Chapter 2

The global energy balance

We consider now the general problem of the radiative equilibrium temperature of the Earth. The Earth is bathed in solar radiation and absorbs much of it. To maintain equilibrium it must warm up and radiate energy away at the same rate as it is received — see Fig.2.1. We will see that the effective, or radiative, temperature of the Earth is 255 K and that a body at this temperature radiates energy in the infra-red. But the atmosphere is strongly absorbing at these wavelengths due to the presence of triatomic molecules — principally H_2O and CO_2 — which absorb and emit in the infrared raising the surface temperature above that of the emission temperature, a mechanism that has become known as the 'greenhouse effect'.

2.1 Effective planetary temperature (emission temperature)

The Earth receives almost all of its energy from the Sun. At the present time in its evolution the Sun emits energy at a rate of $Q = 3.87 \times 10^{26}\mathrm{W}$. The flux of solar energy at the Earth — called the 'solar constant' — depends on the distance of the Earth from the Sun, r, and is given by the inverse square law: $S_0 = \frac{Q}{4\pi r^2}$. Of course, because of variations in the Earth's orbit (see Section 5.1.1) the solar constant is not really constant — the value $S_0 = 1367\ \mathrm{W\,m^{-2}}$ set out in Table 2.1 is an average corresponding to the average distance of Earth from the Sun, $r = 150 \times 10^9\ \mathrm{m}$.

The way in which radiation interacts with an atmosphere depends on the wavelength as well as the intensity of the radiative flux. The relation

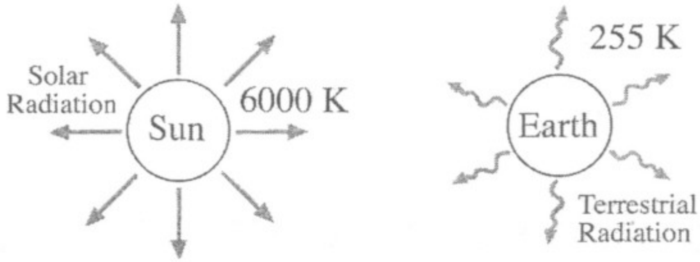

Figure 2.1: The Earth radiates energy away at the same rate as it is received from the Sun. The Earth's emission temperature is 255 K; that of the Sun, 6000 K. The outgoing terrestrial radiation peaks in the infrared; the incoming solar radiation peaks at shorter wavelengths, in the visible.

	r	S_0	α_p	T_e	T_m	T_s	τ
	10^9 m	$W\,m^{-2}$		K	K	K	Earth days
Venus	108	2632	0.77	227	230	760	243
Earth	150	1367	0.30	255	250	288	1.00
Mars	228	589	0.24	211	220	230	1.03
Jupiter	780	51	0.51	103	130	134	0.41

Table 2.1: Properties of some of the planets. S_0 is the solar constant at a distance r from the Sun, α_p is the planetary albedo, T_e is the emission temperature computed from Eq.(2.4), T_m is the measured emission temperature and T_s is the global mean surface temperature. The rotation period, τ, is given in Earth days.

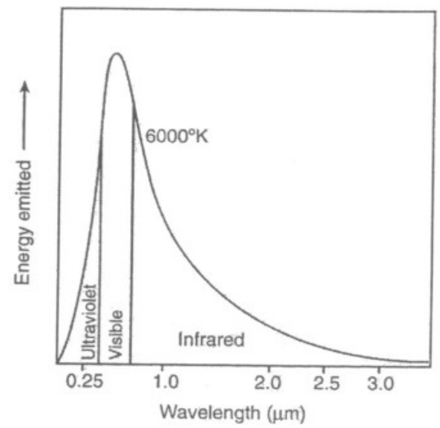

Figure 2.2: The energy emitted from the sun plotted against wavelength based on a black body curve with $T = T_{Sun}$. Most of the energy is in the visible and 95% of the total energy lies between 0.25 and 2.5 μm (10^{-6} m).

between the energy flux and wavelength — the spectrum — is plotted in Fig.2.2. The Sun emits light that is primarily in the visible part of the spectrum, corresponding to the colors of the rainbow — red, orange, yellow, green, blue, indigo and violet — with the energy flux decreasing toward longer (infrared, IR) and shorter (ultraviolet, UV) wavelengths.

Why does the spectrum have this pattern? Such behavior is characteristic of the radiation emitted by incandescent material, as can be observed, for example, in a coal fire. The hottest parts of the fire are almost white and emit the most intense radiation, with a wavelength that is shorter than that coming from the warm parts of the fire, which glow red. The coldest parts of the fire do not seem to be radiating at all, but are, in fact, radiating in the infrared. Experiment and theory show that the wavelength at which the intensity of radiation is a maximum, and the flux of emitted radiation, depend only on the temperature of the source. The theoretical spectrum,

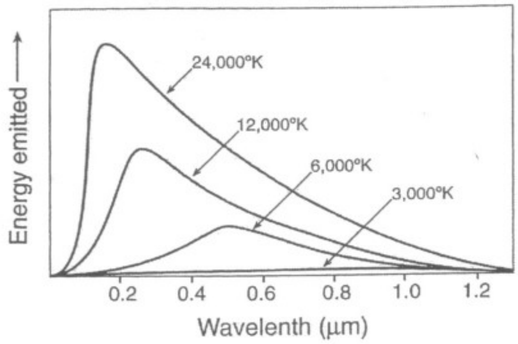

Figure 2.3: The energy emitted at different wavelengths for black bodies at several temperatures.

one of the jewels of Physics, was worked out by Planck[1], and is known as the 'Planck' or 'blackbody' spectrum. (A brief theoretical background to the Planck spectrum is given in an Appendix). It is plotted as a function of temperature in Fig.2.3. If the observed radiation spectrum of the Sun is fitted to the black body curve by using T as a free parameter, we deduce that the blackbody temperature of the sun is 6000 K.

Let us consider the energy balance of the Earth as in Fig.2.4, which shows the Earth intercepting the solar energy flux and radiating terrestrial energy away. If at the location of the (mean) Earth orbit, the incoming solar energy flux is $S_0 = 1367\ \mathrm{W\,m^{-2}}$, then, given that the cross-sectional area of the Earth intercepting the solar energy flux is πa^2 where a is the radius of the

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[1] In 1900 Max Planck (1858-1947) combined the formulae of Wien and Rayleigh describing the distribution of energy as a function of wavelength of the radiation in a cavity at temperature T, to arrive at what is now known as Planck's radiation curve. He went on to a complete theoretical deduction, introduced quanta of energy and set the scene for the development of Quantum Mechanics.

Type of surface	Albedo (%)
Ocean	2 – 10
Forest	6 – 18
Cities	14 – 18
Grass	7 – 25
Soil	10 – 20
Grassland	16 – 20
Desert (sand)	35 – 45
Ice	20 – 70
Cloud (thin, thick stratus)	30, 60 – 70
Snow (old)	40 – 60
Snow (fresh)	75 – 95

Table 2.2: Albedos for different surfaces. Note that the albedo of clouds is highly variable and depend on the type and form.

Earth (Fig. 2.4),

$$\text{Solar power incident on the Earth} = S_0 \pi a^2 = 1.74 \times 10^{17}\ \mathrm{W}$$

using the data in Table 1.1. Not all of this radiation is absorbed by the Earth; a significant fraction is reflected. The ratio of reflected to incident solar energy is called the *albedo*, α. As set out in Table 2.2, α depends on the nature of the reflecting surface and is large for clouds, light surfaces such as deserts and (especially) snow and ice. Under the present terrestrial conditions of cloudiness and snow and ice cover conditions, of the incoming solar radiation at the Earth, on average a fraction $\alpha_p \simeq 0.30$ is reflected back to space; α_p is known as the *planetary albedo*. Thus

$$\text{Solar radiation absorbed by the Earth} = (1 - \alpha_p) S_0 \pi a^2 = 1.22 \times 10^{17}\ \mathrm{W}\ . \tag{2.1}$$

Because the mean temperature of the Earth is neither increasing nor decreasing, the total terrestrial flux radiated to space must balance the solar radiation absorbed by the Earth. If, in total, the spinning Earth radiates in all directions like a blackbody of uniform temperature T_e (known as the '*effective planetary temperature*', or '*emission temperature*' of the Earth) the emitted radiation per unit area is given by the Stefan-Boltzmann law:

$$\text{Power/unit area} = \sigma T_e^4 \tag{2.2}$$

where $\sigma = 5.67 \times 10^{-8}\mathrm{W\,m^{-2}\,K^{-4}}$ is the Stefan-Boltzmann constant. So

$$\text{Emitted terrestrial radiation} = 4\pi a^2 \sigma T_e^4 \ . \tag{2.3}$$

Note that Eq.(2.3) is a **definition** of *emission temperature* T_e — it is the temperature one would infer by looking back at Earth if a black body curve were fitted to the measured spectrum of outgoing radiation.

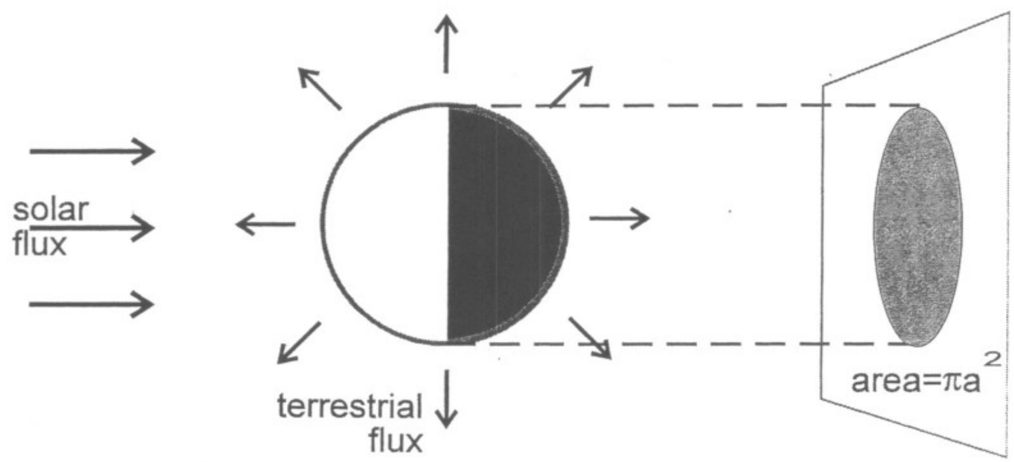


Figure 2.4: The spinning Earth is imagined to intercept solar energy over a disk of radius 'a' and radiate terrestrial energy away isotropically from the sphere. Modified from Hartmann, 1994.

Equating Eq.(2.1) with Eq.(2.3) gives

$$T_e = \left[\frac{S_0(1-\alpha_p)}{4\sigma}\right]^{\frac{1}{4}} \ . \tag{2.4}$$

Note that the radius of the Earth has cancelled out: T_e depends only on the planetary albedo and the distance of the Earth from the Sun. Putting in numbers we find that the Earth has an effective temperature of 255 K. Table 2.1 lists the various parameters for some of the planets and compares approximate measured values, T_m, with T_e computed from Eq.(2.4). The agreement is very good, except for Jupiter where it is thought that $\sim \frac{1}{2}$ of the energy input comes from the gravitational collapse of the planet.

However, as can be seen from Table 2.1, the effective temperature of Earth is nearly 40 K cooler than the globally averaged observed surface temperature which is $T_s = 288\,\mathrm{K}$. As discussed in Section 2.3, $T_s \neq T_e$ because 1) radiation is absorbed within the atmosphere, principally by its water vapor blanket and 2) fluid motions — air currents — carry heat both vertically and horizontally.

2.2 The atmospheric absorption spectrum

A property of the black body radiation curve is that the wavelength of maximum emission, λ_m, satisfies

$$\lambda_m T = \text{constant} . \tag{2.5}$$

This is known as *Wien's displacement law.* Since the solar emission temperature is 6000 K and the maximum of the solar spectrum is at about $0.6\,\mu\mathrm{m}$ — i.e. in the visible, see Fig. 2.2 — and we have determined $T_e = 255\,\mathrm{K}$ for the Earth, it follows that the peak of the terrestrial spectrum is at

$$\lambda_m^{earth} = 0.6\,\mu\mathrm{m} \times \frac{6000}{255} \simeq 14\,\mu\mathrm{m}, \text{ in the infrared.}$$

Thus the Earth radiates to space primarily in the infrared. Normalized black-body spectra for the Sun and Earth are shown in Fig.2.5. The two spectra hardly overlap at all, which greatly simplifies thinking about radiative transfer.

Also shown in Fig.2.5 is the atmospheric absorption spectrum; this is the fraction of radiation at each wavelength that is absorbed on a single vertical path through the atmosphere. From it we see that:

- the atmosphere is almost completely transparent in the visible, at the peak of the solar spectrum.
- the atmosphere is very opaque in the UV.
- the atmosphere has variable opacity across the IR spectrum — it is almost completely opaque at some wavelengths, transparent at others.

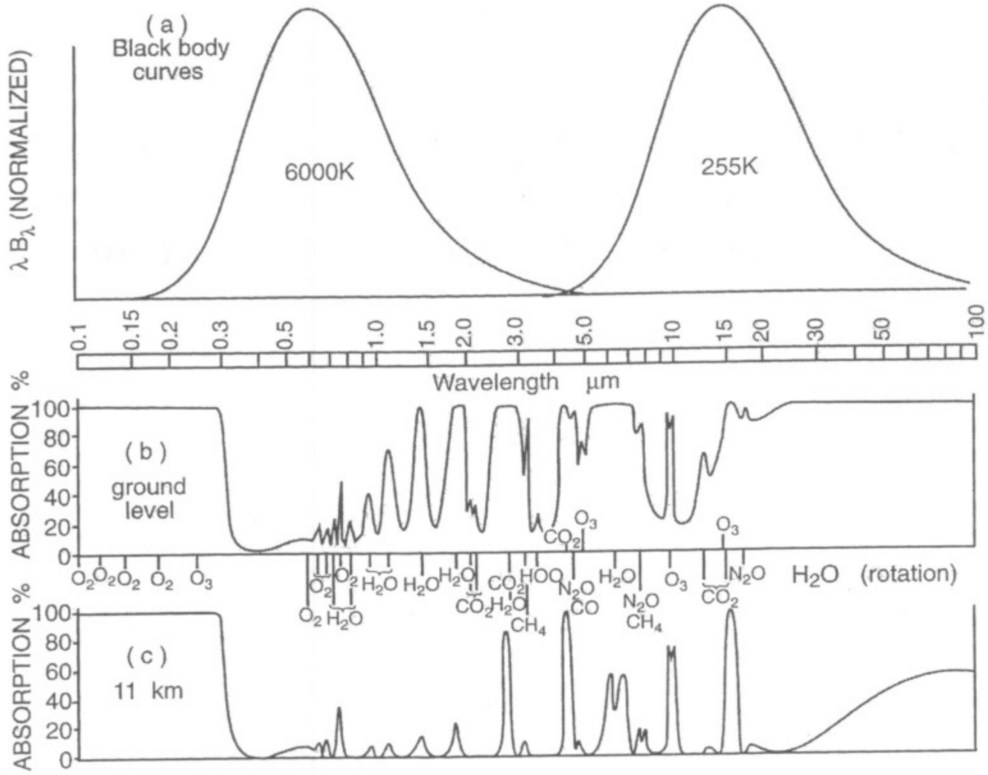


Figure 2.5: (a) The normalized blackbody emission spectra, $T^{-4}\lambda B_\lambda$, for the Sun ($T = 6000\,\mathrm{K}$) and Earth ($T = 255\,\mathrm{K}$) as a function of $ln\lambda$ (top) where B_λ is the black body function and λ is the wavelength — see Appendix for discussion. (b) The fraction of radiation absorbed while passing from the surface to the top of the atmosphere as a function of wavelength. (c) The fraction of radiation absorbed from the tropopause (typically at a height of 11 km) to the top of the atmosphere as a function of wavelength. The atmospheric molecules contributing the important absorption features at each frequency are also indicated. After Goody and Yung: "Atmospheric Radiation", Oxford Univ. Press, 1989.

- N_2 does not figure at all in absorption, and O_2 absorbs only in the far UV (where there is little solar energy flux) and, a little, in the near IR: the dominant constituents of the atmosphere are incredibly transparent across almost the whole spectral range of importance.

- the absorption is dominated by triatomic molecules — O_3 in the UV, H_2O, CO_2 and others in the IR because it so happens that triatomic molecules have rotational and vibrational modes that can easily be excited by radiation with wavelengths in the IR. These molecules are present in tiny concentrations (see Table 1.2) but play a key role in the absorption of terrestrial radiation (see Fig.2.5). They are known as Greenhouse gases. This is the fundamental reason why atmospheric radiation is so vulnerable to human-induced changes in composition.

2.3 The greenhouse effect

The global average mean surface temperature of the earth is 288 K — see Table 2.1. Above we deduced that the emission temperature of the Earth is 255 K, considerably lower. Why? We saw from Fig.2.5 that the atmosphere is rather opaque to IR, so we cannot think of terrestrial radiation as being radiated into space directly from the surface. Much of the radiation emanating from the surface will be absorbed, primarily by H_2O, before passing through the atmosphere. On average, the emission to space will emanate from some level in the atmosphere (typically about 5 km, in fact) such that the region above that level is mostly transparent to IR. It is this region of the atmosphere, rather than the surface, that must be at the emission temperature. Thus radiation from the atmosphere will be directed downward, as well as upward, and hence the surface will receive not only the net solar radiation, but IR from the atmosphere as well. Because the surface feels more incoming radiation than if the atmosphere were not present (or were completely transparent to IR) it becomes warmer than T_e. This has become known as the 'greenhouse effect'[2].

[2] It is interesting to note that the domestic greenhouse does not work in this manner! A greenhouse made of plastic window panes, rather than conventional glass, is effective even though plastic (unlike glass) does not have significant absorption bands in the IR. The greenhouse works because its windows allow energy in and its walls prevent the warm air from blowing away.

2.3.1 A simple greenhouse model

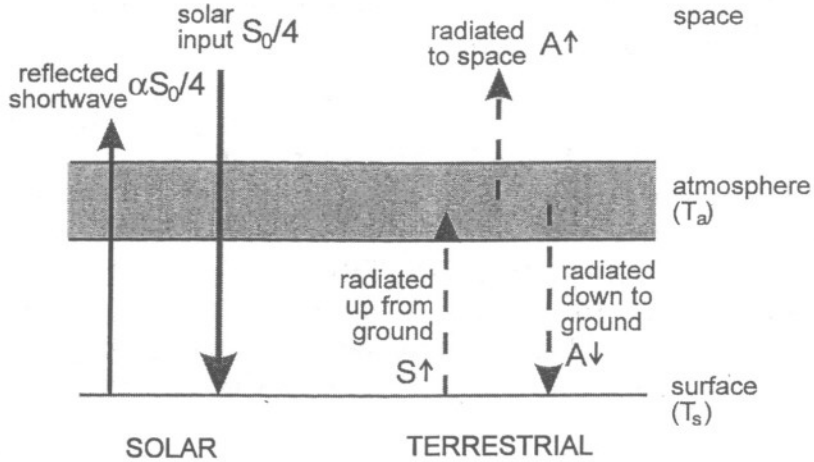


Figure 2.6: The simplest greenhouse model, comprising a surface at temperature T_s, and an atmospheric layer at temperature T_a, subject to incoming solar radiation $\frac{S_o}{4}$. The terrestrial radiation upwelling from the ground is assumed to be completely absorbed by the atmospheric layer.

Consider Fig.2.6. Since the atmosphere is thin, let us simplify things by considering a planar geometry, in which the incoming radiation per unit area is equal to the average flux per unit area striking the Earth. This average incoming solar energy *per unit area of the Earth's surface* is

$$\text{average solar energy flux} = \frac{\text{intercepted incoming radiation}}{\text{Earth's surface area}} = \frac{S_0 \pi a^2}{4\pi a^2} = \frac{1}{4} S_0 \,. \tag{2.6}$$

We will represent the atmosphere by a single layer of temperature T_a, and, in this first calculation, assume 1) that it is completely transparent to shortwave solar radiation, and 2) that it is completely opaque to IR (*i.e.*, it absorbs all the IR radiating up from the ground) so that the layer that is emitting to space is also "seen" by the ground. Now, since the whole Earth-atmosphere system must be in equilibrium (on average), the net flux into the system must vanish. The average net solar flux per unit area is, from Eq.(2.6), and allowing for reflection, $\frac{1}{4}(1 - \alpha_p) S_0$, while the terrestrial radiation being emitted to space per unit area is, using Eq.(2.2):

$$A \uparrow = \sigma T_a^4 \,.$$

Equating them, we find:

$$\sigma T_a^4 = \frac{1}{4}(1-\alpha_p)\,S_0 = \sigma T_e^4\ , \tag{2.7}$$

from the definition of T_e, Eq.(2.4). We see that the atmosphere is at the emission temperature (naturally, because it is this region that is emitting to space).

At the surface, the average incoming shortwave flux is also $\frac{1}{4}(1-\alpha_p)\,S_0$, but there is also a downwelling flux emitted by the atmosphere,

$$A\downarrow = \sigma T_a^4 = \sigma T_e^4\ .$$

The flux radiating upward from the ground is

$$S\uparrow = \sigma T_s^4\ ,$$

where T_s is the surface temperature. Since, in equilibrium, the net flux at the ground must be zero,

$$S\uparrow = \frac{1}{4}(1-\alpha_p)\,S_0 + A\downarrow\ ,$$

whence

$$\sigma T_s^4 = \frac{1}{4}(1-\alpha_p)\,S_0 + \sigma T_e^4 = 2\sigma T_e^4\ , \tag{2.8}$$

where we have used Eq.(2.7). Therefore

$$T_s = 2^{\frac{1}{4}} T_e\ . \tag{2.9}$$

So the presence of an absorbing atmosphere, as depicted here, increases the surface temperature by a factor $2^{\frac{1}{4}} = 1.19$. This arises as a direct consequence of absorption of terrestrial radiation by the atmosphere, which, in turn, re-radiates IR back down to the surface, thus increasing the net downward radiative flux at the surface.

Applying this factor to our calculated value $T_e = 255\,\mathrm{K}$, we predict $T_s = 2^{\frac{1}{4}} \times 255 = 303\,\mathrm{K}$. This is closer to the actual mean surface temperature (288 K — see Table 2.1) but is now an overestimate! The model we have discussed is clearly an oversimplification:

- For one thing, not all the solar flux incident on the top of the atmosphere reaches the surface—typically, some 20-25% is absorbed within the atmosphere (including by clouds).

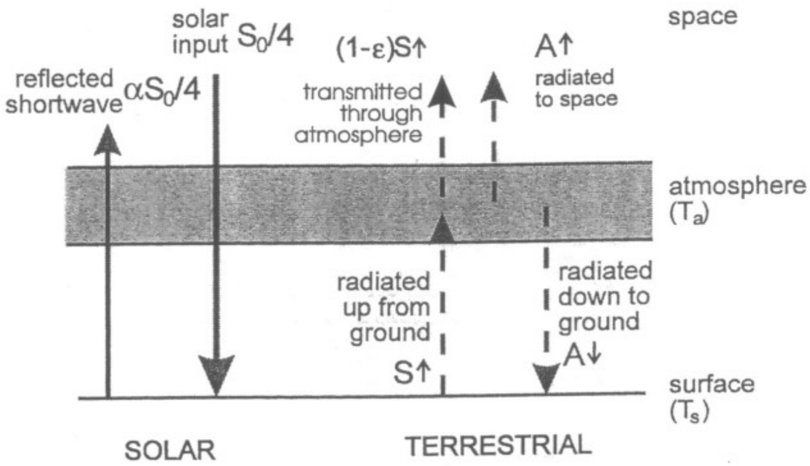


Figure 2.7: A leaky greenhouse. In contrast to Fig.2.6, the atmosphere now absorbs only a fraction, ε, of the terrestrial radiation upwelling from the ground.

- For another, we saw in Section (2.2) that IR absorption by the atmosphere is incomplete. The greenhouse effect is actually less strong than in the model above and T_s will be less than the value implied by Eq.(2.9). We shall analyze this by modifying Fig.2.6 to permit partial transmission of IR through the atmosphere — a leaky greenhouse model.

2.3.2 A leaky greenhouse

Consider Fig.2.7. We suppose the atmosphere has absorptivity ϵ; *i.e.*, a fraction ϵ of the IR upwelling from the surface is absorbed within the atmosphere (so the case of Fig. 2.6 corresponds to $\epsilon = 1$). Now, again insisting that, in equilibrium, the net flux at the top of the atmosphere vanishes gives

$$\frac{1}{4}(1-\alpha_p)S_0 = A\uparrow + (1-\epsilon)\,S\uparrow\ . \tag{2.10}$$

Zero net flux at the surface gives

$$\frac{1}{4}\,(1-\alpha_p)\,S_0 + A\downarrow = S\uparrow\ . \tag{2.11}$$

Since at equilibrium, $A \uparrow = A \downarrow$, as before, we have

$$S \uparrow = \sigma T_s^4 = \frac{1}{2(2-\epsilon)}(1-\alpha_p) S_0 = \frac{2}{(2-\epsilon)}\sigma T_e^4. \tag{2.12}$$

Therefore,

$$T_s = \left(\frac{2}{2-\epsilon}\right)^{\frac{1}{4}} T_e . \tag{2.13}$$

So in the limit $\epsilon \to 0$ (transparent atmosphere), $T_s = T_e$, and for $\epsilon \to 1$ (opaque atmosphere), $T_s = 2^{\frac{1}{4}} T_e$, as found above. In general, when $0 < \epsilon < 1$, $T_e < T_s < 2^{\frac{1}{4}} T_e$. So, of course, partial transparency of the atmosphere to IR radiation—a "leaky" greenhouse—reduces the warming effect we found in Eq.(2.9).

To find the atmospheric temperature, we need to invoke *Kirchhoff's law*[3], *viz.*, that the emissivity of the atmosphere is equal to its absorptivity. Thus,

$$A \uparrow = A \downarrow = \epsilon \sigma T_a^4. \tag{2.14}$$

We can now use Eqs. (2.14), (2.10), (2.11) and (2.12) to find

$$T_a = \left(\frac{1}{2-\epsilon}\right)^{\frac{1}{4}} T_e = \left(\frac{1}{2}\right)^{\frac{1}{4}} T_s .$$

So the atmosphere is, for $\epsilon < 1$, cooler than T_e (since the emission is then only partly from the atmosphere). Note, however, that according to the above the atmosphere is *always* cooler than the ground.

2.3.3 A more opaque greenhouse

Above we considered a leaky greenhouse. To take the other extreme, suppose that the atmosphere is so opaque that even a shallow layer will absorb *all* the IR passing through it. Now the assumption implicit in Fig.2.6—that space and the surface both "see" the same atmospheric layer—is wrong. We can elaborate our model to include a second, totally absorbing, layer in the atmosphere, as illustrated in Fig.2.8. Of course, to do the calculation correctly

[3]Kirchoff's law states that the emittance of a body — the ratio of the actual emitted flux to the flux that would be emitted by a black body at the same temperature — equals its absorptance.

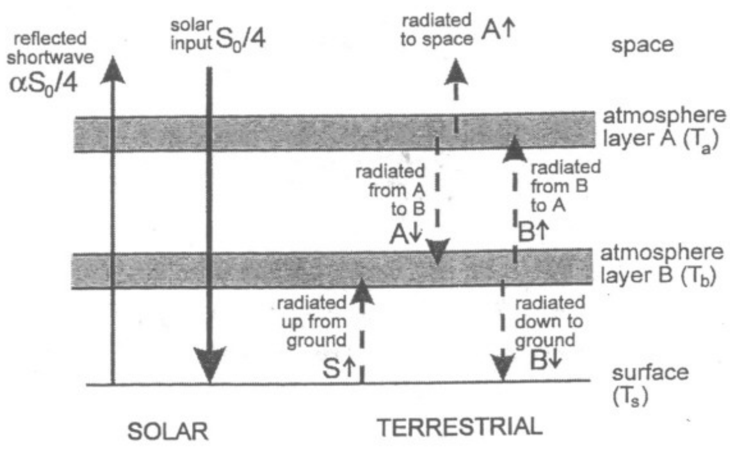


Figure 2.8: An 'opaque' greenhouse made up of two layers of atmosphere. Each layer completely absorbs the IR radiation impinging on it.

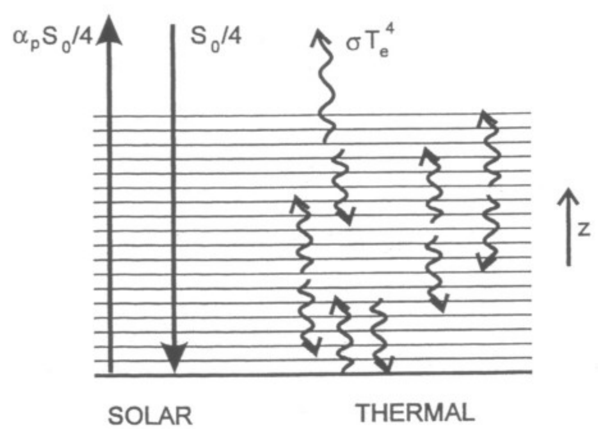


Figure 2.9: Schematic of radiative transfer model with many layers.

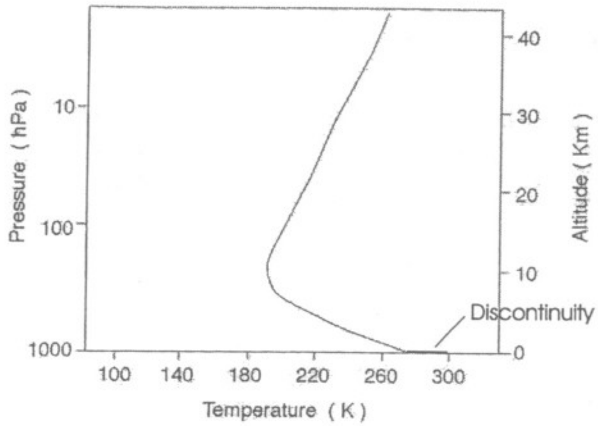


Figure 2.10: The radiative equilibrium profile of the atmosphere obtained by carrying out the calculation schematized in Fig.2.9. The absorbers are H_2O, O_3 and CO_2. The effects of both terrestrial radiation and solar radiation are included. Note the discontinuity at the surface. Modified from Wells, 1997.

(rather than just to illustrate the principles) we would divide the atmosphere into an infinite number of infinitesimally thin layers, allow for the presence of cloud, treat each wavelength in Fig.2.5 separately, allow for atmospheric absorption layer-by-layer — which depends on the vertical distribution of absorbers, particularly H_2O, CO_2 and O_3 — and do the required budgets for each layer and at the surface (we are not going to do this). An incomplete schematic of how this might look for a rather opaque atmosphere is shown in Fig.2.9.

The resulting profile—which would be the actual mean atmospheric temperature profile *if heat transport occurred only through radiative transfer*—is known as the **radiative equilibrium temperature profile.** It is illustrated schematically in Fig.2.10. In particular, note the presence of a large temperature discontinuity at the surface in the radiative equilibrium profile which is not observed in practice. (Recall from our analysis of Fig.2.7 that we found that the atmosphere in our slab model is always colder than the surface.) The reason this discontinuity is produced in radiative equilibrium is that, while there is some absorption within the troposphere, both of solar and terrestrial radiation, most solar radiation is absorbed at the surface. The reason such a discontinuity is not observed in nature is that it would (and does) leads to *convection* in the atmosphere, which introduces an additional mode of *dynamical* heat transport. This is discussed at some length in Chapter 4.

As we will see in Chapter 3, most of the atmosphere above 10km is close to radiative equilibrium; below 10km, however, the observed profile differs substantially from that obtained by the radiative calculation described above.

2.4 References

2.5 Problems

1. (a) The emission temperature of the Earth is 255 K. Way back in the early history of the solar system, the radiative output of the Sun was thought to be 25% less than it is now. Assuming all else (Earth-Sun distance, Earth albedo) has remained fixed, *and using only the data given in this paragraph*, determine the emission temperature of the Earth at that time.

   (b) At present the emission temperature of the Earth is 255 K, and its albedo is 30%. How would the emission temperature change if

      i. the albedo were reduced to 10% (and all else were held fixed);

      ii. the infra-red absorptivity of the atmosphere — ϵ in Fig.2.7 — were doubled, but albedo remains fixed at 30%.

2. Suppose that the Earth is, after all, flat. Specifically, consider it to be a thin circular disk (of radius 6370 km), orbiting the Sun at the same distance as the Earth; the planetary albedo is 30%. The vector normal to one face of this disk always points directly towards the Sun, and the disk is made of perfectly conducting material, so both faces of the disk are at the same temperature. Calculate the emission temperature of this disk, and compare with Eq.(2.4) for a spherical Earth.

3. Consider the thermal balance of Jupiter.

   (a) Assuming a balance between incoming and outgoing radiation, calculate the emission temperature for Jupiter.

(b) In fact, Jupiter has an internal heat source resulting from its gravitational collapse. The measured emission temperature T_e defined by

$$\sigma T_e^4 = (\text{outgoing flux of planetary radiation per unit surface area})$$

is 130 K. Comment in view of your theoretical prediction in part (a). Calculate the magnitude of Jupiter's internal heat source.

You will need the following information about Jupiter: mean planetary radius = 69500 km; mean radius of orbit around the Sun = 5.19A.U. (where 1A.U. is the mean radius of the Earth's orbit); planetary albedo = 0.51.

4. For the one-layer "leaky greenhouse" model considered in Fig.2.7, suppose that, all else being fixed, the atmospheric absorption depends linearly on atmospheric CO_2 concentration as

$$\epsilon = \epsilon_0 + [CO_2]\,\epsilon_1\,,$$

where $[CO_2]$ is CO_2 concentration (in ppm), $\epsilon_0 = 0.734$, and $\epsilon_1 = 1.0 \times 10^{-4}(\text{ppm})^{-1}$. Calculate, for this model, the surface temperature:

(a) for the present atmosphere, with $[CO_2] = 360$ppm (see Table 1.2);

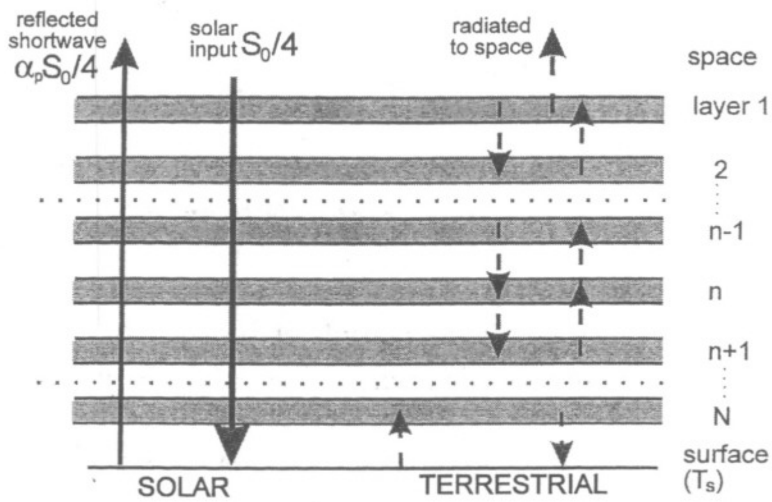
(b) in pre-industrial times, with $[CO_2] = 280$ppm; and

(c) in a future atmosphere with $[CO_2]$ doubled from its present value.

5. Consider the "two-slab" greenhouse model illustrated in Fig.2.8 in which the atmosphere is represented by two perfectly absorbing layers of temperature T_a and T_b.

Determine T_a, T_b, and the surface temperature T_s in terms of the emission temperature T_e.

6. Consider an atmosphere that is completely transparent to shortwave (solar) radiation, but very opaque to infrared (IR) terrestrial radiation. Specifically, assume that it can be represented by N slabs of atmosphere, each of which is completely absorbing of IR, as depicted in the following schematic figure (not all layers are shown).



(a) By considering the radiative equilibrium of the surface, show that the surface must be warmer than the lowest atmospheric layer.

(b) By considering the radiative equilibrium of the n^{th} layer, show that, in equilibrium,

$$2T_n^4 = T_{n+1}^4 + T_{n-1}^4 \,, \tag{2.15}$$

where T_n is the temperature of the n^{th} layer, for $n > 1$. Hence argue that the equilibrium surface temperature is

$$T_s = (N+1)^{\frac{1}{4}}\, T_e \,,$$

where T_e is the planetary emission temperature. [Hint: Use your answer to part (a); determine T_1 and use Eq.(2.15) to get a relationship for temperature differences between adjacent layers.]

7. Determine the emission temperature of the planet Venus. You may assume the following: the mean radius of Venus' orbit is 0.72 times that of the Earth's orbit; the solar flux S_o decreases like the square of the distance from the sun and has a value of $1367\,\mathrm{W\,m^{-2}}$ at the mean Earth orbit; Venus planetary albedo = 0.77; Stefan-Boltzmann constant = $5.67 \times 10^{-8}\,\mathrm{W\,m^{-2}\,K^{-4}}$.

The observed mean surface temperature of the planet Venus is about 750 K — see Table 2.1. How many layers of the N−layer model considered in Question 6 would be required to achieve this degree of warming? Comment.