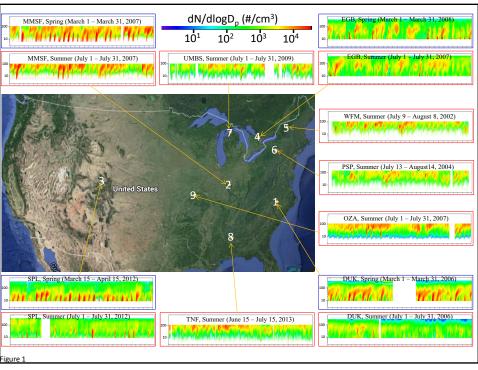
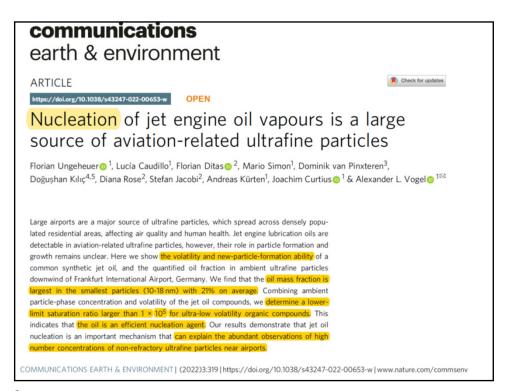
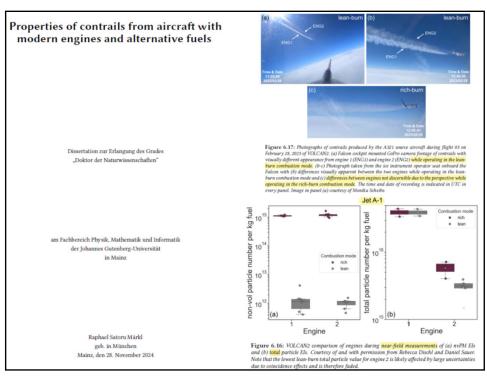
# Lecture 10: Nucleation or new particle formation in the atmosphere







# **Nucleation**

- 1. Homogeneous-homomolecular: self-nucleation of a single species. No foreign nuclei or surfaces involved.
- 2. Homogeneous-heteromolecular: self-nucleation of two or more species. No foreign nuclei or surfaces involved.
- 3. Heterogeneous-homomolecular: nucleation of a single species on a foreign substance.
- 4. Heterogeneous-heteromolecular: nucleation of two or more species on a foreign substance.

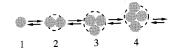
## **Classical Homogeneous Nucleation**

Kinetic approach

Constrained equilibrium approach (or classical approach)

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#### Kinetic approach



$$\frac{dN_{i}}{dt} = \beta_{i-1}N_{i-1}(t) - \gamma_{i}N_{i}(t) - \beta_{i}N_{i}(t) + \gamma_{i+1}N_{i+1}(t)$$

**Net Flux**  $J_{i+1/2} = \beta_i N_i - \gamma_{i+1} N_{i+1}$ 

Steady State 
$$J_{i+1/2} = J$$
, all  $i$ 

Let us define a quantity  $f_i$  by

setting 
$$f_1 = 1$$

$$f_i = \frac{\beta_1}{\gamma_2} \frac{\beta_2}{\gamma_3} \cdots \frac{\beta_{i-1}}{\gamma_i}$$

$$= \prod_{i=1}^{i-1} \frac{\beta_i}{\gamma_{j+1}}$$

$$\frac{J}{\beta_i f_i} = \frac{N_i}{f_i} - \frac{N_{i+1}}{f_{i+1}}$$

$$J \sum_{i=1}^{i_{\max}} \frac{1}{\beta_i f_i} = N_1 - \frac{N_{i_{\max}}}{f_{i_{\max}}}$$

$$J = N_1 \left( \sum_{i=1}^{i_{\text{max}}} \frac{1}{\beta_i f_i} \right)^{-1}$$

$$J = N_1 \left( \sum_{i=1}^{\infty} \frac{1}{\beta_i f_i} \right)^{-1}$$

## The Forward Rate Constant $\beta_i$

Collision rate (# cm<sup>-3</sup> s<sup>-1</sup>) between a monomer and an i-mer

$$Z_{AB} = \left(\frac{8\pi kT}{m_{AB}}\right)^{1/2} \sigma_{AB}^2 N_A N_B$$

$$Z_{1i} = (8\pi kT)^{1/2} \left(\frac{1}{m_1} + \frac{1}{m_i}\right)^{1/2} (r_1 + r_i)^2 N_1 N_i$$

$$m_i = i m_1 \qquad v_i = i v_1 \qquad a_1 = 4\pi \left(\frac{3v_1}{4\pi}\right)^{2/3}$$

$$Z_{1i} = \left(\frac{kT}{2\pi m_1}\right)^{1/2} a_1 \left(1 + \frac{1}{i}\right)^{1/2} \left(1 + i^{1/3}\right)^2 N_1 N_i \qquad \text{(# cm-3 s-1)}$$

$$\beta_i = \left(\frac{kT}{2\pi m_1}\right)^{1/2} \left(1 + \frac{1}{i}\right)^{1/2} (1 + i^{1/3})^2 \ a_1 N_1 \tag{# s-1}$$

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## The Reverse Rate Constant $\gamma_i$

$$\gamma_i = \gamma_i^s$$

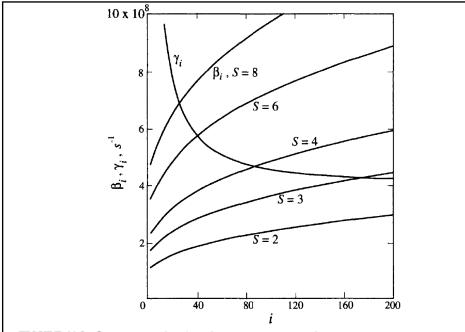


Kelvin Equation:

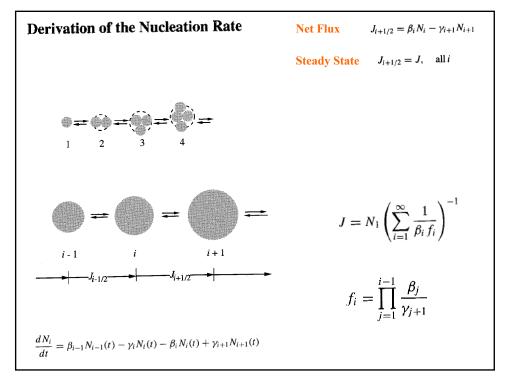
$$p_1 = p_1^s \exp\left(\frac{2\sigma v_1}{kT r_i}\right)$$

i-mer in equilibrium with surrounding vapor

$$\begin{split} \gamma_i &= \beta_i \\ &= \frac{p_1}{(2\pi m_1 kT)^{1/2}} \left(1 + \frac{1}{i}\right)^{1/2} (1 + i^{1/3})^2 a_i \\ &= \frac{p_1^s}{(2\pi m_1 kT)^{1/2}} \left(1 + \frac{1}{i}\right)^{1/2} (1 + i^{1/3})^2 a_i \exp\left(\frac{2\sigma v_1}{kTr_i}\right) \end{split}$$



**FIGURE 11.3**  $\beta_i$  and  $\gamma_i$  as a function of *i* for various values of the saturation ratio *S*. The two quantities are equal at the critical *i* values.



#### **Derivation of the Nucleation Rate**

$$i - 1 \qquad i \qquad i + 1$$

$$- I_{i-1/2} \qquad - I_{i+1/2} \qquad - I_{i+1$$

$$J = N_1 \left( \sum_{i=1}^{J} \frac{\beta_i}{\beta_i} f_i \right)$$

$$\stackrel{i-1}{\longrightarrow} \beta_i$$

$$f_i = \prod_{j=1}^{i-1} \frac{\beta_j}{\gamma_{j+1}}$$

$$f_i = S^{i-1} \prod_{j=1}^{i-1} \frac{\beta_j^s}{\gamma_{j+1}^s}$$

 $\beta_1^s$  is forward rate at saturation

$$\begin{array}{ccc} A_1 + A_1 \rightleftharpoons A_2 & & \left(\frac{\beta_1^s}{\gamma_2^s}\right) \left(\frac{\beta_2^s}{\gamma_3^s}\right) \end{array}$$

equilibrium constant for

$$3A_1 \rightleftharpoons A_3$$

$$iA_1 \rightleftharpoons A_i$$

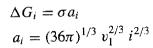
$$f_i = S^{i-1} \exp(-\Delta G_i / kT)$$

 $\Delta G_i$  is the Gibbs free energy change for formation of a cluster of size i at saturation (S=1)

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How to calculate  $\Delta G_i \longrightarrow$  Major assumption of classical nucleation theory

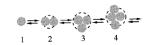
Capillarity Approximation:
Clusters of a small number of
molecules exhibit the same
surface tension as the bulk liquid

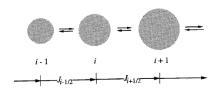


dimensionless surface tension,

$$\theta = (36\pi)^{1/3} v_1^{2/3} \sigma/kT$$

$$\Delta G_i = \theta k T i^{2/3}$$



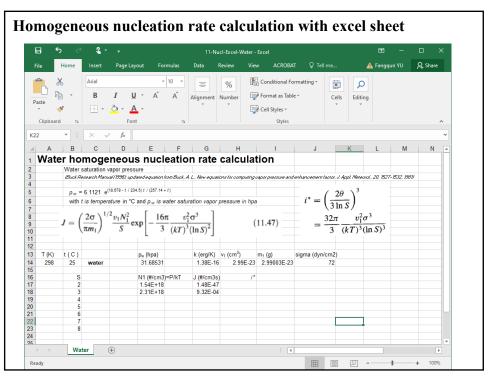


 $a_i$  is the surface area of a cluster of size i.

$$J = N_1 \left( \sum_{i=1}^{\infty} \frac{1}{\beta_i^3 S^i \exp(-\theta i^{2/3})} \right)^{-1}$$
(see textbook 11.1.3 for mathematical derivation)
$$J = \left( \frac{2\sigma}{\pi m_1} \right)^{1/2} \frac{v_1 N_1^2}{S} \exp\left[ -\frac{16\pi}{3} \frac{v_1^2 \sigma^3}{(kT)^3 (\ln S)^2} \right]$$

$$i^* = \left( \frac{2\theta}{3 \ln S} \right)^3$$

$$= \frac{32\pi}{3} \frac{v_1^2 \sigma^3}{(kT)^3 (\ln S)^3}$$
(11.35)



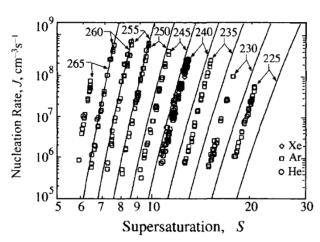


FIGURE 11.7 Nucleation rates measured in supersaturated *n*-butanol vapor as a function of saturation ratio for various temperatures ranging from 225 to 265 K (Viisanen and Strey 1994). Different symbols indicate different carrier gases at 240 K. Predictions of classical nucleation theory are shown by the lines. (Reprinted with permission from Viisanen, Y., and Strey, R., V. M. Homogeneous Nucleation Rates for *n*-Butanol, *J. Chem. Phys.* 101. Copyright 1994 American Institute of Physics.)

TABLE 11.1 Critical Number and Radius for Water Droplets

S	$T = 273 \text{ K}^a$		$T=298 \text{ K}^b$		
	r*, Å	i*	r*, Å	i*	
1	$\infty$	$\infty$	$\infty$	$\infty$	
2	17.3	726	15.1	482	
3	10.9	182	9.5	121	
4	8.7	87	7.6	60	
5	7.5	58	6.5	39	

 $<sup>^</sup>a\sigma = 75.6 \; {\rm dyn \, cm^{-1}}; \; v_1 = 2.99 \times 10^{-23} \; {\rm cm^3 \, molecule^{-1}}. \ ^b\sigma = 72 \; {\rm dyn \, cm^{-1}}; \; v_1 = 2.99 \times 10^{-23} \; {\rm cm^3 \, molecule^{-1}}.$ 

TABLE 11.3 Cri	itical Number and	Radius for Five	<b>Organic Species at</b>	298 K
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		S			
Species	2	3	4	5	
Acetone	i*	265	67	33	21
	r* (Å)	19.8	12.5	9.9	8.5
Benzene	<i>i</i> *	706	177	88	56
	$r^*$ (Å)	29.2	18.4	14.6	12.6
Carbon tetrachloride	i*	678	170	85	54
	r* (Å)	29.6	18.7	14.8	12.7
Ethanol	i*	147	37	18	12
	r* (Å)	15.1	9.5	7.5	6.5
Styrene	i*	1646	413	206	132
•	<i>r</i> * (Å)	42.2	26.6	21.1	18.2

## Constrained equilibrium approach (or classical approach)

#### Free Energy of i-mer Formation

$$\Delta G_i = (\mu_l - \mu_v) i + 4\pi\sigma r^2$$

$$\mu_v - \mu_l = kT \ln S$$

$$\Delta G_i = 4\pi\sigma r^2 - \frac{4\pi}{3} \frac{kT \ln S}{v_1} r^3$$

$$r^* = \frac{2\sigma v_1}{kT \ln S}$$

$$\Delta G^* = \frac{4\pi}{3} \sigma r^{*2}$$

$$= \frac{16\pi}{3} \frac{v_1^2 \sigma^3}{(kT \ln S)^2}$$

#### **Constrained Equilibrium Cluster Distribution**

obeys the usual Boltzmann distribution

$$N_i^e = N_1 \exp(-\Delta G_i/kT)$$

$$\beta_i N_i^e = \gamma_{i+1} N_{i+1}^e$$

$$J = \beta_i N_i - \gamma_{i+1} N_{i+1}$$

$$= \beta_i N_i^e \left[ \frac{N_i}{N_i^e} - \frac{N_{i+1}}{N_{i+1}^e} \right]$$

$$\sum_{j=1}^{i_{\max}-1} \frac{J}{\beta_{j} N_{j}^{e}} = \frac{N_{1}}{N_{1}^{e}} - \frac{N_{i_{\max}}}{N_{i_{\max}}^{e}}$$

$$\sum_{j=1} \frac{}{\beta_j N_j^e} = \frac{}{N_1^e}$$

$$\sum_{j=1}^{n} \beta_j N_j^{c} \qquad N_1 \qquad N_{i_{\text{max}}}$$

FIGURE 11.2 Constrained equilibrium and steady-state cluster distributions,  $N_i^e$  and  $N_i$ ,

Thus the unknown steady-state cluster distribution  $N_p$  disappears, allowing the nucleation rate to be expressed in terms of the constrained equilibrium distribution  $N_i^e$ 

$$J = \left(\sum_{j=1}^{i_{\text{max}}-1} \frac{1}{\beta_j N_j^e}\right)^{-1}$$

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Cluster Concentration

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#### Simple correction to the classical theory of homogeneous nucleation

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It is well known that the classical nucleation theory (CNT),2 which is the most common tool to analyze the nucleation phenomena, fails in predicting the temperature dependence and absolute values of the critical supersaturations of a number of substances including water, alcohols, and high alkanes.  $^{5-13,17}$  A large number of theories  $^{7-9,14,15,18-23}$  to test against the experimental data have been reported in the literature in the last two decades, yet a major source of the discrepancies is not clearly identi-

$$J = K \exp\left(\frac{-\Delta G_i}{kT}\right)$$

$$\Delta G_i^{\rm CNT} = \Delta \mu i + \gamma A(i)$$

The Becker-Doring theory expresses the nucleation

$$J^{\text{BD}} = v N_i^2 \left[ \frac{2\gamma}{\pi m} \right]^{0.5} \exp \left[ \frac{-\Delta G_i^{\text{CNT}}}{kT} \right]$$
$$= K^{\text{BD}} \exp \left[ \frac{-\Delta G_i^{\text{CNT}}}{kT} \right], \tag{3}$$

Nucleation rates in the kinetically consistent version of CNT are given by the following equation:

$$\mathcal{J}^{\text{CNT}} = vN_1^2 \left[ \frac{2\gamma}{\pi m} \right]^{0.5} \frac{1}{S} \exp \left[ \frac{-\Delta G_t^{\text{CNT}}}{kT} \right] = \frac{1}{S} J^{\text{BD}}. \tag{4}$$

$$J^{\text{SCC}} = \frac{1}{S} \exp\left(\frac{\gamma A(1)}{kT}\right) J^{\text{BD}} = \exp\left(\frac{\gamma A(1)}{kT}\right) J^{\text{CNT}}.$$
 (5)

Finally, the present study (CCNT) express the nucleation

$$J^{\text{CCNT}} = \frac{1}{S^2} \exp\left(\frac{\gamma A(1)}{kT}\right) J^{\text{CNT}} = \frac{1}{S} \exp\left(\frac{\gamma A(1)}{kT}\right) J^{\text{KCNT}}$$
$$= \frac{1}{S} J^{\text{SCC}}. \tag{11}$$