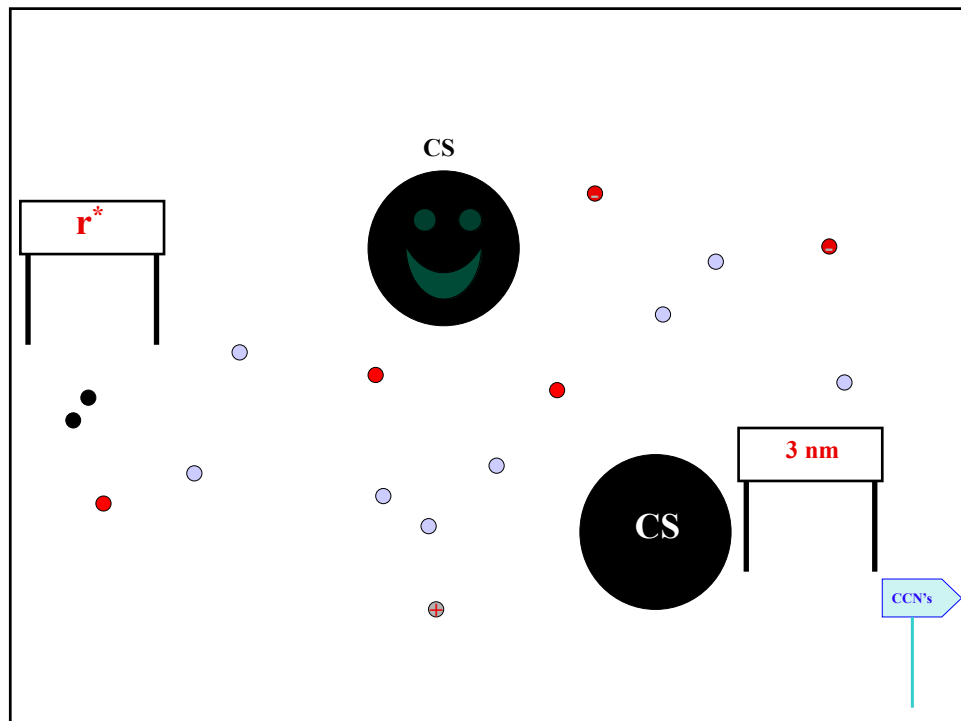


Lecture 12: Mass transport to or from atmospheric particles : Condensation and Evaporation

1



2

Nucleation and growth events

in the boundary atmosphere

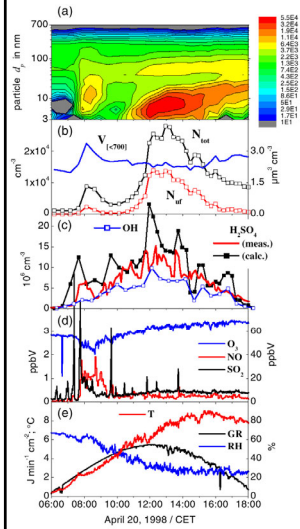


Figure 1. Time evolution of aerosol, gas phase and meteorological parameters at Hohenpeissenberg on April 20, 1998. (a) Particle size distribution $dN/d\log D_p$ in cm^{-3} . (b) Total particle concentration N_{tot} (3–11 nm), UF particle concentration N_{uf} (3–11 nm), and particle volume $< 700 \text{ nm}$ in $\mu\text{m}^3 \cdot \text{cm}^{-3}$ (assuming spherical particles). (c) Measured H_2SO_4 and OH concentrations, and $[\text{H}_2\text{SO}_4]$ derived from steady-state calculation. (d) NO , SO_2 (left axis), and O_3 (right axis). (e) Dry temperature T , RH, and global radiation "GR".

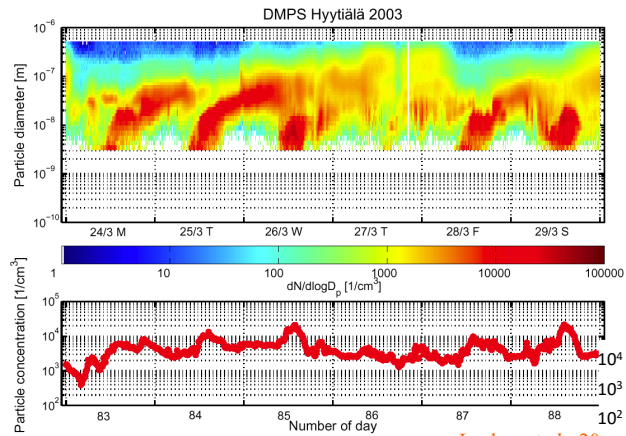


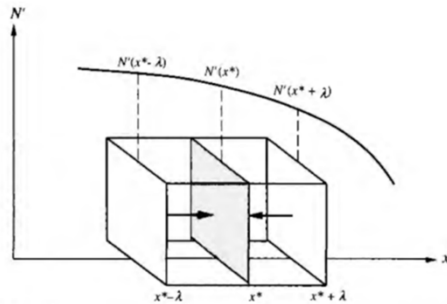
Fig. 6. Particle concentration measured by DMPS during the days 83-89

Laakso et al., 2011

Birmili et al., 2000

3

Mass transfer: Diffusion flux



$$J = -D(\partial N' / \partial x)$$

FIGURE 9.4 Control surfaces for molecular diffusion as envisioned in the elementary kinetic theory of gases.

4

Knudsen Number and three regimes of gas-particle interactions

Knudsen number:

$$Kn = \frac{2\lambda}{D_p} = \frac{\lambda}{R_p}$$

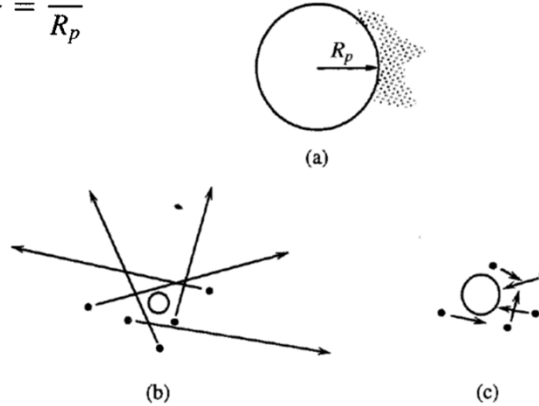
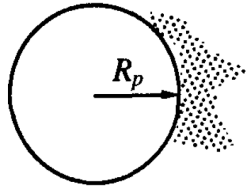


FIGURE 8.1 Schematic of the three regimes of suspending fluid-particle interactions: (a) continuum regime ($Kn \rightarrow 0$), (b) free molecule (kinetic) regime ($Kn \rightarrow \infty$), and (c) transition regime ($Kn \sim 1$).

5

The Continuum Regime



$$\frac{\partial c}{\partial t} = D_g \left(\frac{\partial^2 c}{\partial r^2} + \frac{2}{r} \frac{\partial c}{\partial r} \right)$$

$$c(r) = c_\infty - \frac{R_p}{r} (c_\infty - c_s)$$

$$\frac{\partial c}{\partial t} = -\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \tilde{J}_{A,r})$$

$$\tilde{J}_{A,r} = -D_g \frac{\partial c}{\partial r}$$

$c(r, t)$ is the concentration of A, and $\tilde{J}_{A,r}(r, t)$ is the molar flux of A (moles area⁻¹ time⁻¹) at any radial position r .

$$c(r, 0) = c_\infty, \quad r > R_p$$

$$c(\infty, t) = c_\infty$$

$$c(R_p, t) = c_s$$

The total flow of A (moles time⁻¹) toward the particle is denoted by J_c , the subscript c referring to the continuum regime, and is given by

$$J_c = -4\pi R_p^2 (\tilde{J}_A)_{r=R_p}$$

$$J_c = 4\pi R_p D_g (c_\infty - c_s)$$

Maxwellian flux

6

A mass balance on the growing or evaporating particle is

$$\frac{\rho_p}{M_A} \frac{d}{dt} \left(\frac{4}{3} \pi R_p^3 \right) = J_c$$

$$\frac{dR_p}{dt} = \frac{D_g M_A}{\rho_p R_p} (c_\infty - c_s)$$

$$R_p^2 = R_{p0}^2 + \frac{2D_g M_A}{\rho_p} (c_\infty - c_s) t$$

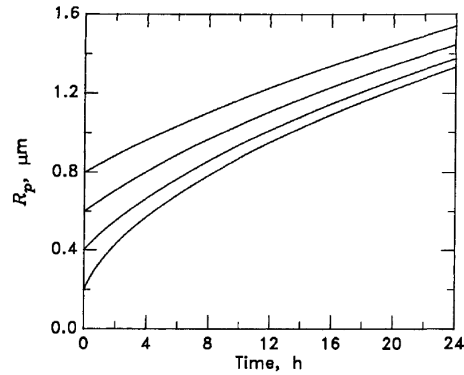
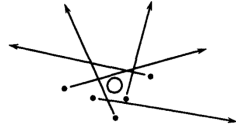


FIGURE 12.1 Growth of aerosol particles of different initial radii as a function of time for a constant concentration gradient of $1 \mu\text{g m}^{-3}$ between the aerosol and gas phases ($D_g = 0.1 \text{ cm}^2 \text{ s}^{-1}$, $\rho_p = 1 \text{ g cm}^{-3}$).

7

The Kinetic Regime



For molecules in three-dimensional random motion the number of molecules Z_N striking a unit area per unit time is (Moore 1962)

$$Z_N = \frac{1}{4} N \bar{c}_A \quad (12.23)$$

where $\bar{c}_A$ is the mean speed of the molecules:

$$\bar{c}_A = \left(\frac{8kT}{\pi m_A} \right)^{1/2} \quad (12.24)$$

Under these conditions the molar flow J_k (moles time^{-1}) to a particle of radius R_p is

$$J_k = \pi R_p^2 \bar{c}_A \alpha (c_\infty - c_s)$$

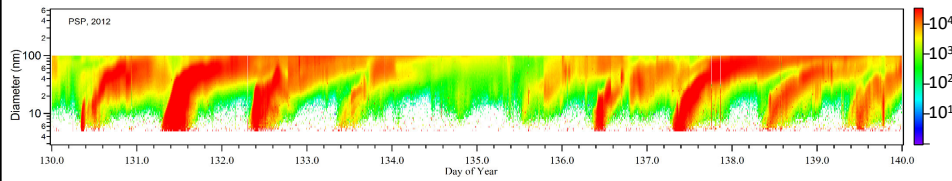
where α is the molecular accommodation coefficient

$$\alpha = \frac{\text{number of molecules entering the liquid phase}}{\text{number of molecular collisions with the surface}}$$

$$\frac{J_k}{J_c} = \frac{\alpha \bar{c}_A}{4D_g} R_p \quad 0 \leq \alpha \leq 1$$

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Growth rates of ultrafine particles



$$\frac{\rho_p}{M_A} \frac{d}{dt} \left(\frac{4}{3} \pi R_p^3 \right) = J_k = \pi R_p^2 \bar{c}_A \alpha (c_\infty - c_s)$$

$$\frac{dR_p}{dt} = ?$$

$$\frac{dD_p}{dt} = 2 \frac{dR_p}{dt} = ?$$

Q: If the observed growth rate (diameter) of 5-10 nm particles is 3 nm/hr and H₂SO₄ is assumed to be the only condensing gas, what is the concentration of H₂SO₄ gas ?

9

The Transition Regime

$$Kn \approx 1$$

Flux matching:

Flux matching assumes that the noncontinuum effects are limited to a region $R_p \leq r \leq \Delta + R_p$ beyond the particle surface and that continuum theory applies for $r \geq \Delta + R_p$. The distance Δ is then of the order of the mean free path λ and within this inner region the basic kinetic theory of gases is assumed to apply.

$$R_p \leq r \leq \Delta + R_p$$

Kinetic theory applies

$$r \geq \Delta + R_p$$

Continuum theory applies

Fuchs Theory

$$4\pi R_p^2 \left(\frac{1}{4} \bar{c}_A \right) [c(R_p + \Delta) - c_s] = D \left(\frac{dc}{dr} \right)_{r=R_p+\Delta} 4\pi (R_p + \Delta)^2$$

Then solving the steady-state continuum transport equation for a dilute system,

$$\frac{d^2 c}{dr^2} + \frac{2}{r} \frac{dc}{dr} = 0$$

$$c(r) = c_\infty - \frac{R_p}{r} (c_\infty - c_s) \beta_F$$

$$\beta_F = \frac{[1 + (\Delta/R_p)] \bar{c}_A R_p}{\bar{c}_A R_p + 4D[1 + (\Delta/R_p)]}$$

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Fuchs Theory

$$\alpha \cdot 4\pi R_p^2 \left(\frac{1}{4} \bar{c}_A\right) [c(R_p + \Delta) - c_i] = D \left(\frac{dc}{dr}\right)_{r=R_p+\Delta} 4\pi (R_p + \Delta)^2$$

Relating the binary diffusivity and the mean free path using $D_{AB}/\lambda_{AB}\bar{c}_A = \frac{1}{3}$ and letting $Kn = \lambda_{AB}/R_p$, one obtains

$$\frac{J}{J_c} = 0.75\alpha \frac{1 + Kn\Delta/\lambda_{AB}}{0.75\alpha + Kn + (\Delta/\lambda_{AB})Kn^2}$$

If $\alpha = 1$

$$\frac{J}{J_c} = 0.75 \frac{1 + Kn\Delta/\lambda_{AB}}{0.75 + Kn + (\Delta/\lambda_{AB})Kn^2}$$

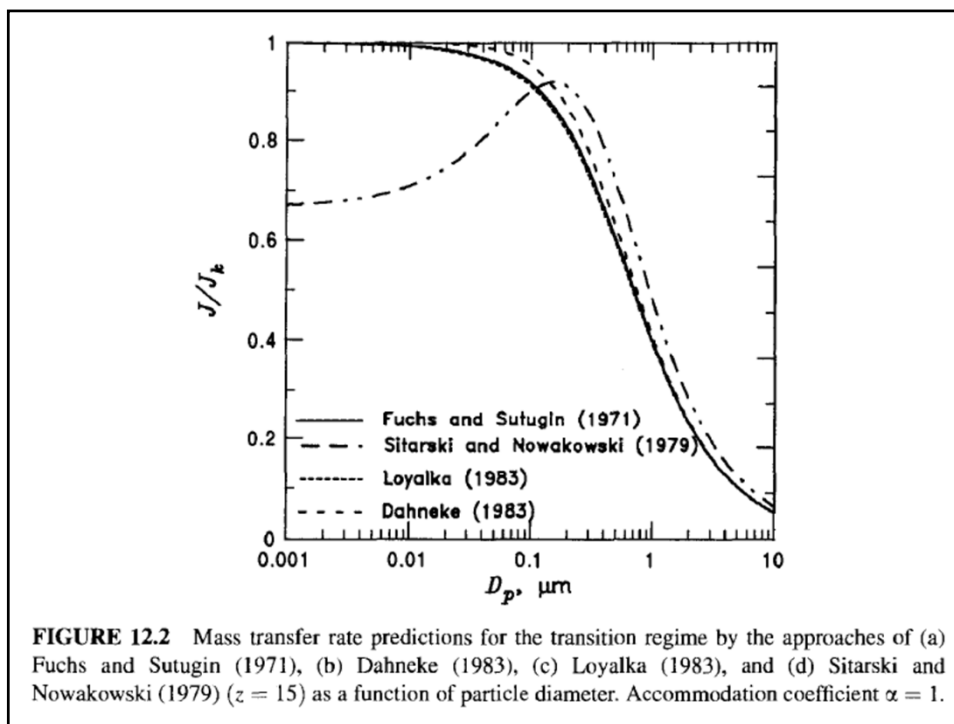
$$\frac{J}{J_k} = \frac{1 + Kn\Delta/\lambda_{AB}}{1 + Kn\Delta/\lambda_{AB} + 0.75 Kn^{-1}} \quad \frac{J_k}{J_c} = \frac{3}{4 Kn}$$

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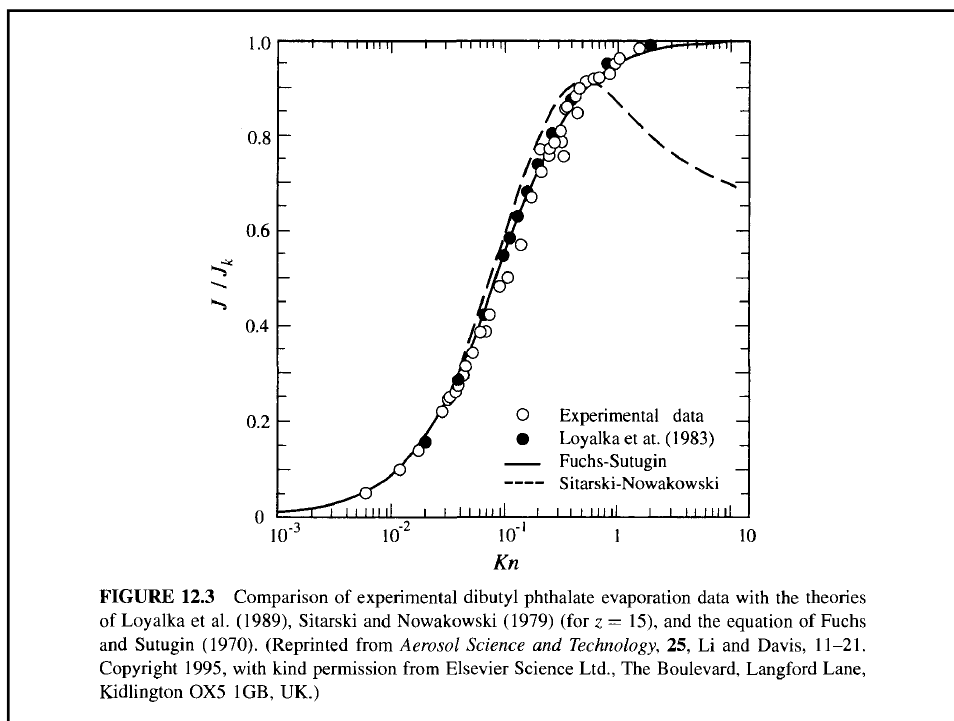
TABLE 12.1 Transition Regime Formulas for Diffusion of Species A in a Background Gas B to an Aerosol

J/J_c	Mean Free Path Definition	Reference
$\frac{0.75 \alpha (1 + Kn\Delta/\lambda_{AB})}{0.75 \alpha + Kn + (\Delta/\lambda_{AB})Kn^2}$	$\frac{3D_{AB}}{\bar{c}_A}$	Fuchs (1964)
$\frac{0.75 \alpha (1 + Kn)}{Kn^2 + Kn + 0.283 Kn\alpha + 0.75 \alpha}$	$\frac{3D_{AB}}{\bar{c}_A}$	Fuchs and Sutugin (1971)
$\frac{1 + Kn}{1 + 2 Kn(1 + Kn)/\alpha}$	$\frac{2D_{AB}}{\bar{c}_A}$	Dahneke (1983)
$\frac{1 + 1.333 Kn}{1 + 1.333 Kn + (1.333\sqrt{\pi}Kn + 1)Kn}$	$\frac{4}{\sqrt{\pi}} \frac{D_{AB}}{\bar{c}_A}$	Loyalka (1983)
$\frac{b(1 + aKn)}{b + cKn + d Kn^2}$	$4b \frac{D_{AB}}{\bar{c}_A}$ [see (12.39) for $a, b, c,$ and d]	Sitarski and Nowakowski (1979)

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The Accommodation Coefficient

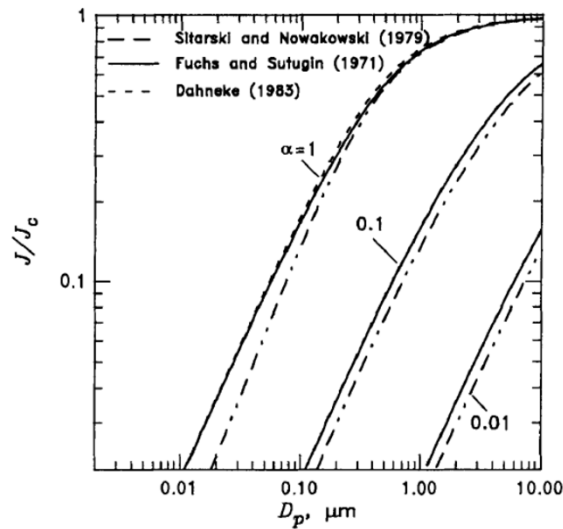
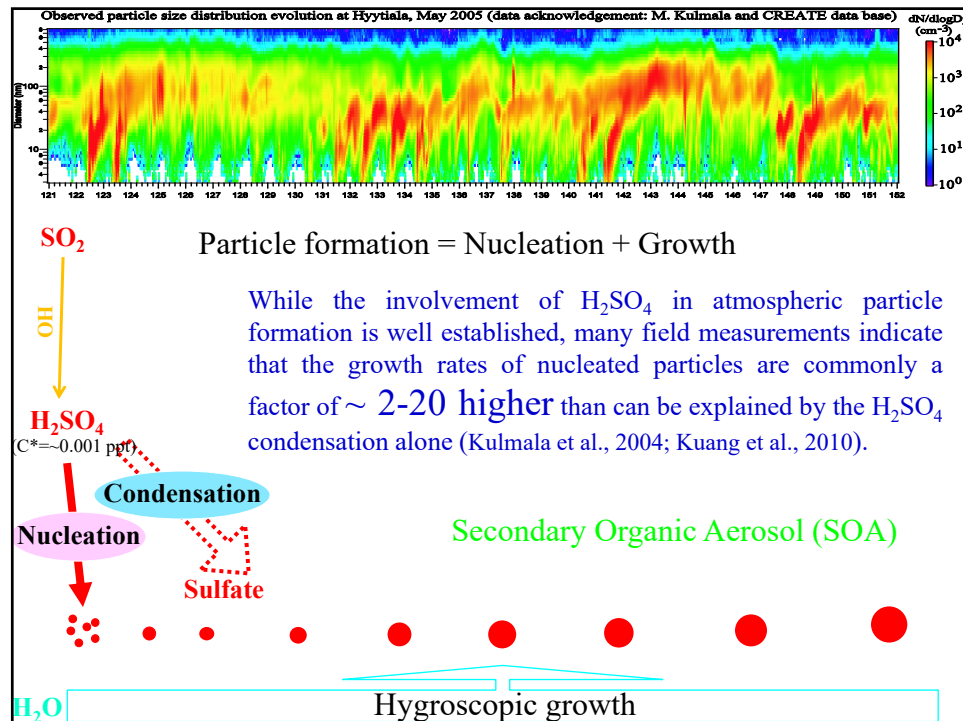


FIGURE 12.4 Mass transfer rates as a function of particle diameter for accommodation coefficient values 1.0, 0.1, and 0.01 for the approaches of Sitariski and Nowakowski (1979), Fuchs and Sutugin (1970), and Dahneke (1983).

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