that is directly coming from the sun flecks (Beer's law) and (2) the diffuse radiation contribution from the solution of the multiscattering process.

[18] The four-stream approximation divides each hemisphere into two sectors. The cosine of the dividing angle is μ_s . *Li and Dobbie* [1998] found that $\mu_s = 0.33998$, which is the Gaussian angle (angle of about 70°), generally gives much better results than the Gaussian angle of $\mu_s = 0.86114$ (angle of about 31°). We tested and confirmed their finding in this study. However, we also found that $\mu_s = 0.501$ (angle of about 60°) gives more accurate transmittance (a slightly worse albedo and absorptance) than $\mu_s = 0.33998$. Therefore, throughout the remaining section of this paper, we consider the angle of $\mu_s = 0.501$ as the dividing angle.

3.1.1.1. Visible Band ($r = t = 0.05$)

[19] Figure 1a shows the relative errors of albedo, transmittance and absorptance from the four-stream and two-

stream approximation schemes in the visible band as a function of LAI and solar zenith angle for direct beam radiation under the black soil condition. The albedo has a relative error of 0~10% depending on sun angle for the two-stream and less than 6% for the four-stream in the LAI- μ_0 space, indicating a substantial reduction of albedo errors in the four-stream. The two-stream scheme exhibits the largest errors at the lowest sun positions, while the four-stream has larger error at very low and very high sun position. For the transmittance, the four-stream scheme has a relative error mostly less than 2%, even for very low sun positions; the relative error of the two-stream scheme is comparable to that of the four-stream for high sun positions or low LAI (less than 2) but becomes extremely large for high LAI and μ_0 less than 0.4, with the maximum error up to 50%. The relative error for the absorption is very small for both schemes but the four-stream still performs better.

3.1.1.2. NIR Band ($r = t = 0.45$)

[20] The relative errors of albedo and transmittance in the NIR band are smaller for both the four- and two-stream schemes (Figure 1b), compared to those in the visible band. The transmittance from the four-stream scheme gives a relative error less than 2% at all sun angles and the two-stream has a slightly larger error. As shown in the visible band, the largest error in the NIR band occurs for large LAI and low sun positions. The relative error of absorption is larger in the NIR band than that in the visible band. Again, the four-stream has more accurate results relative to the two-stream.

[21] As we previously discussed, the canopy transmittance consists of contributions from direct and diffuse radiation. For small LAI, if μ_0 is large (overhead sun), the contribution from the direct radiation is dominant and that from the diffuse radiation is minor and thus the transmittance is accurate. The substantially large transmittance error from the two-stream (both in the visible and NIR bands) occurs when sun is low and LAI is large, representing the situation that the contribution from the direct radiation is small. These results suggest that the four-stream is more capable in description of diffuse radiation [*Li and Dobbie*, 1998]. The four-stream outperforms the two-stream because it uses two more sectors to evaluate the angular integration of the radiative transfer equation. The light within the canopy is not isotropic. By adding one more sector in each hemisphere, we represent better the directionality of the radiation, hence obtain a more accurate description of the radiation transfer within the canopy.

3.1.2. Soil Albedo $\rho > 0$

[22] When soil reflectance is considered, the basic equations remain the same as equation (3) but the boundary condition is changed as,

$$I_1^\downarrow(0) = I_2^\downarrow(0) = 0, \quad (8a)$$

$$I_1^\uparrow(LAI) = I_2^\uparrow(LAI) = \frac{\rho}{\pi} \left[F^\downarrow(LAI) + \mu_0 \pi I_0 \exp\left(-\frac{G \cdot LAI}{\mu_0}\right) \right]. \quad (8b)$$

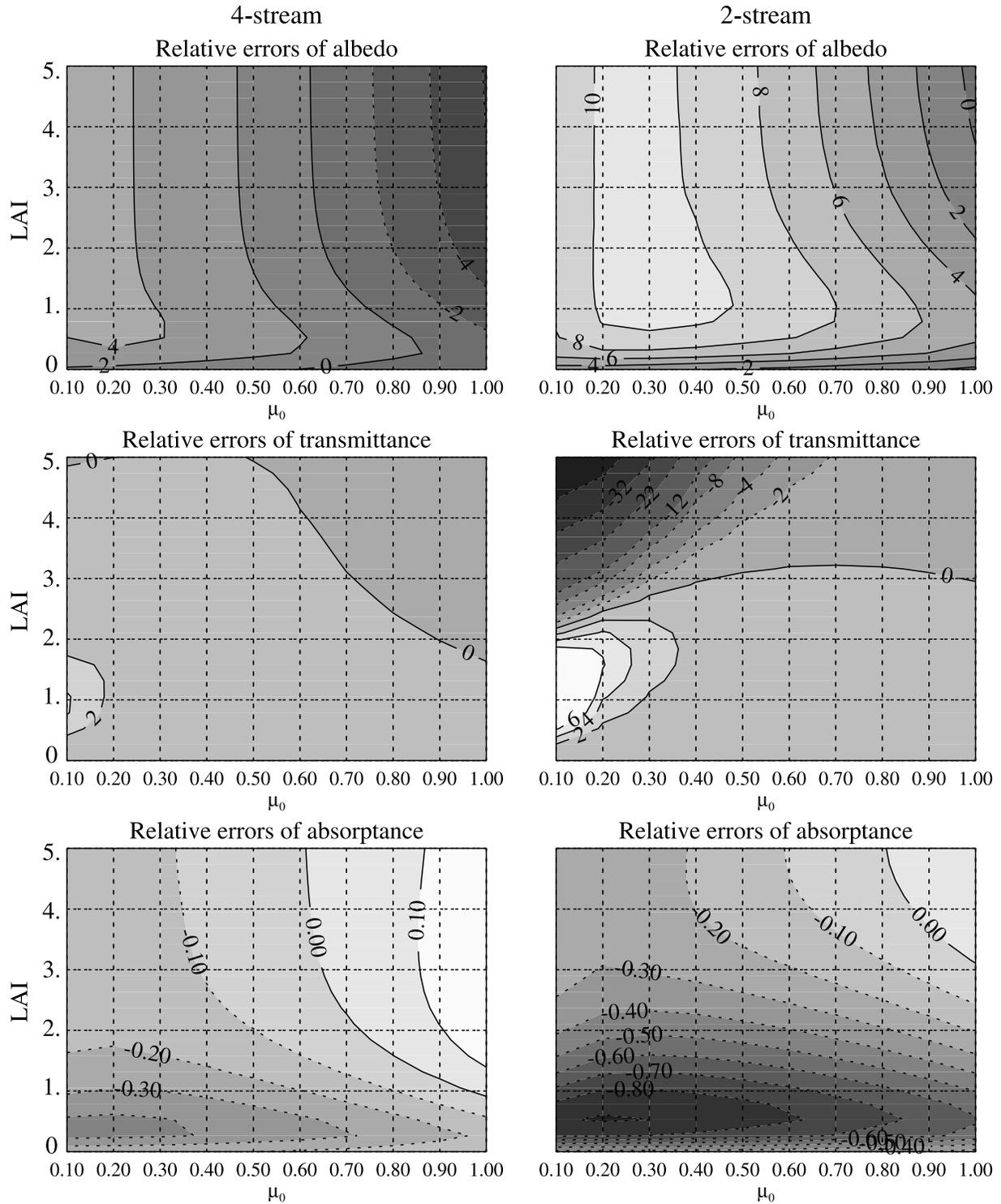


Figure 1a. Relative errors of albedo, transmittance and absorptance from the four-stream and two-stream approximation schemes as a function of LAI and the cosine of the solar zenith angle μ_0 in the visible band for direct beam radiation under the black soil condition (soil albedo $\rho = 0$).

[23] We calculate the relative errors of albedo, transmittance and absorptance from the four-stream and two-stream approximation schemes as a function of LAI and solar zenith angle for direct beam radiation when the soil albedo

is set as 0.1 (0.2) for the visible (NIR) band (figure not shown). The pattern and magnitude of the relative errors are very similar to those under the black soil condition (Figure 1),

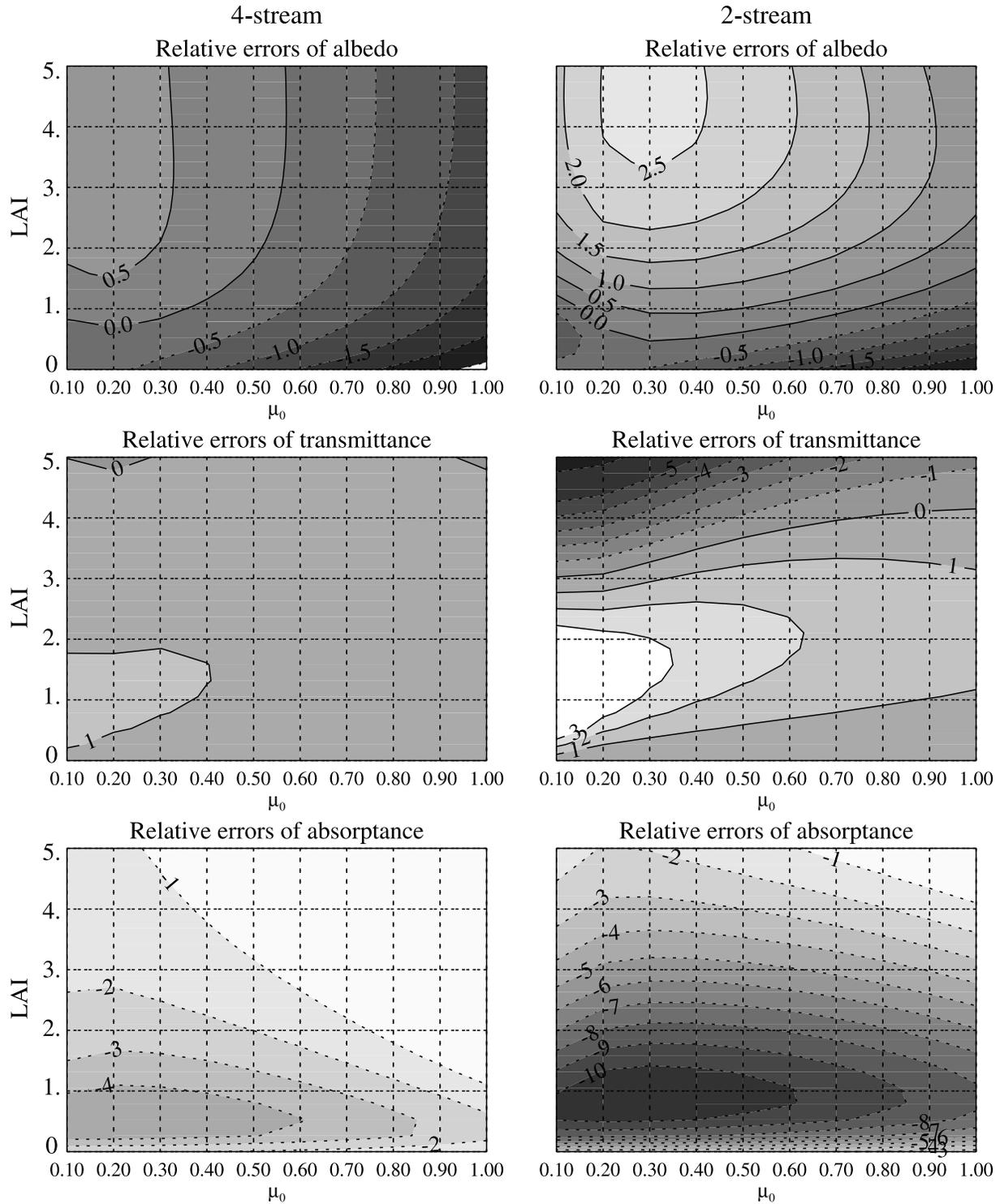


Figure 1b. Same as in Figure 1a but for the NIR band.

and the four-stream scheme makes a marked improvement over the two-stream scheme.

3.2. Diffuse Incident Radiation

3.2.1. Black Soil (Soil Reflectance $\rho = 0$)

[24] For diffuse incident radiation, the direct radiation terms of the right-hand sides of the four-stream equations in

equation (3) should be dropped from the basic equations. The boundary condition is,

$$I_1^{\downarrow}(0) = I_2^{\downarrow}(0) = I_0, \quad (9a)$$

$$I_1^{\uparrow}(LAI) = I_2^{\uparrow}(LAI) = 0. \quad (9b)$$

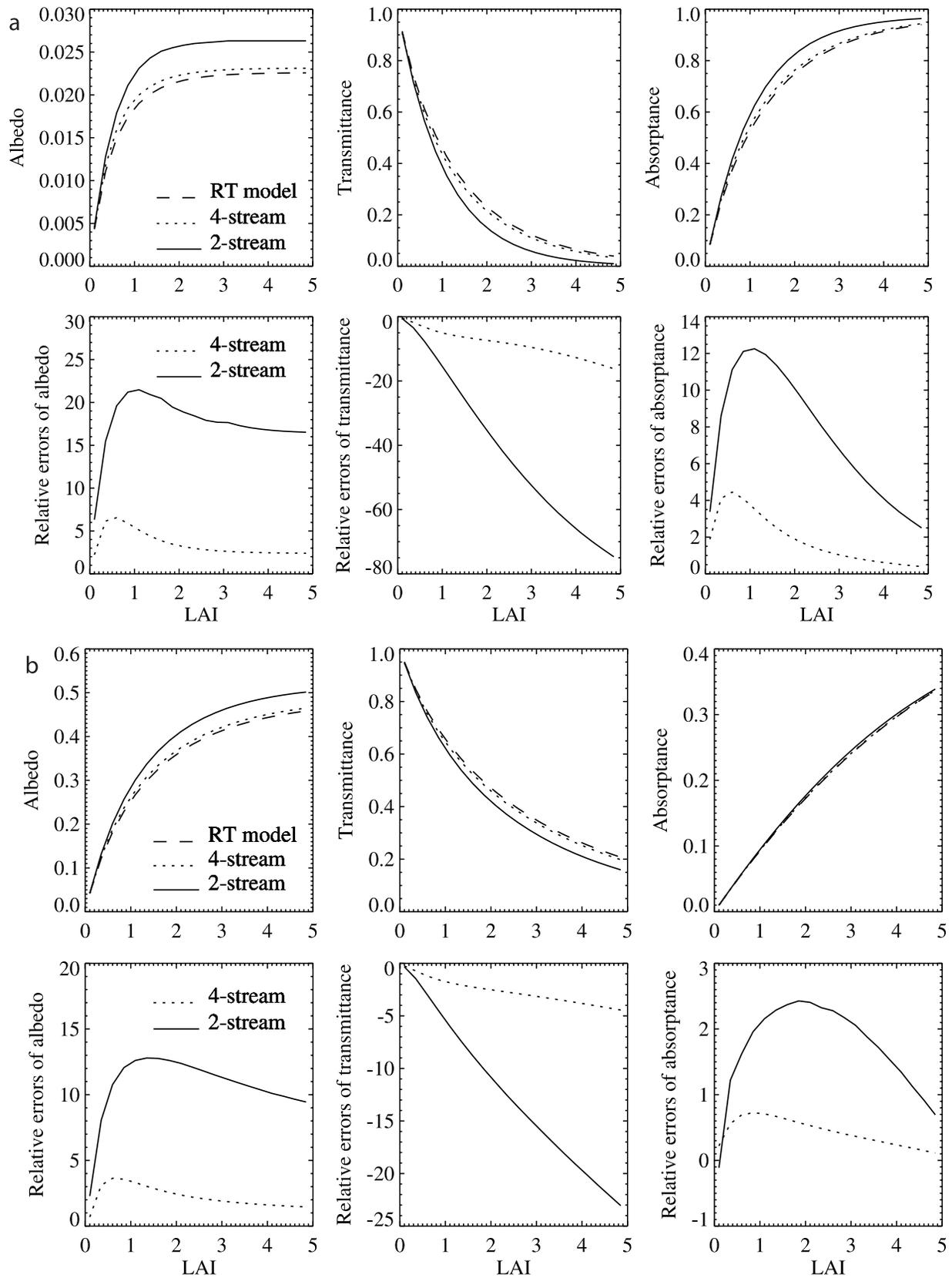


Figure 2. (a) Albedo, transmittance, and absorptance and their relative errors from the four-stream and two-stream approximation schemes in the visible band for incident diffuse solar radiation under the black soil condition (soil reflectance $\rho = 0$). (top) Values of albedo, transmittance and absorptance calculated from the RT model, the four-stream scheme, and the two-stream scheme, respectively, and (bottom) corresponding relative errors. (b) Same as in Figure 2a but for the NIR band.

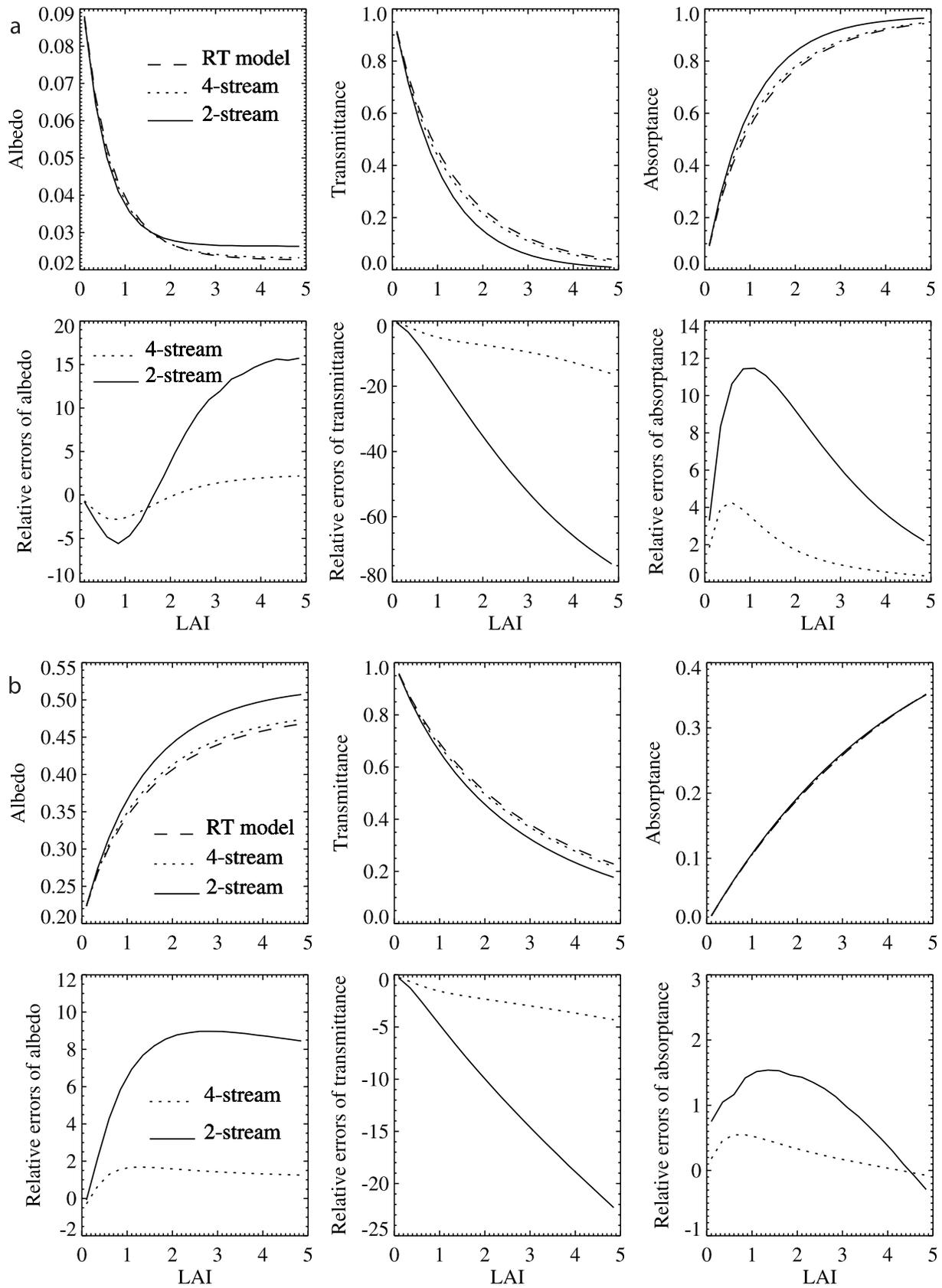


Figure 3. (a) Same as in Figure 2a but for soil albedo $\rho = 0.1$. (b) Same as in Figure 3a but for the NIR band and soil albedo $\rho = 0.2$.

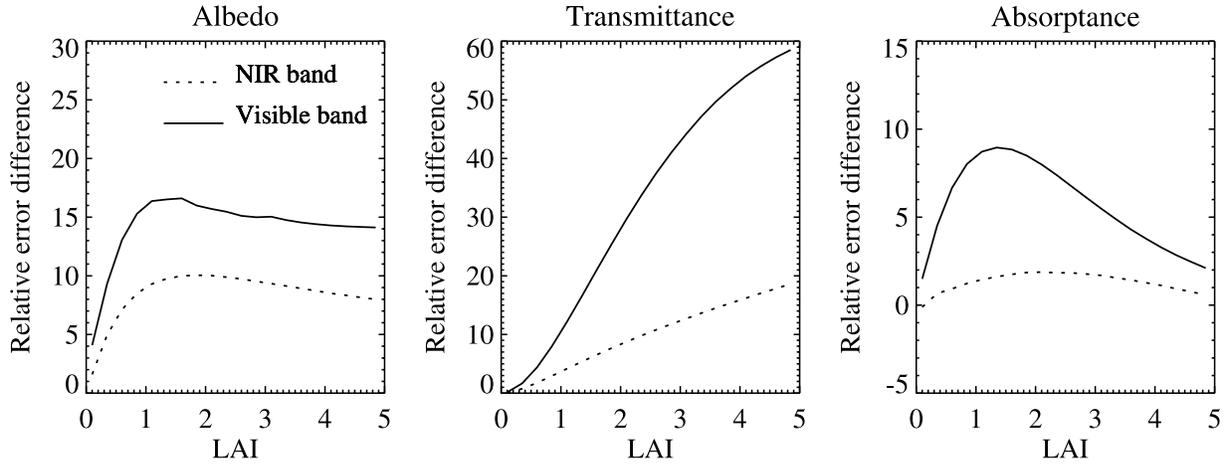


Figure 4. Difference of relative error (DRE) of albedo, transmittance and absorptance between the four- and two-stream approximation schemes in the visible band and the NIR band for diffuse solar radiation under the black soil condition (soil albedo $\rho = 0$).

[25] Since the upward and downward fluxes are the same as equation (6), the albedo, transmittance, and absorptance are,

$$r_d = F^\uparrow(0)/(\pi I_0), \quad (10a)$$

$$t_d = F^\downarrow(LAI)/(\pi I_0), \quad (10b)$$

$$a_d = 1 - r_d - t_d(1 - \rho). \quad (10c)$$

3.2.1.1. Visible Band ($r = t = 0.05$)

[26] The albedos calculated from the two-stream have a low accuracy, with a relative error in the order of 15~20% (Figure 2). The relative errors larger than 10% occur for the transmittance with LAI higher than 0.5 and such errors increase almost linearly with LAI (e.g., 55% for LAI = 3.0 and 75% for LAI = 5.0). In general, the albedos computed from the four-stream have a relative error less than 3% (about 5% for LAI between 0.3 and 1.3). The transmittance of the four-stream has a much smaller relative error than that of the two-stream (e.g., less than 20% for LAI = 5.0 and about 10% for LAI = 3.0). For the absorption, the two-stream generally produces an error of 2–12% while the error of the four-stream is within 4%.

3.2.1.2. NIR Band ($r = t = 0.45$)

[27] The NIR band shows similar features in the relative error as the visible band but with a smaller magnitude.

3.2.2. Soil Albedo $\rho > 0$

[28] When soil reflectance is considered, the boundary condition is,

$$I_1^\downarrow(0) = I_2^\downarrow(0) = I_0, \quad (11a)$$

$$I_1^\uparrow(LAI) = I_2^\uparrow(LAI) = \frac{\rho}{\pi} F^\downarrow(LAI). \quad (11b)$$

[29] The results for reflective soils (Figure 3) are very similar to those for black soils (Figure 2).

4. Discussion

[30] *Li and Dobbie* [1998] stated that the four-stream isosector approximation addresses radiative transfer for optical thickness less than unity. When applied to vegetation canopies by considering the G function, the four-stream method is expected to perform well for LAI less than 2. However, our study indicates that it is still accurate for middle and high LAI ranging from 3 to 5, even when sun position is very low.

[31] In order to quantify the relative improvement of the four-stream versus the two-stream with respect to the visible band versus the NIR band, and direct versus diffuse incident radiation, we compare the difference of relative error (DRE) between the four- and two-stream schemes. The DRE is defined as $|\text{RN}_{2\text{-stream}}| - |\text{RN}_{4\text{-stream}}|$, where RN represents the relative error of the two- (four-) stream approximation for albedo, transmittance, or absorptance.

[32] For direct incident solar radiation (figure not shown), the improvement from the four-stream relative to the two-stream is the largest for the albedo and transmittance in the visible band and the transmittance and absorptance in the NIR band; for diffuse incident solar radiation (Figure 4), the improvement to these three variables is significantly higher in the visible band, especially for the transmittance. In addition, more improvement is seen for diffuse incident radiation than for direct incident radiation. The fraction of solar radiation that is diffuse may vary from not much more than 10% for very clear skies to nearly 100% in the presence of clouds. An accurate description of the impacts of diffuse solar fluxes in canopy radiation is needed for determining how changing aerosol and cloudiness affects surface energy balance and carbon assimilation.

[33] Previous studies have often used the value of clear sky condition at the sun angle of 60° to represent the value of diffuse condition when data is not available.

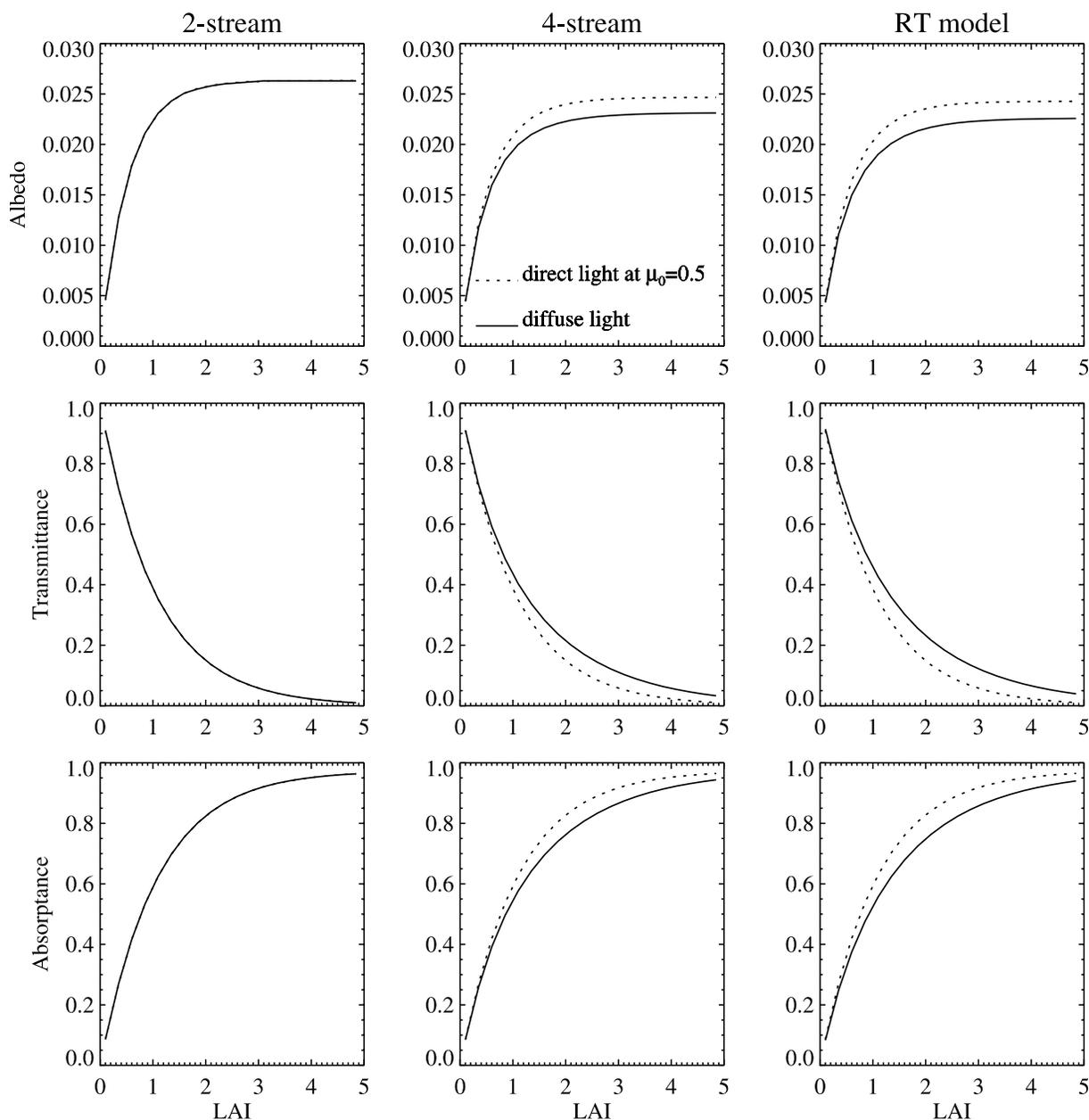


Figure 5. Canopy albedo, transmittance, and absorption in the visible band for diffuse incident solar radiation and for direct beam incident radiation at the sun angle of 60° from the four-stream and two-stream approximation schemes and the RT model.

Figure 5 shows the canopy albedo, transmittance, and absorption in the visible band for diffuse incident solar radiation, together with those for direct beam incident radiation at the sun angle of 60° . Evidently, the values under these two conditions are exactly the same for the two-stream scheme but not for the four-stream scheme and the RT model. The values from the direct beam radiation are higher (lower) in albedo and absorption (transmittance) than those from the diffuse incident radiation in the visible band, and so are they in the NIR band. Because of the distribution of path length there is no proper angle to best represent diffuse beam albedo,

transmittance, and absorption under all conditions. Rather, the “best angle” depends not only on LAI but also the single scattering albedo. The four-stream compensates for this complication. Our analysis shows that $50\sim 52^\circ$ maybe is a better choice depending on LAI values.

[34] The most important elements of the four-stream are the black soil terms, i.e., the direct incident and diffuse incident beam albedo, transmittance and absorption when soil is black (soil reflectance is 0). Its solutions for a bottom boundary with diffuse reflection ρ also can be constructed by an adding method using the black soil solutions [Coakley and Chylek, 1975; Liou, 1980;

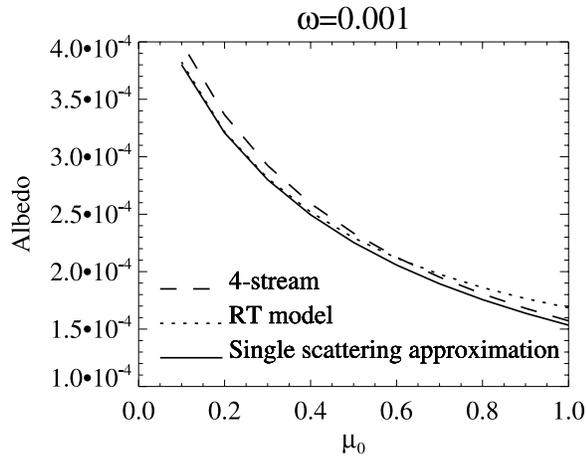


Figure 6. Albedos as a function of the cosine of the solar zenith angle μ_0 from the four-stream scheme (LAI = 4.85), the RT model, and the single scattering approximation with the infinite LAI limit, the single scattering albedo ω is 0.001.

[Knyazikhin et al., 1998a, 1998b; Pinty et al., 2006]. For reference, they should be:

$$r_s = r_{BS} + \frac{\rho}{1 - \rho r_{bd}} t_{BS} t_{bd}, \quad (12a)$$

$$t_s = t_{BS} + \frac{\rho}{1 - \rho r_{bd}} t_{BS} r_{bd} = \frac{t_{BS}}{1 - \rho r_{bd}}, \quad (12b)$$

$$a_s = a_{BS} + \frac{\rho}{1 - \rho r_{bd}} t_{BS} a_{bd}. \quad (12c)$$

$$r_d = r_{bd} + \frac{\rho}{1 - \rho r_{bd}} t_{bd} t_{bd}, \quad (12d)$$

$$t_d = t_{bd} + \frac{\rho}{1 - \rho r_{bd}} t_{bd} r_{bd} = \frac{t_{bd}}{1 - \rho r_{bd}}, \quad (12e)$$

$$a_d = a_{bd} + \frac{\rho}{1 - \rho r_{bd}} t_{bd} a_{bd}. \quad (12f)$$

[35] In the above, r_{BS} and r_{bd} , t_{BS} and t_{bd} , a_{BS} and a_{bd} are vegetation albedo, transmittance, and absorptance for direct solar incident (subscript “BS”) and diffuse solar incident (subscript “bd”) condition when soil is black as discussed in sections 3.1.1 and 3.2.1. The results from sections 3.1.2 and 3.2.2 (when soil reflectance is considered) were checked with those from equation (12). They are exactly the same (figure not shown).

[36] As pointed out in earlier sections, the accuracy of the four-stream is dependent on the choice of the separating angle for the isosectors. Generally, a larger angle (for example, 60° for $\mu_s = 0.501$ or 70° for $\mu_s = 0.33998$) gives

better results compared to those of smaller angle (for example, 31° for $\mu_s = 0.86114$), because the radiation field changes more smoothly from local zenith angle 0° to about $60\sim 70^\circ$, and then more rapidly as local zenith angle approaches 90° [Li and Dobbie, 1998]. By using a larger angle, we separate the radiation field in the upper and lower hemisphere each into two regions with distinct properties. The isotropic assumption of the light intensity for each sector of the hemisphere is more realistic than that of using a smaller angle and therefore gives better results.

[37] The difference in the choice of μ_s in our study compared to that of Li and Dobbie [1998] lies in the requirement for the accuracy of transmittance. When the value of μ_s changes from 0.3 to 0.9 (including 0.33998 and 0.86114), with an interval of 0.1, the relative error of albedo and absorptance from the four-stream scheme keeps increasing, while the relative error of transmittance reaches a significant minimum value around $\mu_s = 0.5$ (tables not shown). The total relative error of albedo, transmittance and absorptance around $\mu_s = 0.5$ is smaller than that of $\mu_s = 0.33998$. It changes little between $\mu_s = 0.501$ and $\mu_s = 0.51$.

[38] The aforementioned results show that the four-stream performs better than the two-stream. One exception is that for direct solar incident radiation near nadir, the apparent albedo error in the visible band is larger in the four-stream than in the two-stream (Figure 1a). The albedo in the limit of large LAI (infinite) and single scattering condition should be $0.5\omega[1 - \mu_0 \ln(1 + 1/\mu_0)]$ [Dickinson, 1983] for direct beam incident radiation and 0.204569ω for diffuse solar incident radiation. The diffuse albedo from the RT model (the four-stream) is 0.0002136 (0.0002188) when ω is 0.001. The albedo as a function of μ_0 for large LAI is shown in Figure 6. When sun position is low ($\mu_0 < 0.4$), albedos from the RT model are almost equal to that of the single scattering approximation, while the four-stream has higher values. However, when the sun position is high (especially for $\mu_0 > 0.6$), the result from the RT model is higher than that of both the four-stream and the single scattering approximation. Possible reasons for these differences are still under investigation.

5. Conclusions

[39] The four-stream isosector approximation scheme presented here approximates the light intensity in a specified angular region by a constant value. Except for its simple treatment of angular dependency, it is an analytical model and can be easily applied to land surface radiation modeling in climate models.

[40] The four-stream isosector approximation substantially improves the accuracy over that of the corresponding two-stream in calculation of albedo, transmittance, and absorptance. For direct incident radiation, the four-stream approximation has relative errors mostly under 5% for leaf area index less than 5, even when sun angles are very low, compared to errors of about 10% or higher for the two-stream method; more improvement is achieved for albedo and transmittance in the visible band, and for transmittance and absorptance in the NIR band. For diffuse incident radiation, the accuracy is always higher in the NIR band than in the visible band for both the two-stream and four-stream approximations, but the improvement of the four-

stream scheme is larger for the visible band than for the NIR band. All radiation terms (i.e., canopy albedo, transmittance, and absorption) are more accurately determined for direct incident radiation than for diffuse incident radiation but the relative improvement with the four-stream is greater for diffuse incident radiation. The inclusion of reflective soils at the low boundary does not degrade the performance of the four-stream approximation.

[41] Although the isosector approximation model is optimum for low optical thickness (small LAI), its accuracy is still satisfactory for all likely larger LAI, i.e., as large as 5, even at very low sun positions. Therefore it can be used as an efficient approach to canopy radiative transfer in climate models.

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