Problem Set #2: Sutcliffe-Trenberth and QGPV Equations

1. Overview:

The approximate Sutcliffe-Trenberth equation (1) relates QG forcing for vertical motion to advection of geostrophic relative vorticity by the thermal wind (Term A), whereas the QG Potential Vorticity (PV) (QGPV) equation (2) relates QG forcing for geopotential height changes to advection of QGPV by the geostrophic wind (Term B).

The purpose of this problem set is to have you use archived GEMPAK plots from Tom Galarneau's Real-Time QG Analysis and Forecast Diagnostics webpage located at:

http://www.atmos.albany.edu/student/tomjr/weather_links/qg_diagnostics.htm

to interpret the synoptic pattern over North America in the context of the approximate Sutcliffe-Trenberth and QGPV equations. Additionally, you will compare forcing associated with the approximate Sutcliffe-Trenberth and QGPV equations (1) and (2) to forcing associated with the traditional QG omega and height tendency equations for a frictionless, dry atmosphere (4) and (5).

$$\begin{pmatrix} \nabla_{p}^{2} + \frac{f_{0}^{2}}{\sigma} \frac{\partial^{2}}{\partial p^{2}} \end{pmatrix} \omega = \frac{f_{0}}{\sigma} 2 \left(\frac{\partial \vec{V}_{g}}{\partial p} \cdot \vec{\nabla}_{p} \zeta_{g} \right)$$

$$\text{Term A} \\
\begin{pmatrix} \nabla_{p}^{2} + \frac{f_{0}^{2}}{\sigma} \frac{\partial^{2}}{\partial p^{2}} \end{pmatrix} \chi = f_{0} \left(-\vec{V}_{g} \cdot \vec{\nabla}_{p} q \right)$$

$$\text{Term B}$$
(1)

where q is QGPV defined as:

$$q = \frac{1}{f_0} \nabla_p^2 \Phi + f + \frac{\partial}{\partial p} \left(\frac{f_0}{\sigma} \frac{\partial \Phi}{\partial p} \right)$$
(3)

$$\begin{pmatrix} \nabla_{p}^{2} + \frac{f_{0}^{2}}{\sigma} \frac{\partial^{2}}{\partial p^{2}} \end{pmatrix} \omega = -\frac{f_{0}}{\sigma} \frac{\partial}{\partial p} \Big[-\vec{V}_{g} \cdot \vec{\nabla}_{p} \big(\boldsymbol{\zeta}_{g} + f \big) \Big] - \frac{R}{\sigma p} \nabla_{p}^{2} \big(-\vec{V}_{g} \cdot \vec{\nabla}_{p} T \big)$$
(4)
Term C Term D

$$\left(\nabla_{p}^{2} + \frac{f_{0}^{2}}{\sigma} \frac{\partial^{2}}{\partial p^{2}} \right) \chi = -f_{0} \vec{V}_{g} \cdot \vec{\nabla}_{p} \big(\boldsymbol{\zeta}_{g} + f \big) - \frac{f_{0}^{2}}{\sigma} \frac{\partial}{\partial p} \Big[\frac{R}{p} \big(-\vec{V}_{g} \cdot \vec{\nabla}_{p} T \big) \Big]$$
(5)
Term E Term F

[Note that the QGPV equation (2) is identical to the traditional QG height tendency equation (5), given that the left hand sides of (2) and (5) are identical, while the right hand sides are written differently.

2. Symbols

- ζ_{g} : geostrophic relative vorticity
- χ : geopotential height tendency $\left(=\frac{\partial \Phi}{\partial t}\right)$
- ω : vertical motion
- $\vec{V}_{_{o}}$: geostrophic wind
- Φ : geopotential height
- f: Coriolis parameter $(2\Omega \sin \phi)$
- f_0 : Coriolis parameter approximation (10⁻⁴ s⁻¹)
- σ : static stability parameter (2x10⁻⁶ m² Pa⁻² s⁻²)
- T : temperature

3. Plots:

Pick a single observed analysis **or** forecast time (this will be time **T=0** h). Print plots of North America (labeled 1-9) displaying:

1) 700-hPa HGHT, 1000–500-hPa THICK, and Term A.

2) 500-hPa HGHT, QGPV, and Term B.

3) 1000-hPa HGHT, 1000–500-hPa THICK, and the total Sutcliffe forcing (sum of the terms on the RHS of the Sutcliffe development equation; as in PS #1).

4) 1000-hPa HGHT, 1000–500-hPa THICK, and the total Sutcliffe-Petterssen forcing (sum of the terms on the RHS of the Sutcliffe-Petterssen development equation).

700-hPa HGHT, 1000-500-hPa THICK, and

5) Term C.

6) Term D.

500-hPa HGHT, 700-300-hPa THICK, and

- 7) Term E.
- 8) Term F.

9) 1000-hPa HGHT, 1000–500-hPa THICK, and 1000-hPa geostrophic relative vorticity 12 h later (this will be time **T=12 h**).

4. Questions:

1. At **T=0 h**, where is the Sutcliffe-Trenberth forcing for rising and sinking motion the largest? Relate the forcing patterns to the synoptic pattern.

2. Discuss how the regions of Sutcliffe-Trenberth forcing you identified in 1) compare to regions of both Sutcliffe forcing and Sutcliffe-Petterssen forcing at T=0 h.

3. Based on QGPV forcing at **T=0 h**, how would you expect the primary troughs and ridges to move? Would you expect them to strengthen or weaken?

4. Discuss how well Sutcliffe-Trenberth forcing and QGPV forcing at **T=0 h** predict the position and strength of the primary troughs and ridges at **T=12 h**.

5. How does QG forcing for vertical motion at **T=0 h**, assessed using the Sutcliffe-Trenberth equation, compare to the forcing derived from the traditional QG omega equation? Explain.

6. How does QG forcing for geopotential height rises and falls at **T=0 h**, assessed using the QGPV equation, compare to the forcing derived from the traditional QG height tendency equation? Explain.