$$\frac{\partial \zeta}{\partial t} = -\vec{V} \bullet \vec{\nabla}\zeta - v\beta - \zeta\left(\vec{\nabla} \bullet \vec{V}\right) - f\left(\vec{\nabla} \bullet \vec{V}\right) - \hat{k} \bullet \vec{\nabla} w \times \frac{\partial \vec{V}}{\partial z} + \hat{k} \bullet \vec{\nabla} \times \overrightarrow{Fr}$$

1 2 3 4 5 6

- The Vorticity Equation says that the local time rate of change of relative vorticity is related to SIX separate physical processes:

1) The advection of relative vorticity

2) The advection of planetary vorticity

3) The divergence acting on relative vorticity

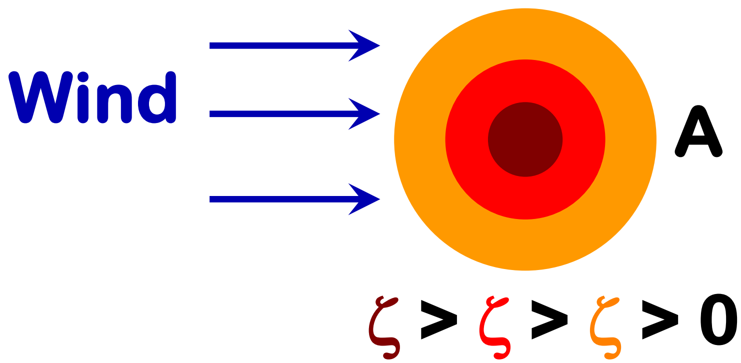
4) Planetary vorticity acting on divergence

5) The tilting of horizontal shear vorticity

6) Friction

1) The advection of relative vorticity: $-\vec{V} \bullet \vec{\nabla} \zeta$

- We've already examined advection, and this term simply represents the transport of relative vorticity by the flow.

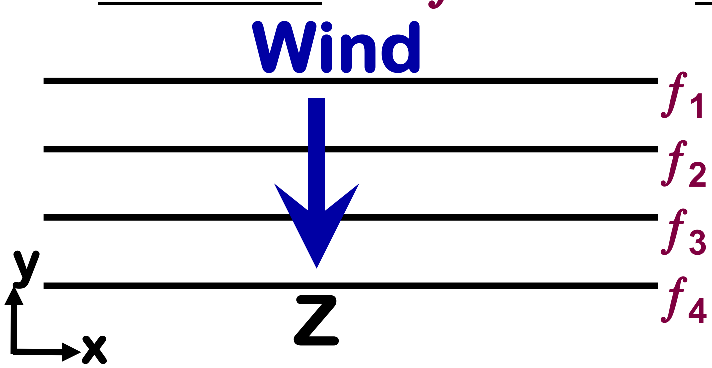


At point A, the gradient of ζ points to the left, so

$$-\vec{V} \bullet \vec{\nabla} \zeta = -\left|\vec{V}\right|\left|\vec{\nabla} \zeta\right| \cos 180^o > 0 \rightarrow \frac{\partial \zeta}{\partial t} > 0$$

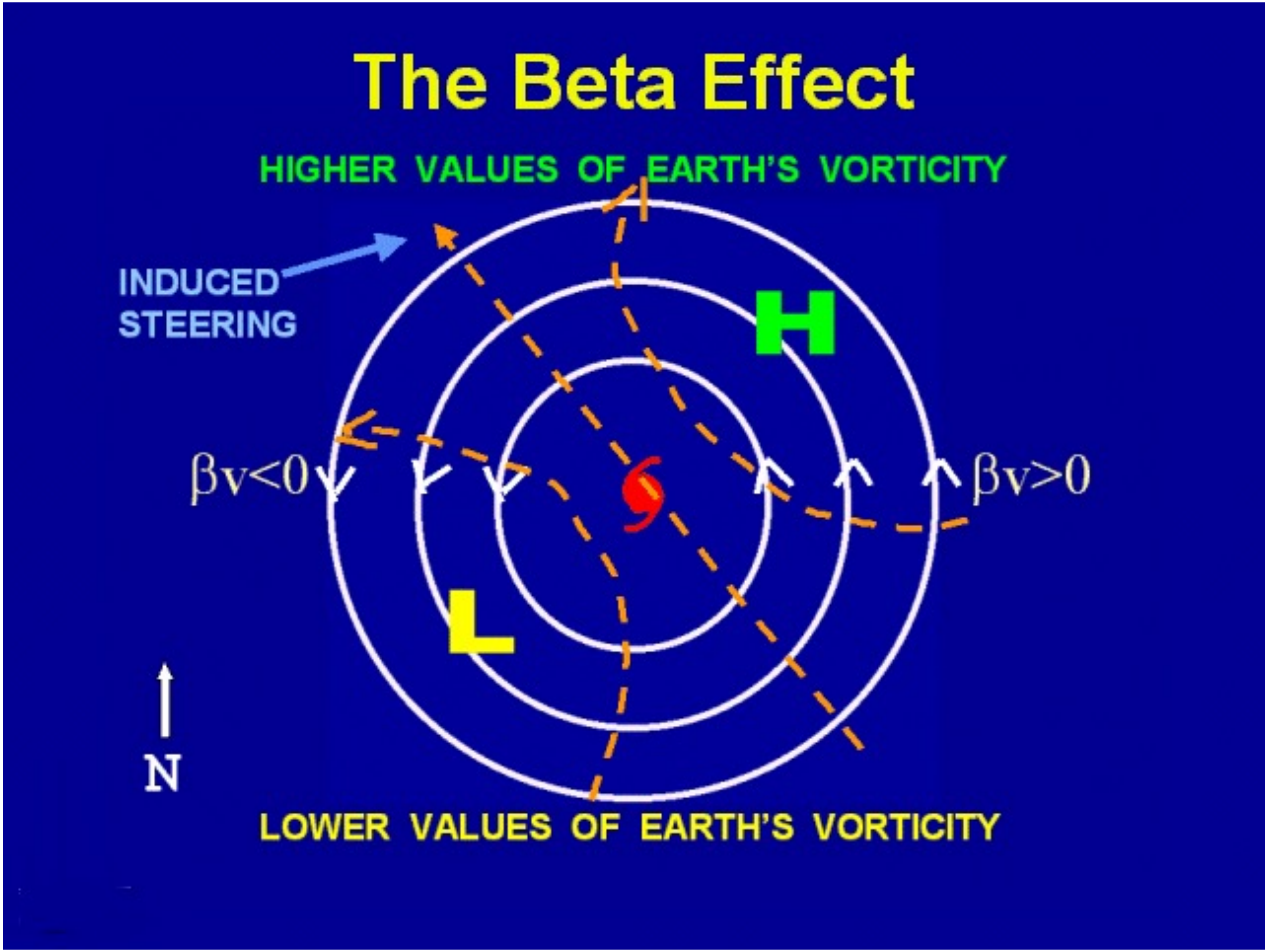
2) The advection of planetary vorticity: $-v\beta$

- We recall that $\beta \equiv \frac{\partial f}{\partial y}$ and $f = 2\Omega \sin\phi$; thus, we have higher values of f to the north and lower to the south ($f_1 > f_4$).



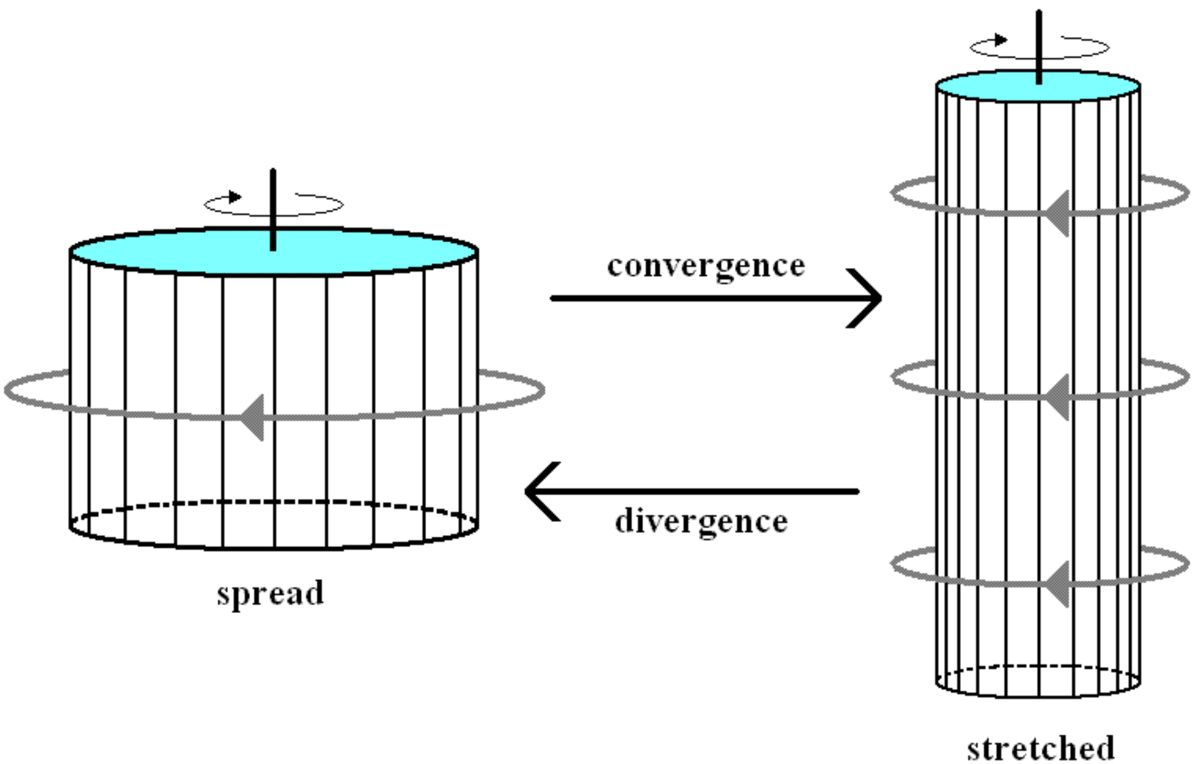
A northerly wind (v < 0) will advect higher f to the south, so

$$-v\beta > 0 \rightarrow \frac{\partial \zeta}{\partial t} > 0$$


The Beta Effect
HIGHER VALUES OF EARTH'S VORTICITY
INDUCED STEERING
H
βv<0
βv>0
L
N
LOWER VALUES OF EARTH'S VORTICITY

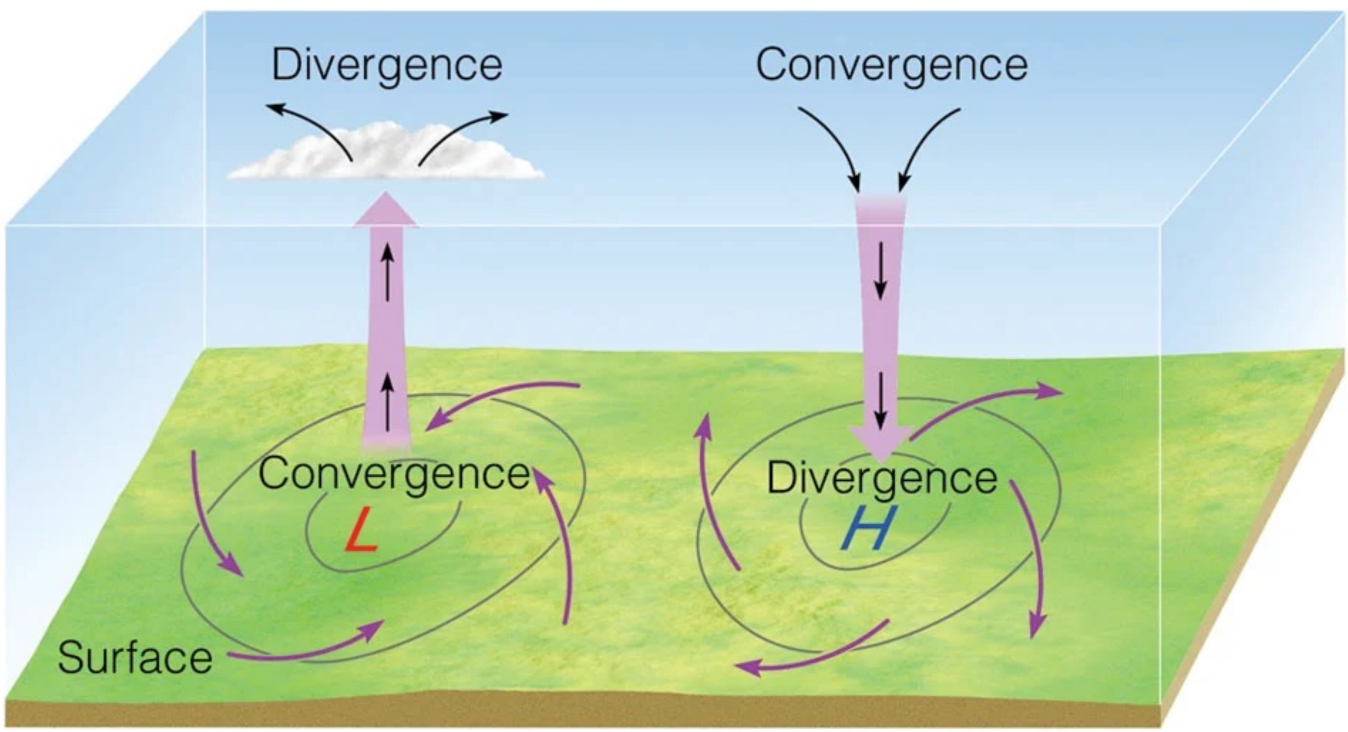
3) The divergence acting on relative vorticity: $-\zeta\left(\vec{\nabla}\bullet\vec{V}\right)$

- This term is often referred to as vortex tube stretching or shrinking.
- Consider a vertical column of air (vortex tube) rotating with some vorticity $\zeta < 0$.
- If the column is squished by convergence, $\left(\vec{\nabla}\bullet\vec{V}\right)<0$, it will stretch vertically and the rotation rate will increase.



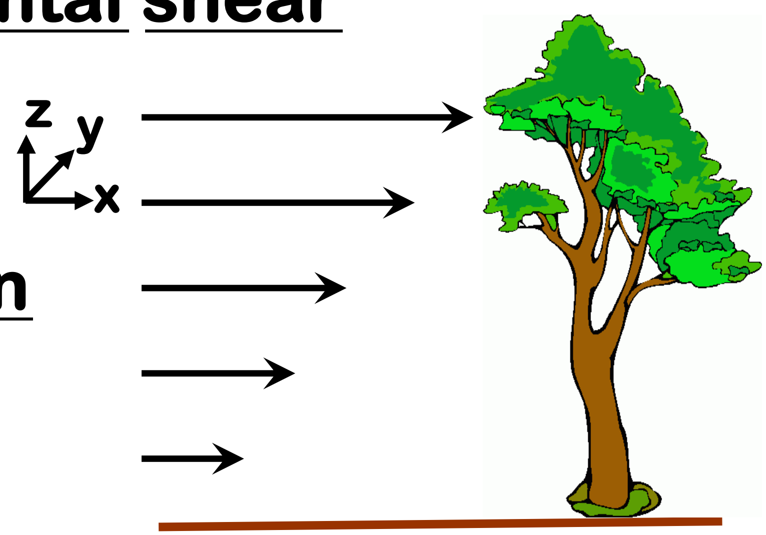
4) Planetary vorticity acting on divergence: $-f\left(\vec{\nabla}\bullet\vec{V}\right)$

- Consider purely convergent $\left(\vec{\nabla}\bullet\vec{V}\right)<0$ and divergent $\left(\vec{\nabla}\bullet\vec{V}\right)>0$ winds, like at the surface below.
- Because the Coriolis force deflects to the right of motion, the winds are turned such that purely convergent flow attains a counterclockwise (cyclonic) spin, and purely divergent flow becomes divergent and clockwise (anticyclonic).



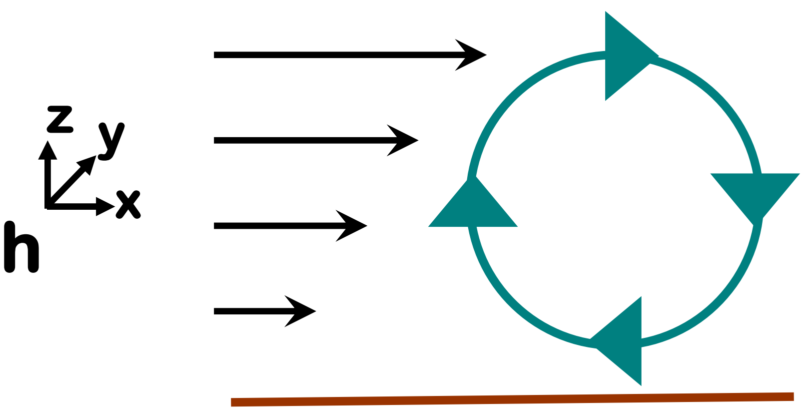
5) Tilting horizontal shear vorticity

- This term is also known as the twisting or tipping term, and represents the production of vertical relative vorticity by literally tipping horizontal shear vorticity into the vertical.



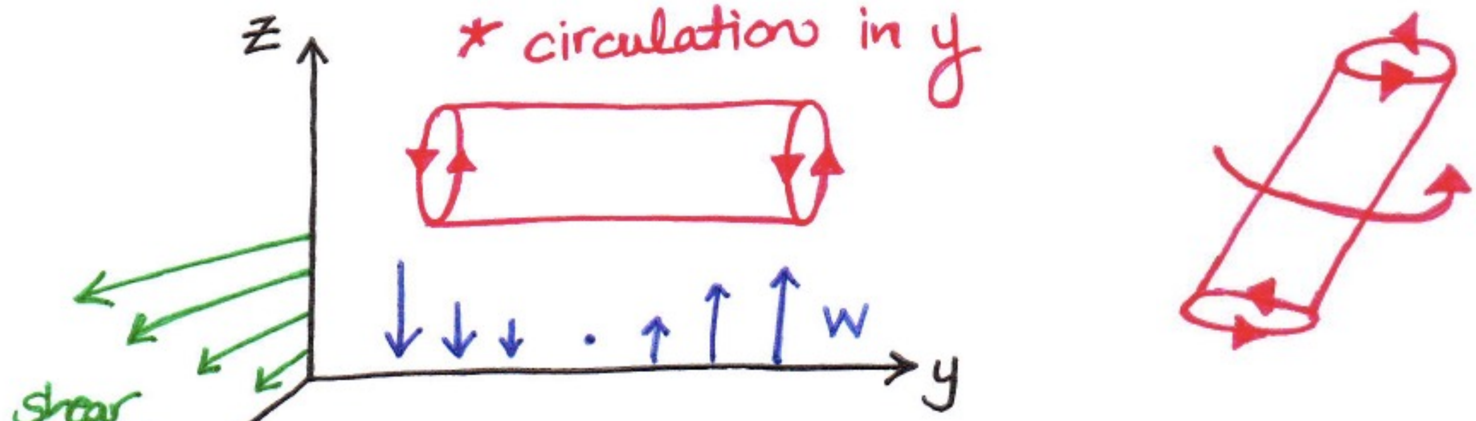
- Suppose, we have vertical speed shear of the wind in the x direction such that du/dz > 0 (westerly winds increase with height).

- From natural coordinates, we know speed shear normal to the direction of the flow generates vorticity.



- In this case, it is a clockwise circulation about the y axis, which is directed into the slide.

5) The tilting of horizontal shear vorticity: $-\hat{k} \bullet \vec{\nabla} w \times \frac{\partial \vec{V}}{\partial z}$



- In the figure, I've imposed vertical motion along the y axis, the same one as our circulation, with dw/dy > 0.

- Thus, the horizontal circulation will be differentially tilted into the vertical, generating vertical vorticity. Here,

$$\frac{\partial \zeta}{\partial t} = -\hat{k} \bullet \vec{\nabla} w \times \frac{\partial \vec{V}}{\partial z} = -\hat{k} \bullet \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & \frac{\partial w}{\partial y} & 0 \\ \frac{\partial u}{\partial z} & 0 & 0 \end{vmatrix} = -\left(-\frac{\partial u}{\partial z} \frac{\partial w}{\partial y} \right) > 0$$

6) Friction: $\hat{k}\bullet\vec{\nabla}\times\overrightarrow{Fr}$

$$\frac{\partial\zeta}{\partial t}=\hat{k}\bullet\vec{\nabla}\times\overrightarrow{Fr}=\hat{k}\bullet\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ F_x & F_y & F_z\end{vmatrix}=\frac{\partial F_y}{\partial x}-\frac{\partial F_x}{\partial y}$$

- **This expression says there needs to be a horizontal gradient of friction, i.e., surface roughness, to generate vertical vorticity, such as along a coastline.**