Angular Momentum Budget Equations

Inflow layer: (u < 0)

$$\frac{\partial M_I}{\partial t} = -\int_0^h \int_0^{r_1} \int_0^{2\pi} r^2 \rho f u \, d\lambda \, dr \, dz - \int_0^{r_1} \int_0^{2\pi} r^2 \left(\tau_{\lambda z}\right) d\lambda dr - \int_0^{2\pi} \int_0^{h} r_1 \left(\rho m u\right)_{r_1} dz d\lambda - \int_0^{r_1} \int_0^{2\pi} r \left(\rho m w\right)_h d\lambda dr$$

(1) (2) (3) (4)

Outflow layer: (u > 0)

$$\frac{\partial M_O}{\partial t} = -\int_h^{z_T} \int_0^{r_1} \int_0^{2\pi} r^2 \rho f u \, d\lambda \, dr \, dz - \int_0^{2\pi} \int_h^{z_T} r_1 \left(\rho m u\right)_{r_1} dz d\lambda + \int_0^{r_1} \int_0^{2\pi} r \left(\rho m w\right)_h d\lambda dr$$

(5) (6) (7)

(1) & (5) = Conversion of radial (*u*) to tangential (*v*) flow & angular momentum (*M* = *rv*) by Coriolis torque

(2) = Loss of angular momentum due to friction

(3) & (6) = Radial flux of momentum across outer radius (r_1)

(4) & (7) = Vertical flux of momentum through BL top (*h*)

Angular Momentum Budget

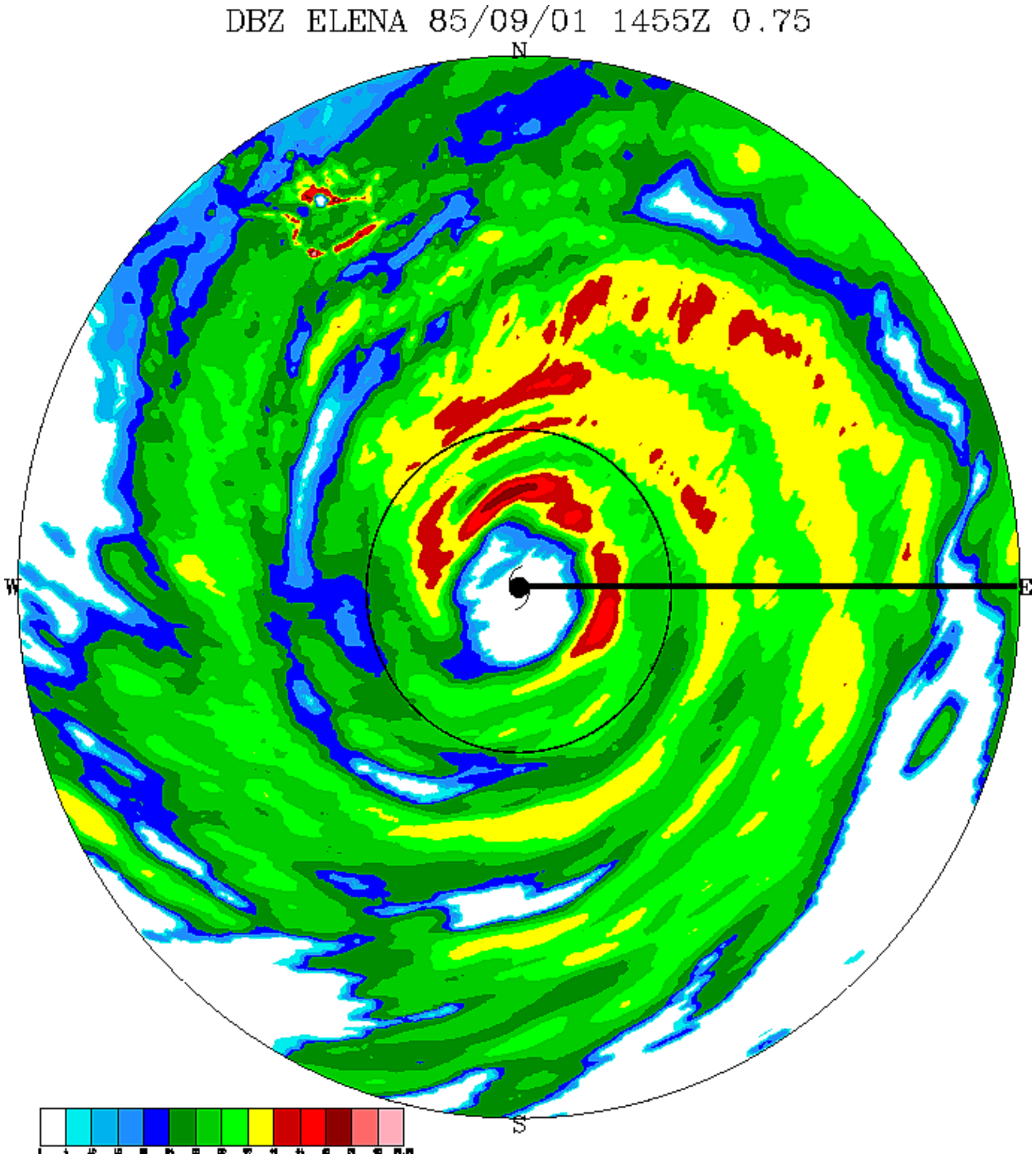
TABLE 5. Empirical Angular Momentum Budgets for Mature Hurricanes

Storm	Radial Domain, km	**1 & 5** Coriolis Torque	Mean Horizontal Flux	Eddy Horizontal Flux	Mean Vertical Flux	Eddy Vertical Flux	Loss to Sea
			3 & 6		**4 & 7**		**2**
Mean storm [*Palmén and Riehl*, 1957]							
Mean storm [*Pfeffer*, 1958]							
Mean storm [*Palmén and Riehl*, 1957]							
Mean storm [*Pfeffer*, 1958]							

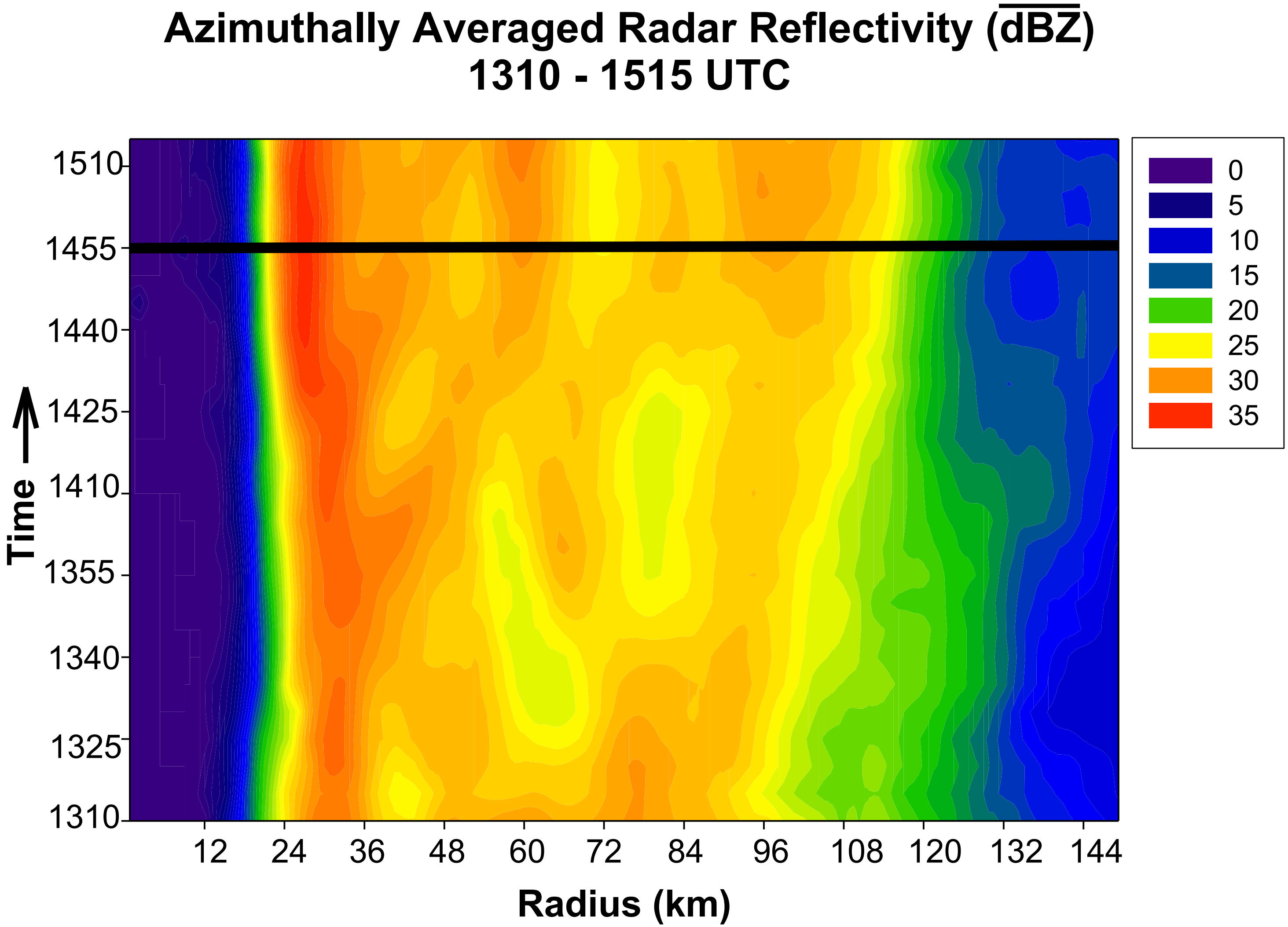
The horizontal (3 & 6) and vertical flux (4 & 7) terms above have been broken down into azimuthal mean (λ) and eddy components:

$$q = \overline{q} + q' \quad \text{with} \quad \overline{q} = \frac{1}{2\pi}\int_0^{2\pi} q\, d\lambda \quad \text{and} \quad \overline{q'} = \frac{1}{2\pi}\int_0^{2\pi} q'\, d\lambda = 0$$

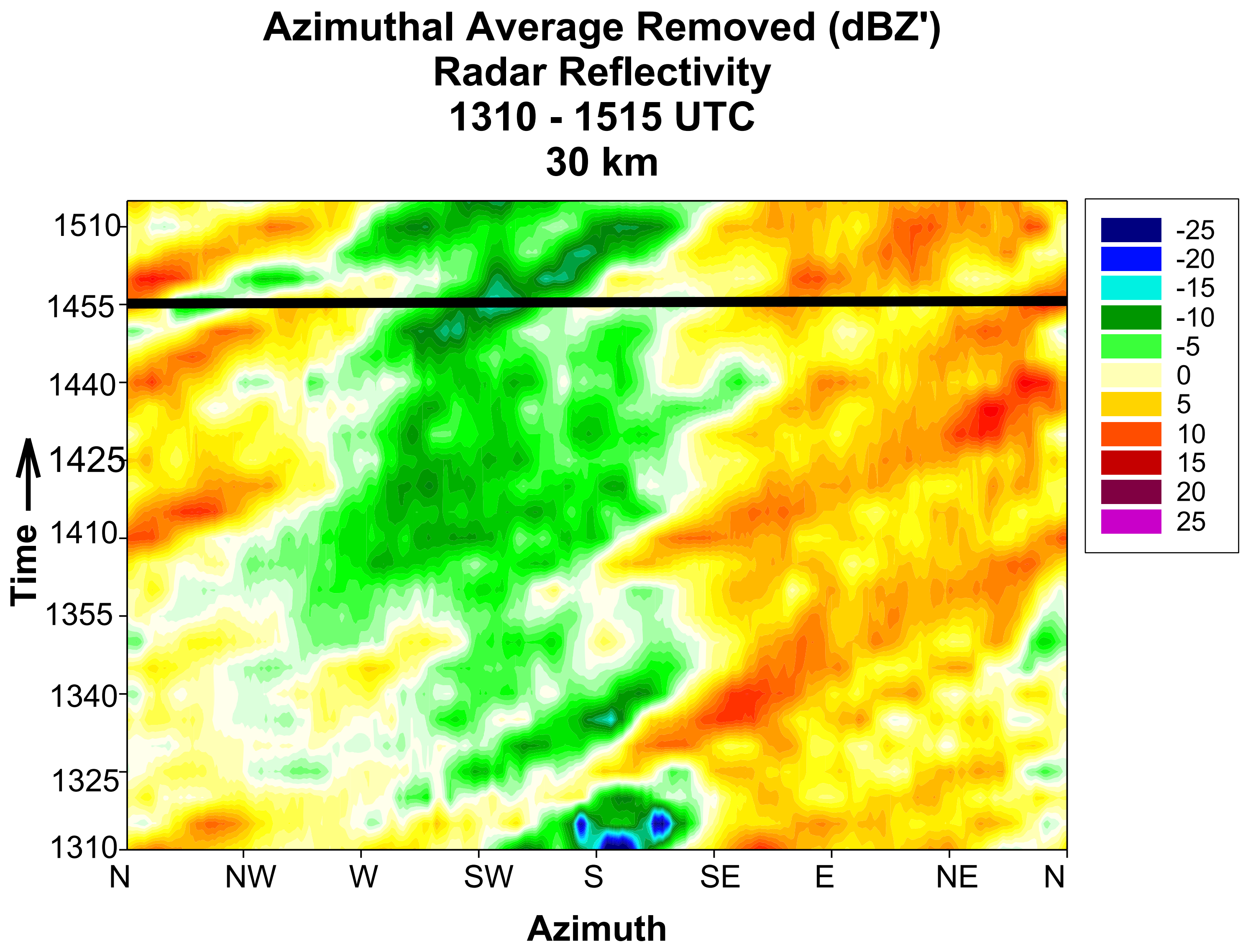
Hurricane Elena (1985): AQQ radar reflectivity



Hurricane Elena (1985): Azimuthally-averaged reflectivity



Hurricane Elena (1985): Azimuthally-asymmetric reflectivity



Angular Momentum Budget

TABLE 5. Empirical Angular Momentum Budgets for Mature Hurricanes

Storm	Radial Domain, km	Coriolis Torque	Mean Horizontal Flux	Eddy Horizontal Flux	Mean Vertical Flux	Eddy Vertical Flux	Loss to Sea
		1 & 5	3 & 6		4 & 7		2
		Lower Layer					
Mean storm [*Palmén and Riehl*, 1957]	0–333	48.0	74.0	12.0	−40.0	−20.0	−74.0
	333–666	143.0	−9.0	26.0	...	−44.0	−116.0
Mean storm [*Pfeffer*, 1958]	222–444	54.0	22.0	8.0	−5.0	...	−8.5
	444–666	78.0	−1.0	26.0	4.0	...	−123.0
		Upper Layer					
Mean storm [*Palmén and Riehl*, 1957]	0–333	−48.0	−20.0	8.0	40.0	20.0	
	333–666	−143.0	76.0	23.0	...	44.0	
Mean storm [*Pfeffer*, 1958]	222–444	−54.0	46.0	20.0	5.0	...	
	444–666	−78.0	−11.0	13.0*	−4.0	...	

Angular Momentum Budget

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Note that the Coriolis torque (terms 1 & 5) and vertical flux terms (4 & 7) exactly cancel over the depth of the troposphere ☹

Angular Momentum Budget

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TABLE 5. Empirical Angular Momentum Budgets for Mature Hurricanes

Storm	Radial Domain, km	Coriolis Torque (1 & 5)	Mean Horizontal Flux (3 & 6)	Eddy Horizontal Flux (3 & 6)	Mean Vertical Flux (4 & 7)	Eddy Vertical Flux (4 & 7)	Loss to Sea (2)
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	444–666	−78.0	−11.0	13.0*	−4.0	. . .	

Note that the Coriolis torque (terms 1 & 5) and vertical flux terms (4 & 7) exactly cancel over the depth of the troposphere ☹

The horizontal flux terms (3 & 6) are the big sources and frictional loss to the sea (2) is the angular momentum sink

Angular Momentum Budget: Mean and Eddy Flux Terms

TABLE 5. Empirical Angular Momentum Budgets for Mature Hurricanes

Storm	Radial Domain, km	1 & 5 Coriolis Torque	Mean Horizontal Flux (3 & 6)	Eddy Horizontal Flux (3 & 6)	Mean Vertical Flux (4 & 7)	Eddy Vertical Flux (4 & 7)	Loss to Sea (2)
		Lower Layer					
Mean storm [*Palmén and Riehl*, 1957]	0–333	48.0	74.0	12.0	−40.0	−20.0	−74.0
	333–666	143.0	−9.0	26.0	. . .	−44.0	−116.0
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	444–666	78.0	−1.0	26.0	4.0	. . .	−123.0
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	333–666	−143.0	76.0	23.0	. . .	44.0	
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The horizontal (3 & 6) and vertical flux (4 & 7) terms above have been broken down into azimuthal mean (λ) and eddy components:

$$q = \overline{q} + q' \quad \text{with} \quad \overline{q} = \frac{1}{2\pi}\int_0^{2\pi} q\, d\lambda \quad \text{and} \quad \overline{q'} = \frac{1}{2\pi}\int_0^{2\pi} q'\, d\lambda = 0$$

Angular Momentum Budget Equations

Inflow layer: (u < 0)

$$\frac{\partial M_I}{\partial t} = -\int_0^h \int_0^{r_1} \int_0^{2\pi} r^2 \rho f u \, d\lambda \, dr \, dz - \int_0^{r_1} \int_0^{2\pi} r^2 \left(\tau_{\lambda z}\right) d\lambda dr - \int_0^{2\pi} \int_0^h r_1 \left(\rho m u\right)_{r_1} dz d\lambda - \int_0^{r_1} \int_0^{2\pi} r \left(\rho m w\right)_h d\lambda dr$$

Outflow layer: (u > 0)

$$\frac{\partial M_O}{\partial t} = -\int_h^{z_T} \int_0^{r_1} \int_0^{2\pi} r^2 \rho f u \, d\lambda \, dr \, dz - \int_0^{2\pi} \int_h^{z_T} r_1 \left(\rho m u\right)_{r_1} dz d\lambda + \int_0^{r_1} \int_0^{2\pi} r \left(\rho m w\right)_h d\lambda dr$$

Angular Momentum Budget: Mean and Eddy Flux Terms

For the product of two variables, like our flux terms with m and u (or m and w):

$$\overline{mu} = \overline{(\bar{m}+m')(\bar{u}+u')} = \overline{\bar{m}\bar{u}} + \overline{\bar{m}u'} + \overline{m'\bar{u}} + \overline{m'u'} = \bar{m}\bar{u} + \overline{m'u'}$$

= the product of the azimuthal means +
the product of the asymmetries averaged around λ

The value of $\overline{m'u'}$ depends on the spatial correlation between m and u:

For example, if m' > 0 when u' < 0 and m' < 0 when u' > 0, then $m'u'$ is always < 0 and it will NOT vanish when averaged around λ.

Angular Momentum Budget: Horizontal Momentum Flux

Considering the horizontal momentum flux term (the large momentum source term):

$$\frac{\partial M}{\partial T} = -\int_0^{z_T}\int_0^{2\pi} r_1 \left(\rho m u\right)_{r_1} d\lambda\, dz = -2\pi \int_0^{z_T} r_1 \rho \left(\overline{m}\overline{u} + \overline{m'u'}\right)_{r_1} dz$$

In the inflow layer, the eddy term is generally small (see later slides); thus, *+m* is fluxed in towards the center:

$$\overline{m} > 0 \text{ and } \overline{u} < 0, \text{ thus } \frac{\partial M_I}{\partial t} > 0$$

Angular Momentum Budget: Outflow Layer Mean Horizontal Momentum Flux

Close to the center (r < 300 km) in the outflow layer, the flow is cyclonic and +*m* is carried away from the storm:

$$\bar{m} > 0 \quad \text{and} \quad \bar{u} > 0, \text{ thus } \frac{\partial M_O}{\partial t} < 0$$

At further distances, the flow turns anticyclonic (WHY?) and −*m* is transported away, helping to strengthen the vortex:

$$\bar{m} < 0 \quad \text{and} \quad \bar{u} > 0, \text{ thus } \frac{\partial M_O}{\partial t} > 0$$

TC outflow layer structure

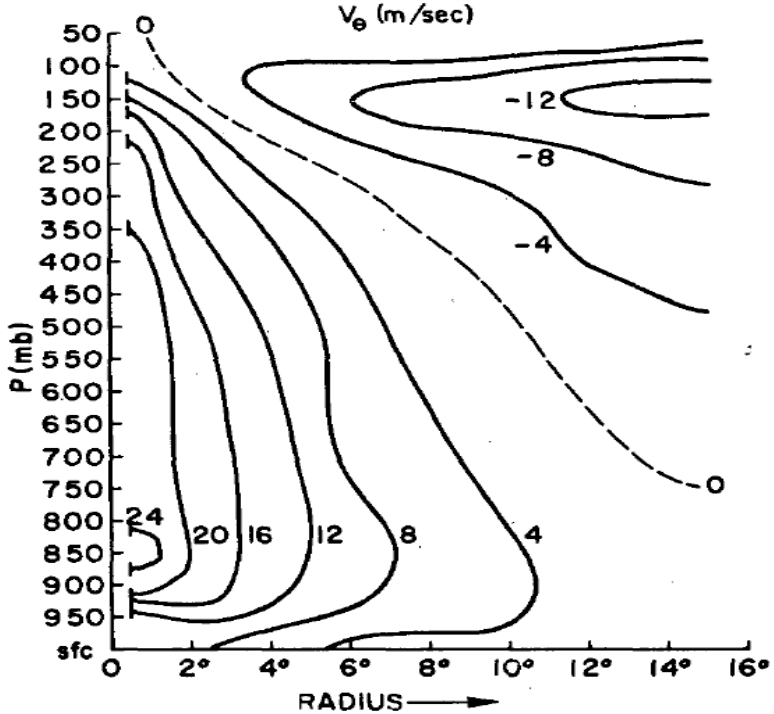
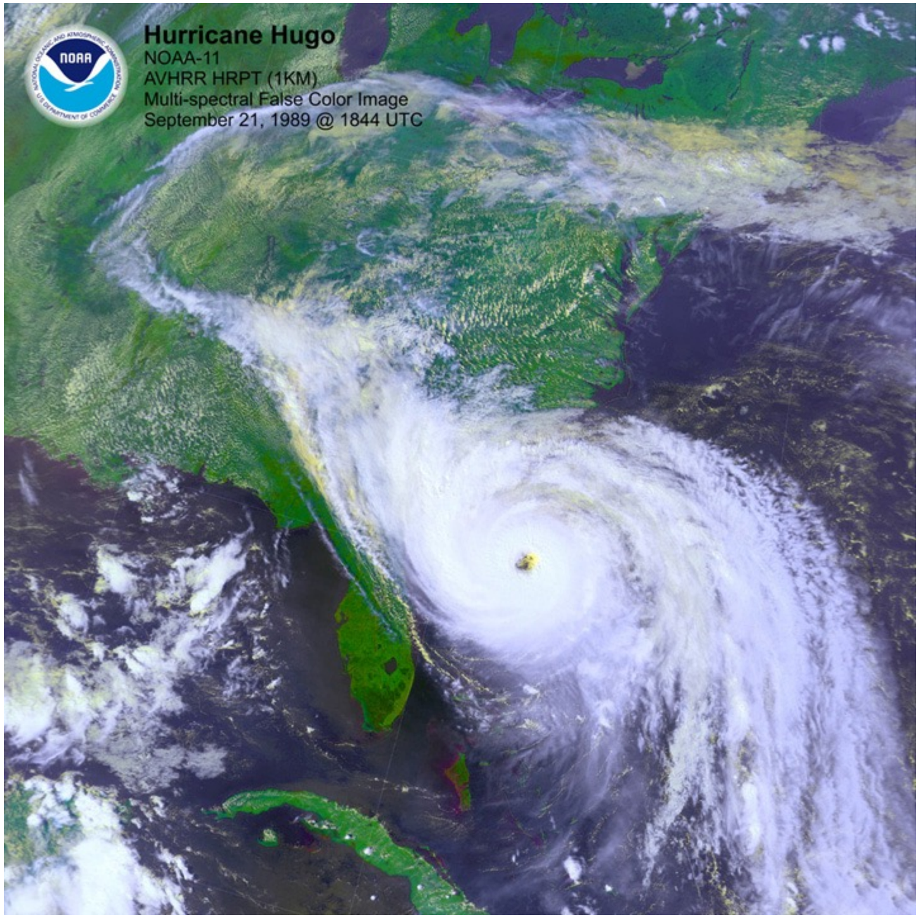


FIG. 9. Two-dimensional cross section of V_θ (m s^{-1}) in stationary (NAT) coordinates. Positive numbers denote cyclonic flow.

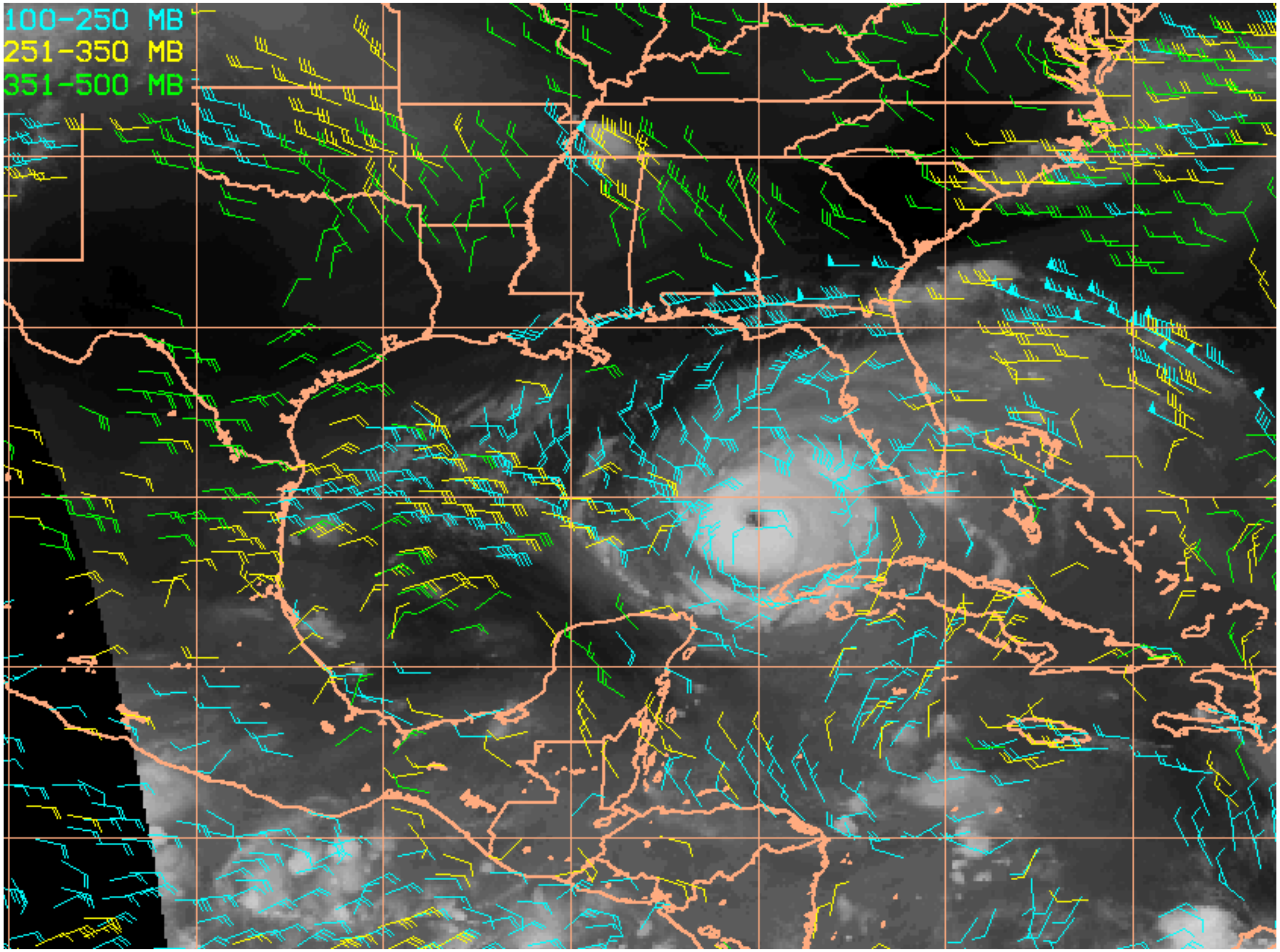
Cyclonic to anticyclonic flow aloft due to friction



Outflow jets (not symmetric!)

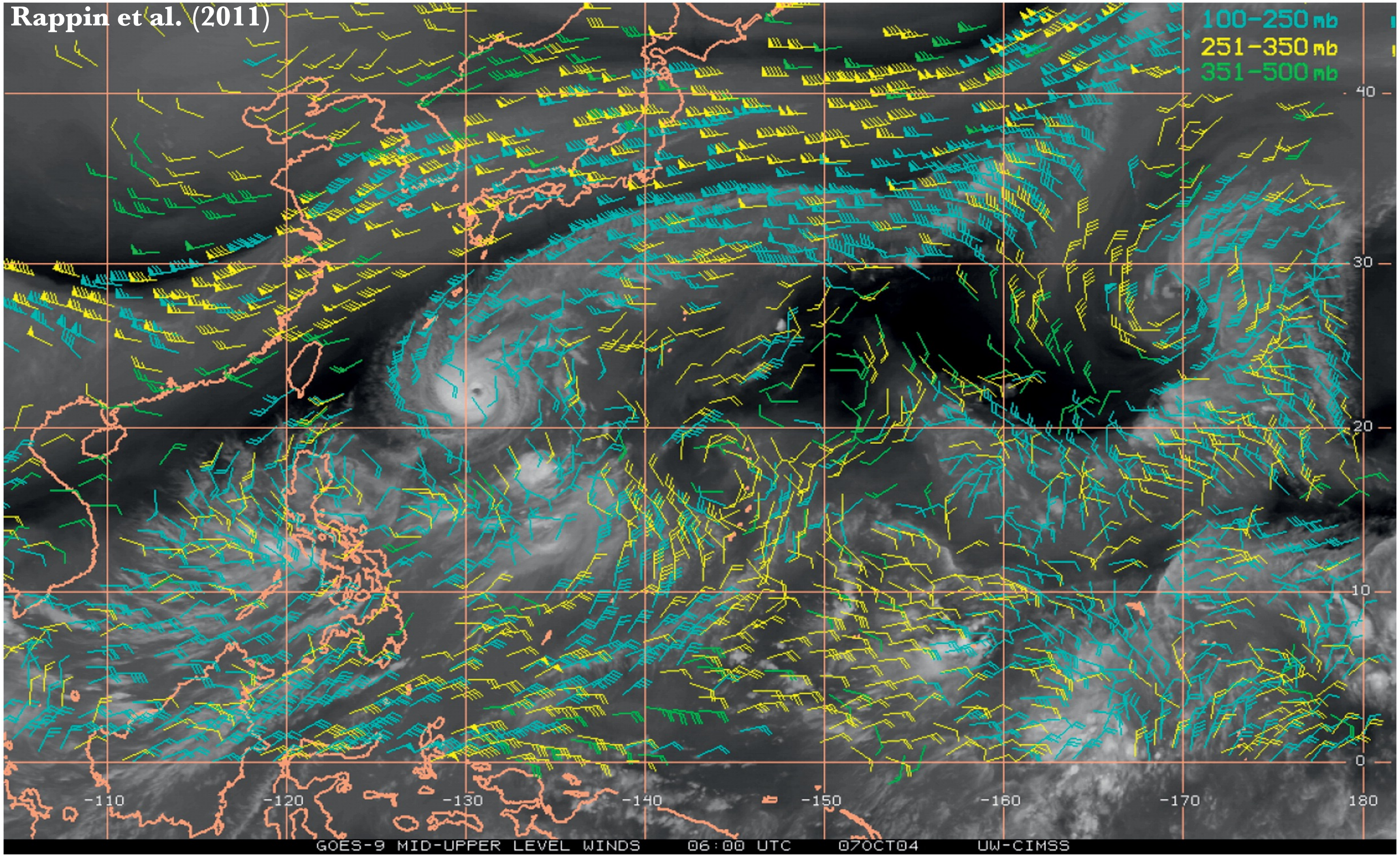
Hurricane Hugo (1989)

TC outflow layer structure: It's not symmetric!



http://cimss.ssec.wisc.edu/tropic/real-time/atlantic/winds/winds.html

TC outflow layer structure: Eddies are important!



Supertyphoon Ma-on (2004) satellite-derived winds at the beginning of rapid intensification

McBride (1981) & McBride and Zehr (1981): Observational analysis of TC development

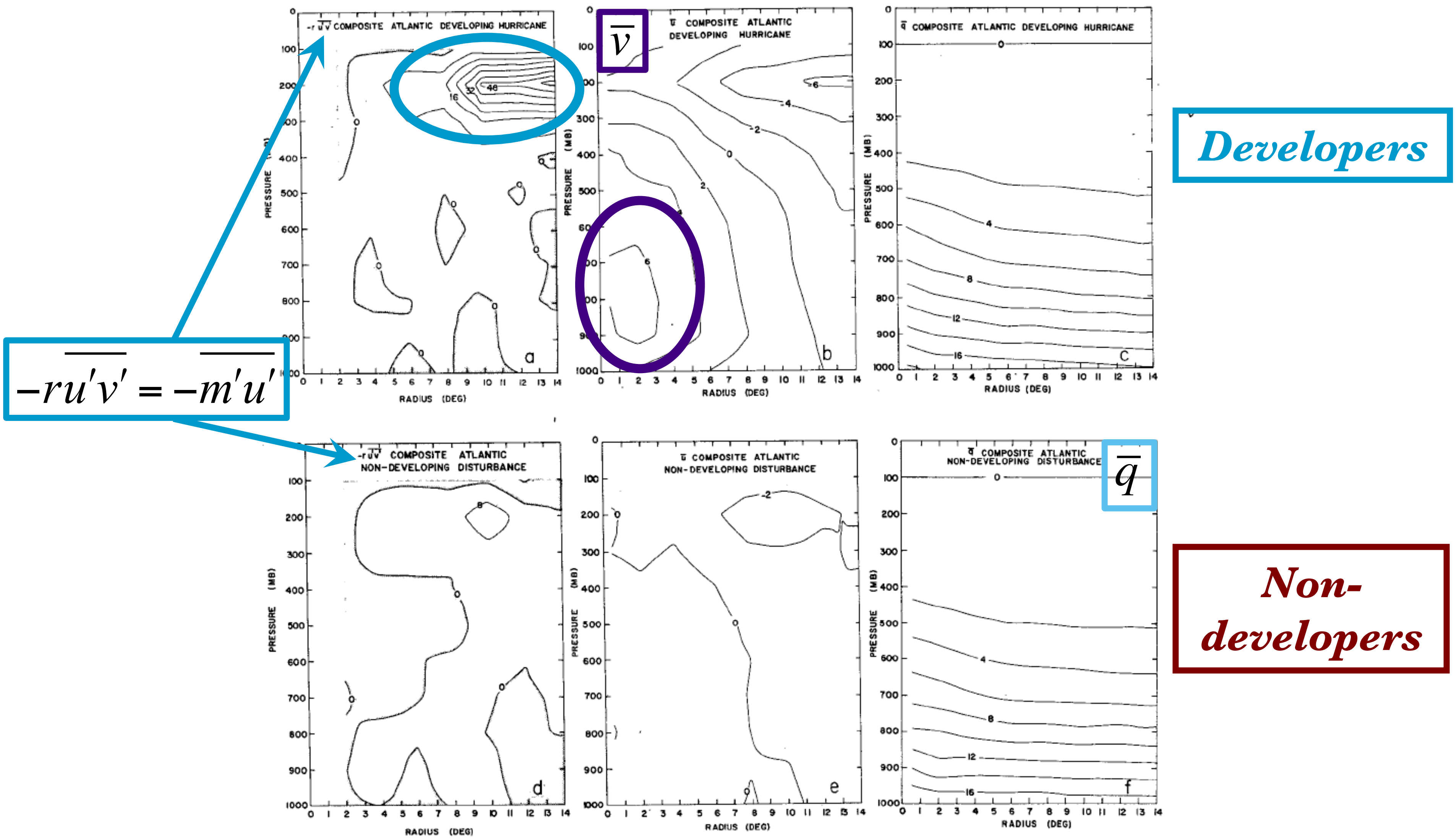


FIG. 1. Distributions of inward eddy flux of momentum ($-r\overline{u'v'}$), tangential wind ($\bar{u}$) and specific humidity ($\bar{q}$) for the composite developing and non-developing Atlantic tropical disturbances. Units are 10 deg m^2 s^{-2}, m s^{-1} and g kg^{-1}, respectively.

McBride (1981) & McBride and Zehr (1981): Observational analysis of TC development

$$-\overline{ru'v'} = -\overline{m'u'}$$

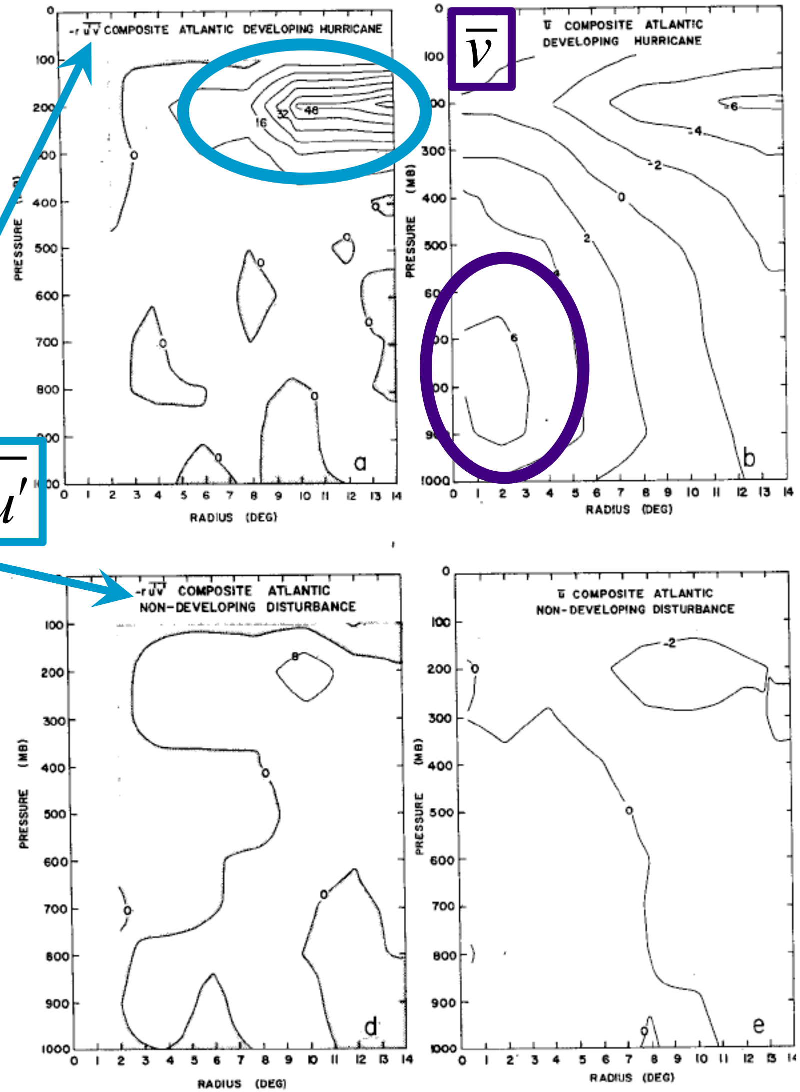


FIG. 1. Distributions of inward eddy flux of momentum ($-\overline{ru'v'}$), tangential wind ($\bar{u}$) and specific humidity ($\bar{q}$) for the composite developing and non-developing Atlantic tropical disturbances. Units are 10 deg m² s⁻², m s⁻¹ and g kg⁻¹, respectively.

Are the eddy momentum fluxes or the stronger initial vortex the more important factor in genesis?

McBride (1981) & McBride and Zehr (1981): Observational analysis of TC development

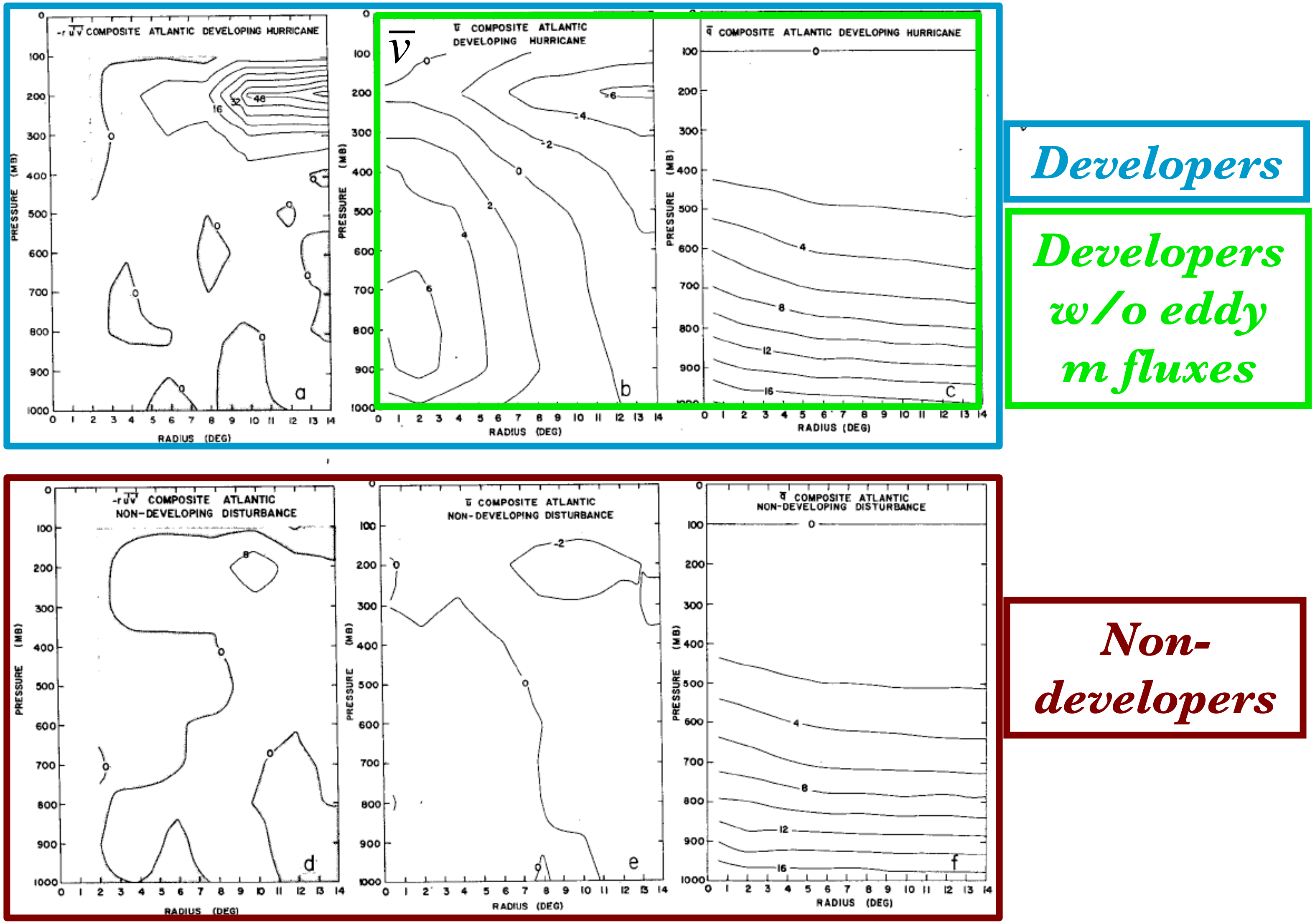


FIG. 1. Distributions of inward eddy flux of momentum ($-r\overline{u'v'}$), tangential wind ($\bar{u}$) and specific humidity ($\bar{q}$) for the composite developing and non-developing Atlantic tropical disturbances. Units are 10 deg m^2 s^{-2}, m s^{-1} and g kg^{-1}, respectively.

Pfeffer & Challa (1981): Role of eddy momentum fluxes in TC development

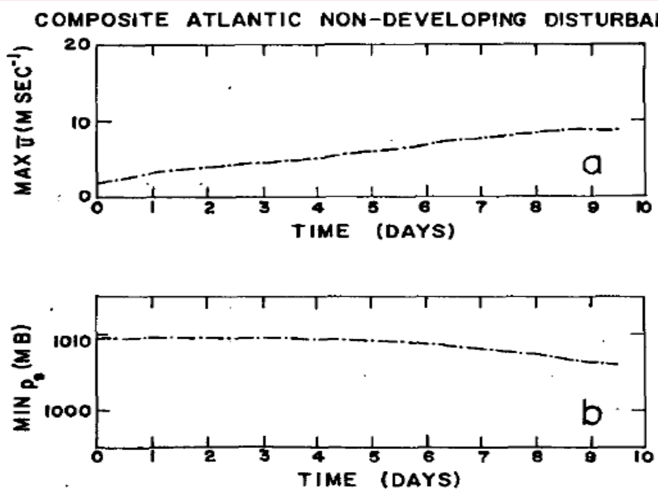


FIG. 3. Time variations of (a) maximum tangential wind velocity at p = 900 mb and (b) minimum surface pressure in the numerical integration corresponding to the composite Atlantic non-developing disturbance.

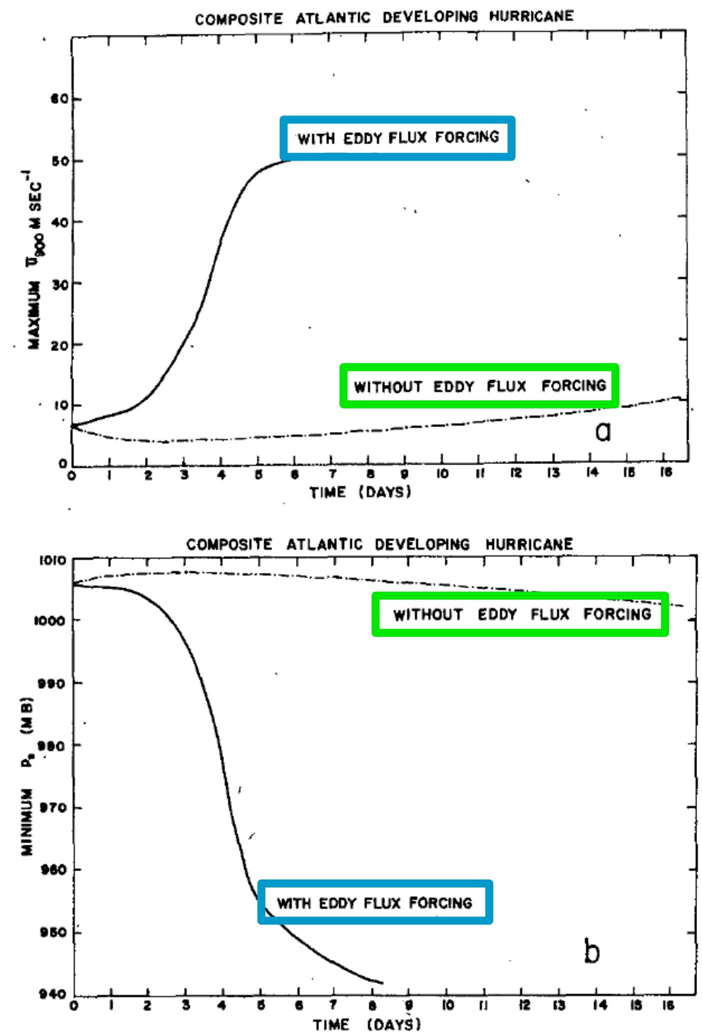


FIG. 2. (a) Time variations of maximum tangential wind velocity at p = 900 mb in the numerical integrations corresponding to the composite Atlantic developing hurricane with and without eddy fluxes of momentum. (b) Time variations of minimum surface pressure in the same integrations.

Pfeffer and Challa (1981) **initialized** a **model** with the **vortex structure** and **eddy momentum forcing** of McBride (1981) to **highlight** the **important role of eddies** in TC development

Eddy momentum fluxes are important!

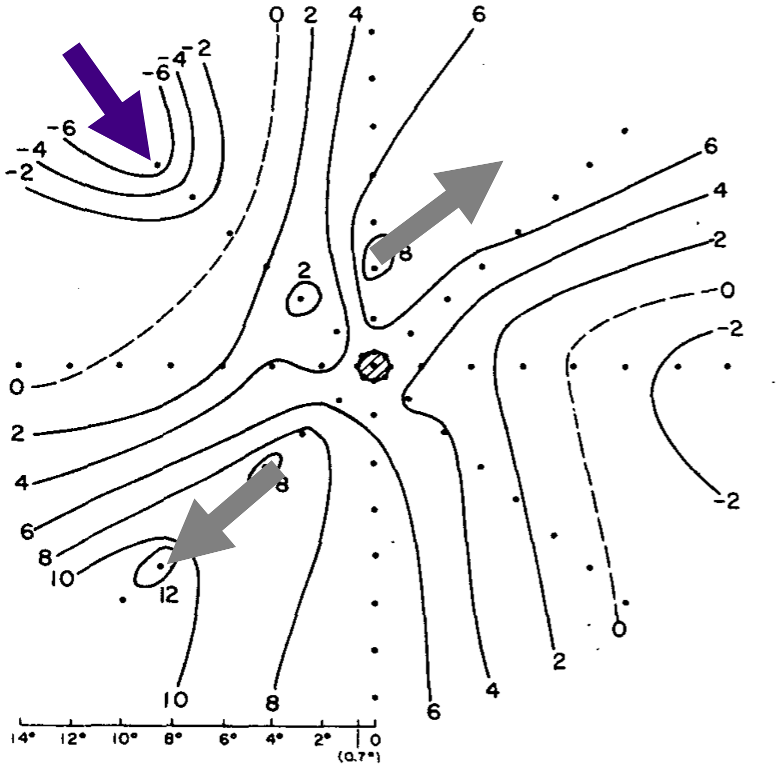
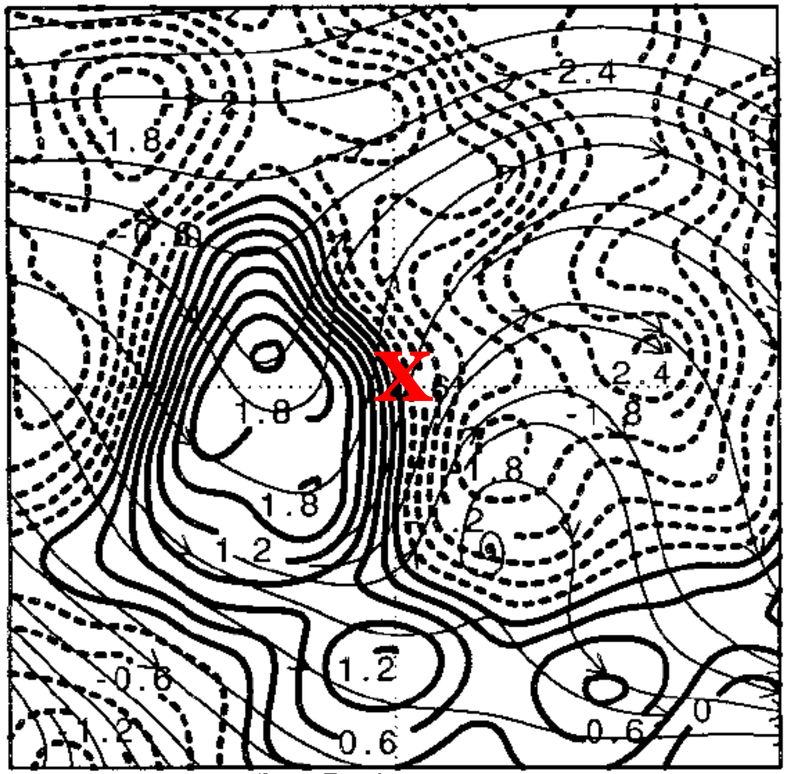


FIG. 15. Plan view of V_r (m s^{-1}) in NAT coordinates at 150 mb.

Frank (1977)*: *Pacific, all TCs



b. 200 hPa Bahamas cases

Bracken and Bosart (2000)*: *Atlantic, just TDs

Across many studies, ocean basins, and TC intensities, there is evidence of a trough upstream of the TC, and thus eddy momentum forcing over the TC.