Problem Set 7

Question 1

The azimuthal velocity in the rotating bucket is

$$V_\theta(r) = \Omega \frac{(r_0^2 - r^2)}{r}$$ where r_0 is the outer radius.

A rossby number for this flow is

$$R_o = \frac{U}{fL} = \frac{V_\theta}{2\Omega r}$$ since $f = 2\Omega$ for the rotating tank

$$= \frac{\cancel{\Omega}(r_0^2 - r^2)}{2\cancel{\Omega} r^2}$$

$$= \frac{r_0^2 - r^2}{2r^2} = \frac{1}{2}\left(\left(\frac{r_0}{r}\right)^2 - 1\right)$$

~~at the outer edge, $r_0 > r$ and so $R_o \to$~~

~~near the outer edge~~

near the outer edge, r is just a little smaller than r_0

and so $\left(\frac{r_0}{r}\right)^2 - 1 \ll 1$

and $R_o \ll 1$ the rossby number is small.

Halfway to center, $r = \frac{r_0}{2}$ and $R_o = \frac{1}{2}\left[(2)^2 - 1\right]$

$= 3/2$

so R_o is $O(1)$ ~~in~~ at the halfway point.

(i.e. $R_o = \frac{3}{2}$ is neither large nor small)

near the center, $r \to 0$

and so $\left(\frac{r_o}{r}\right)^2 - 1 \gg 1$

and $R_o \gg 1$

The Rossby number becomes large near the center.

b) Near center: $R_o \gg 1$, flow is approximately cyclostrophic, which is a balance between PGF (toward center) and centrifugal force (radially outward)

Near outer edge: $R_o \ll 1$, flow is approximately geostrophic, which is a balance between PGF (toward center) and Coriolis force (to the right of flow, radially outward)

Halfway: $R_o \simeq 1$, flow is in gradient balance. There is a three-way force balance between PGF, Coriolis and Centrifugal forces.

c) Rossby numbers for the Hurricane:

$$R_o = \frac{V}{fR} \quad \text{and} \quad f = 2\Omega \sin(20^\circ)$$

$$= 2\left(\frac{2\pi}{1\,\text{day}}\right)\sin(20^\circ)\left(\frac{1\,\text{day}}{3600 \times 24\,\text{s}}\right)$$

$$= 4.97 \times 10^{-5}\ \text{s}^{-1}$$

$$\doteq 5 \times 10^{-5}\ \text{s}^{-1}$$

at $R = 20$ km:

$$R_o = \frac{(50\ \text{m s}^{-1})}{\cancel{4.97}\,(5 \times 10^{-5}\,\text{s}^{-1})\,(2 \times 10^{4}\ \text{m})}$$

$$= 50 \gg 1$$

at $R = 500$ km:

$$R_o = \frac{(25\ \text{m s}^{-1})}{(5 \times 10^{-5}\,\text{s}^{-1})(5 \times 10^{5}\,\text{m})} = 1$$

at $R = 1000$ km:

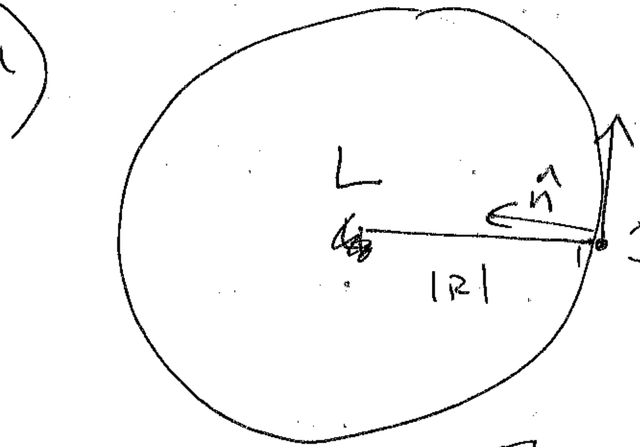
$$R_o = \frac{(5\ \text{m s}^{-1})}{(5 \times 10^{-5}\,\text{s}^{-1})(1 \times 10^{6}\,\text{m})} = 0.1$$

d) Based on these Rossby numbers, the balanced flow in the hurricane looks much like the rotating bucket:

The flow is approximately geostrophic at the outer edges. Near the center, it is approximately cyclostrophic. In between, the Rossby numbers are $O(1)$ and the flow is in gradient balance (i.e. PGF, Coriolis and Centrifugal forces are all important).

Problem 2

a)



$$\phi = 26°N$$

$$f = 2\Omega \sin\phi$$

$$= 2\left(\frac{2\pi}{1\,\text{day}}\right)(3600 \times 24\ \text{s/day})^{-1} \sin(26°)$$

$$= 6.37 \times 10^{-5}\ \text{s}^{-1}$$

station is here

$V = 60\ \text{m s}^{-1}$ @ 950 hPa

The radial distance is ~~|R| = 200 km~~ ~~but because the flow is counter~~ $R = 200$ km (positive because flow is counterclockwise)

and $Z(950\ \text{hPa}) = 367$ m at the station.

We can use the balanced wind speed to calculate the horizontal geopotential gradient. The balanced wind speed must obey

$$\frac{V^2}{R} + fV = -\frac{\partial \Phi}{\partial n}$$

Here V, R and f are known.

Thus
$$\frac{\partial \Phi}{\partial n} = -\frac{V^2}{R} - fV$$

$$= -\frac{(60\ \text{m s}^{-1})^2}{(2 \times 10^5\ \text{m})} - (6.37 \times 10^{-5}\ \text{s}^{-1})(60\ \text{m s}^{-1})$$

$$\frac{\partial \Phi}{\partial n} = -0.022\ \text{m s}^{-2}$$

So Φ decreases toward the storm center at a rate of $\approx 0.022\ ms^{-2}$

or the geopotential height ~~decreases~~ decreases at a rate

$$\frac{\partial Z}{\partial n} = \frac{1}{g}\frac{\partial \Phi}{\partial n} = -\frac{0.022\ ms^{-2}}{9.8\ ms^{-2}}$$

$$= -0.0022\ m/m$$

$$= -2.2\ m/km$$

The height decreases by 2.2 m for every km toward the storm center.

We can use this rate to extrapolate to the center, a distance $R = 200$ km away.

Thus the height of the 950 hPa surface at the center is approximately

$$Z_{center} = Z_{station} - 2.2\ m/km\ (200\ km)$$

$$= 367\ m - 445\ m$$

$$= -78\ m$$

So the 950 hPa surface lies below sea level!

This means that the SLP must be less than 950 hPa.

We need to use hydrostatic balance to extrapolate the pressure from 78 m below the surface to $Z = 0$.

Recall that we have done this before, using the "hypsometric equation", which is really just hydrostatic balance integrated vertically.

The thickness of a layer is

$$Z_2 - Z_1 = H \ln\left(\frac{p_1}{p_2}\right)$$

where $H = \frac{R_d \overline{T}}{g}$ is the scale height, which depends on the average temperature in the layer.

Here let $Z_2 = 0$ (the surface or sea level) and $Z_1 = -78$ m

then $p_2 =$ unknown surface pressure and $p_1 = 950$ hPa

then $$0 - (-78\text{ m}) = H \ln\left(\frac{950\text{ hPa}}{p_2}\right)$$

$$\ln\left(\frac{950\text{ hPa}}{p_2}\right) = +\frac{78\text{ m}}{H}$$

take exponentials:

$$\frac{950\text{ hPa}}{p_2} = \exp\left(\frac{78\text{ m}}{H}\right)$$

So $$p_2 = (950\text{ hPa}) \exp\left(-\frac{78\text{ m}}{H}\right)$$

So to calculate the sea level pressure we just need the scale height.

Temperature is not given in the problem, so let's make a reasonable assumption. Let $\overline{T} = 300\ K$ (a warm tropical temperature)

Then $$H = \frac{R_d \overline{T}}{g} = \frac{(287\ J\ kg^{-1}\ K^{-1})(300\ K)}{9.8\ m\ s^{-2}}$$

$$= 8785\ \cancel{\frac{J\ kg^{-1}}{m\ s^{-2}}} \times \frac{N\ m}{\cancel{J}} \times \frac{\cancel{kg\ m\ s^{-2}}}{\cancel{N}}$$

$$= 8.8 \times 10^3\ m$$

Then the SLP is

$$P_2 = (950\ hPa)\ \exp\left(\frac{-78\ m}{8.8 \times 10^3\ m}\right)$$

$$= 942\ hPa$$

This is the approximate sea level pressure at the eye of the storm.

b) Calculate the ageostrophic wind speed at the station:

$$V_{ag} = V - V_g$$

and $V = 60\ m\ s^{-1}$

We can calculate the geostrophic wind speed directly from the geopotential gradient that we already worked out:

$$V_g = -\frac{1}{f}\frac{\partial \Phi}{\partial n}$$

$$= \frac{-1}{(6.37 \times 10^{-5}\ s^{-1})}\left(-0.022\ m\,s^{-2}\right)$$

$$= 345\ m\,s^{-1}$$

The ageostrophic wind speed is simply the difference between the actual wind speed and the geostrophic wind:

$$V_{ag} = V - V_g$$

$$= 60\ ms^{-1} - 345\ ms^{-1}$$

$$= -282\ m\,s^{-1}$$

So the ageostrophic wind speed is $282\ ms^{-1}$ in the opposit direction

(or in other words, the ~~wind is~~ actual wind (gradient wind) is much less than the geostrophic wind associated with this pressure gradient.

Question 3

a) Prove that $\underset{\sim}{V}_g$ is non-divergent, so long as f is constant:

$$\underset{\sim}{V}_g = \frac{\hat{k}}{f} \times \nabla_p \Phi$$ is the geostrophic wind vector

~~To prove that~~ we just need to show that

$$\nabla \cdot \underset{\sim}{V}_g = 0$$

Let's write it out in components:

$$\underset{\sim}{V}_g = -\frac{1}{f}\frac{\partial \Phi}{\partial y}\hat{i} + \frac{1}{f}\frac{\partial \Phi}{\partial x}\hat{j}$$ (there is no $\hat{k}$ component)

So $$\nabla \cdot \underset{\sim}{V}_g = \frac{\partial}{\partial x}\left(-\frac{1}{f}\frac{\partial \Phi}{\partial y}\right) + \frac{\partial}{\partial y}\left(\frac{1}{f}\frac{\partial \Phi}{\partial x}\right)$$

$\Rightarrow$ If f is constant we can take it out of the derivatives:

$$\nabla \cdot \underset{\sim}{V}_g = -\frac{1}{f}\frac{\partial^2 \Phi}{\partial x \partial y} + \frac{1}{f}\frac{\partial^2 \Phi}{\partial y \partial x}$$

we are free to reverse the order of differentiation:

$$\nabla \cdot \underset{\sim}{V}_g = -\frac{1}{f}\frac{\partial^2 \Phi}{\partial x \partial y} + \frac{1}{f}\frac{\partial^2 \Phi}{\partial x \partial y} = 0$$

b) since ω is the rate of change of pressure of an air parcel, and pressure always decreases upward, ω is negative for upward motion

c) Continuity equation in height coords:

$$\frac{1}{\rho}\frac{D\rho}{Dt} + \nabla \cdot \underset{\sim}{u} = 0$$

(the velocity divergence form, involving Lagrangian changes in density)

or

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \left(\rho \underset{\sim}{u}\right) = 0$$

(the flux divergence form, involving Eulerian change in density)

In pressure coords, the continuity eq. is

$$\left.\frac{\partial u}{\partial x}\right|_p + \left.\frac{\partial v}{\partial y}\right|_p + \frac{\partial \omega}{\partial p} = 0$$

where the horizontal derivatives are taken at constant p.

This form is considerably simpler because

1) there is no time derivative
2) there is no explicit dependence on density.

d) If horizontal winds are convergent near the surface (on an isobaric surface) then

$$\left.\frac{\partial u}{\partial x}\right|_p + \left.\frac{\partial v}{\partial y}\right|_p < 0$$

thus $$\frac{\partial \omega}{\partial p} = -\left(\left.\frac{\partial u}{\partial x}\right|_p + \left.\frac{\partial v}{\partial y}\right|_p \right) > 0$$

~~so~~ ω increases with increasing pressure

or ω gets more negative with increasing height (since p decreases upward)

So $\omega < 0$ aloft

and negative ω implies upward motion.

There is upward motion aloft above a region of convergence near the surface.

e) The total ~~wind f~~ horizontal wind field is the sum of a geostrophic and ageostrophic part:

$\underset{\sim}{V} = \underset{\sim}{V}_g + \underset{\sim}{V}_{ag}$ or in components, $u = u_g + u_{ag}$
$v = v_g + v_{ag}$

We have seen in part (a) that the geostrophic part is non-divergent:

$$\frac{\partial u_g}{\partial x} + \frac{\partial v_g}{\partial y} = 0 \qquad \left[\text{where derivatives are understood to be taken on } p \text{ surfaces}\right]$$

The vertical motion is determined by horizontal divergence/convergence through the continuity equation:

$$\frac{\partial \omega}{\partial p} = -\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)$$

write in terms of geo- and ageostrophic parts:

$$\frac{\partial \omega}{\partial p} = -\left(\frac{\partial u_g}{\partial x} + \frac{\partial u_{ag}}{\partial x} + \frac{\partial v_g}{\partial y} + \frac{\partial v_{ag}}{\partial y}\right)$$

$$= -\left(\frac{\partial u_g}{\partial x} + \frac{\partial v_g}{\partial y}\right) - \left(\frac{\partial u_{ag}}{\partial x} + \frac{\partial v_{ag}}{\partial y}\right)$$

$$= 0 - \left(\frac{\partial u_{ag}}{\partial x} + \frac{\partial v_{ag}}{\partial y}\right)$$

Thus it is the ageostrophic part of the wind that determines the vertical motion.

Problem 4   [Holton 3.2]

The actual wind is 30° to the right of the geostrophic wind. $|\mathbf{V}_g| = 20 \text{ m s}^{-1}$

What is $\frac{DV}{Dt}$, the rate of change of wind speed?

Use the momentum equations in natural coordinates:

$$\frac{DV}{Dt} = -\frac{\partial \Phi}{\partial s} \quad \text{along the flow}$$

$$\frac{V^2}{R} + fV = -\frac{\partial \Phi}{\partial n} \quad \text{across the flow.}$$

To determine the change in wind speed, we need the along-flow component of the geopotential gradient, $\frac{\partial \Phi}{\partial s}$.

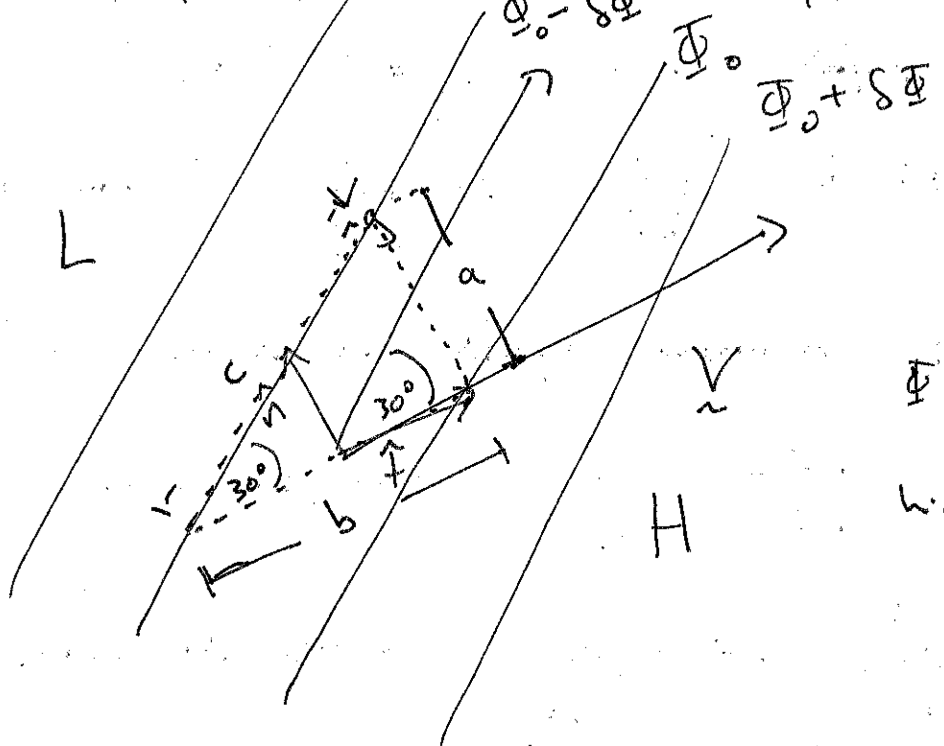
We can calculate this from knowledge of the geostrophic wind, which is related to the cross-flow component of the geopotential gradient:

$$|\mathbf{V}_g| = -\frac{1}{f}\frac{\partial \Phi}{\partial n} = 20 \text{ m s}^{-1}$$

so
$$\frac{\partial \Phi}{\partial n} = -f\,(20 \text{ m s}^{-1}) = -(10^{-4}\text{ s}^{-1})(20 \text{ m s}^{-1}) = -2\times 10^{-3} \text{ m s}^{-2}$$

Sketch the wind vectors and the geopotential contours:

The Φ contours must be parallel to $\underset{\sim}{v}_g$



The actual wind is crossing Φ contours toward higher heights

consider the dotted triangle, with lengths a, b and c (c is the hypotenuse)

We want to find the rate of change of Φ in the $\hat{t}$ direction. We know the rate of change of Φ in the $\hat{n}$ direction

$$\frac{\delta\Phi}{\delta n} = -2 \times 10^{-3} \text{ m s}^{-2} = -\frac{\delta\Phi}{a}$$

and $$\frac{\delta\Phi}{\delta s} = \frac{\delta\Phi}{b}$$

and these lengths a and b are related by

$$\tan(30°) = \frac{a}{b} \quad \text{so} \quad b = \frac{a}{\tan(30°)}$$

So $$\frac{\partial \Phi}{\partial s} = \frac{\delta \Phi}{a} \tan(30^\circ) = \frac{\tan(30^\circ)}{\cancel{a}}\left(-\cancel{a}\,\frac{\partial \Phi}{\partial n}\right)$$

$$= -\tan(30^\circ)\,\frac{\partial \Phi}{\partial n}$$

$$= -0.577\,\frac{\partial \Phi}{\partial n}$$

$$= (-0.577)\left(-2 \times 10^{-3}\ \mathrm{m\,s^{-2}}\right)$$

$$= 1.15 \times 10^{-3}\ \mathrm{m\,s^{-2}}$$

a positive value, meaning that heights are <u>increasing</u> in the direction of the flow as required in the sketch.

Therefore the change in wind speed is

$$\frac{DV}{Dt} = -\frac{\partial \Phi}{\partial s} = -1.15 \times 10^{-3}\ \mathrm{m\,s^{-2}}$$

the wind must be ~~slowing~~ slowing down.

Problem 5 [Holton 3.4]

~~PE~~ Geopotential height gradient is 100 m per 1000 km

so
$$\left|\frac{\partial \Phi}{\partial n}\right| = g\left|\frac{\partial z}{\partial n}\right|$$
$$= g\left(\frac{100\text{ m}}{1000\text{ km}}\right)$$
$$= g\left(\frac{10^2\text{ m}}{10^6\text{ m}}\right) = 9.8\times10^{-4}\text{ m s}^{-2}$$

the geostrophic wind in natural coords. is
$$V_g = -\frac{1}{f}\frac{\partial \Phi}{\partial n}$$

So the geostrophic wind speed is
$$|V_g| = \frac{1}{|f|}\left|\frac{\partial \Phi}{\partial n}\right| = \frac{1}{10^{-4}\text{ s}^{-1}}\left(9.8\times10^{-4}\text{ m s}^{-2}\right)$$
$$= \underline{9.8\text{ m s}^{-1}}$$

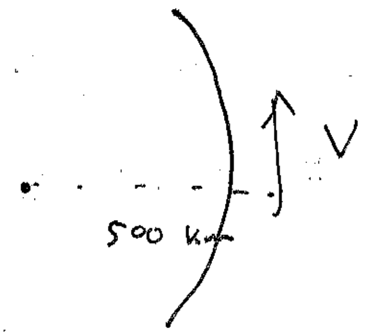
Now consider the gradient wind equation
$$V = -\frac{fR}{2} \pm \sqrt{\frac{f^2R^2}{4} - R\frac{\partial \Phi}{\partial n}}$$

or
$$V = -\frac{fR}{2} \pm \sqrt{\frac{f^2R^2}{4} + fRV_g}$$

where V_g is positive if it's in the same direction as the gradient wind.

Consider $R = +\ 500$ km

positive means counterclockwise curvature



one possibility is that this is flow around a low, so that V and V_g are in the same direction.

(i.e. $V_g = +\,9.8\ \mathrm{m\,s^{-1}}$)

then $$V = -\frac{(10^{-4}\,\mathrm{s^{-1}})(5\times10^{5}\,\mathrm{m})}{2} \pm \sqrt{\left(\frac{(10^{-4}\,\mathrm{s^{-1}})(5\times10^{5}\,\mathrm{m})}{2}\right)^2 + (10^{-4}\,\mathrm{s^{-1}})(5\times10^{5}\,\mathrm{m})(9.8\,\mathrm{m\,s^{-1}})}$$

$\cancel{V = -2.5\times10^{1}\ \mathrm{m\,s^{-1}} \pm \sqrt{}}$

$$V = -25\ \mathrm{m\,s^{-1}} \pm 33.4\ \mathrm{m\,s^{-1}}$$

since we must have $V > 0$ for a physically meaningful solution, only the positive root is relevant.

~~The only possi~~ In this case $V = 8.4\ \mathrm{m\,s^{-1}}$

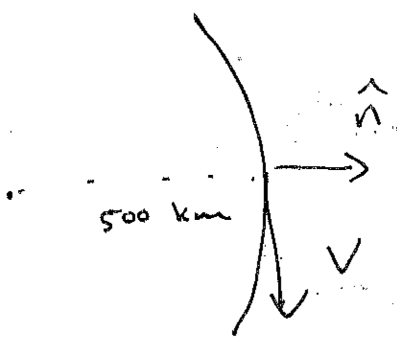
So in this case [the "regular low"] the gradient wind is slightly sub-geostrophic.

We have to also consider the possibility of counterclockwise flow around a <u>high</u>. In that case, $V_g = -9.8 \text{ m s}^{-1}$ since the geostrophic wind (which would be clockwise) is in the opposite direction of the actual wind.

Plugging into to our gradient wind formula gives $V = -25 \text{ m s}^{-1} \pm 11.6 \text{ m s}^{-1}$

So in this case $V < 0$ for both the positive and negative roots, so there are no possible physical solutions.

Now consider $R = -500$ km, which is clockwise flow



If this is ~~?~~ clockwise around a <u>high</u> then $V_g = +9.8 \text{ m s}^{-1}$. Plugging this and $R = -5 \times 10^5$ m into the gradient wind solution gives

$$V = +25 \text{ m s}^{-1} \pm 11.6 \text{ m s}^{-1}$$

~~The negative root~~

The positive root gives gradient wind speed

$$V = 36.6 \text{ ms}^{-1}$$

This is much larger than the geostrophic wind. This is the "anomalous high", in which excess coriolis force is balanced by excess centrifugal force.

The negative root gives $V = 13.4 \text{ ms}^{-1}$

This is the "regular high". The gradient wind speed is slightly super-geostrophic.

Finally we have to consider the possibility of clockwise flow around a low.

In this case $V_g = -9.8 \text{ ms}^{-1}$, $R = -500$ km

The gradient wind speed is thus

$$V = +25 \text{ ms}^{-1} \pm 33.4 \text{ ms}^{-1}$$

The negative root gives $V < 0$ and is therefore ~~ap~~ unphysical. The positive root yields

$$V = 58.4 \text{ ms}^{-1}$$

This is the "anomalous low". It is a possible balanced wind in which the flow is very rapid $(|V| \gg |V_g|)$ and in the "wrong" direction,

Problem 6 (Holton 3.14)

Gradient wind classification scheme for the S.H. $(f<0)$

the gradient wind speed solution is

$$V = -\frac{fR}{2} \pm \sqrt{\frac{f^2R^2}{4} - R\frac{\partial \Phi}{\partial n}}$$

~~Case~~ There are a total of 8 possible cases to consider: all possible combinations of

$$\frac{\partial \Phi}{\partial n} < 0, \quad \frac{\partial \Phi}{\partial n} > 0, \quad R>0, \quad R<0$$

and the positive and negative roots (i.e. taking the + or − sign in the above expression)

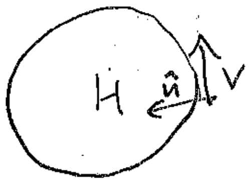
We'll go through those systematically:

First assume $\frac{\partial \Phi}{\partial n} > 0$

i.e. the geopotential field increase to the left of the flow direction

and $R > 0$

i.e. the ~~Φ~~ curvature is counter-clockwise



As we saw in class, ~~this~~ there is a condition on the shape of the geopotential field near the center of the high: $\left|\frac{\partial \Phi}{\partial n}\right| < \frac{f^2|R|}{4}$

which guarantees that the ~~solution~~ wind speed V is a <u>real</u> number.

But we also need V to be a <u>positive</u> number for the solution to be physically meaningful

(recall V is <u>defined</u> as the speed in the <u>direction of motion</u>, so it can't be negative)

In ~~these cases~~ this case with $R > 0$ and $f < 0$,

$-\frac{fR}{2}$ is a positive number

and $\sqrt{\frac{f^2R^2}{4} - R\frac{\partial \Phi}{\partial n}}$ is a <u>smaller</u> positive number

So there are physically meaningful solutions with <u>both</u> the positive and negative roots:

positive root: $$V = -\frac{fR}{2} + \sqrt{\frac{f^2R^2}{4} - R\frac{\partial \Phi}{\partial n}}$$

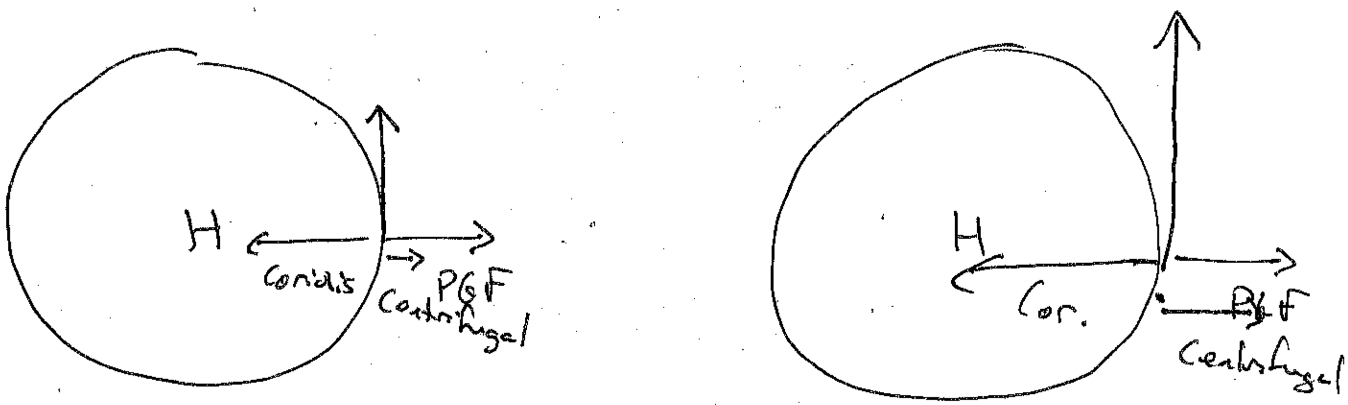
So $V > -\frac{fR}{2}$ This is the "anomalous high" and with negative root,

$$V = -\frac{fR}{2} - \sqrt{\frac{f^2R^2}{4} - R\frac{\partial\Phi}{\partial n}}$$

So $\quad V < -\frac{fR}{2}$

This is the "regular high" in the SH.

The force balances look like

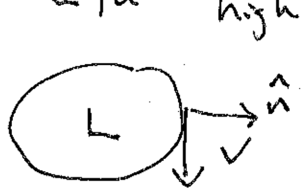


regular high in the SH. anomalous high

i.e. for the same pressure gradient / geopotential height gradient, the anomalous high has faster wind speeds. The extra Coriolis force is balanced by extra centrifugal force, compared to the regular high.

Second case: $\frac{\partial\Phi}{\partial n} > 0$ but $R < 0$

clockwise curvature with high heights to the left of the flow.



Thus this case is clockwise flow around a Low.

In this case there is no restriction on the shape of the geopotential field since

$$\frac{f^2R^2}{4} - R\frac{\partial \Phi}{\partial n} > 0 \quad \text{for any } \cancel{\text{geopotential}} \text{ value of } \frac{\partial \Phi}{\partial n}$$

but in this case $-\dfrac{fR}{2} < 0$

so if we take the negative root

$$V = -\frac{fR}{2} - \sqrt{\frac{f^2R^2}{4} - R\frac{\partial \Phi}{\partial n}}$$

gives $V < 0$
which is unphysical

the only possible solution is the positive root:

$$V = -\frac{fR}{2} + \sqrt{\frac{f^2R^2}{4} - R\frac{\partial \Phi}{\partial n}}$$

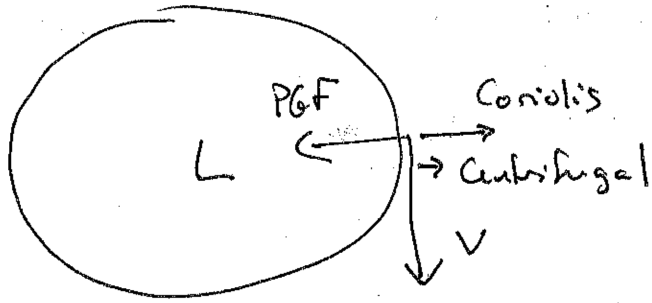
notice that since $-R\dfrac{\partial \Phi}{\partial n} > 0$

it follows that $\sqrt{\dfrac{f^2R^2}{4} - R\dfrac{\partial \Phi}{\partial n}} > \left|\dfrac{fR}{2}\right|$

so in this case the solution is always positive $V > 0$

This is the "regular low" in the SH.

The force balance looks like ⟶

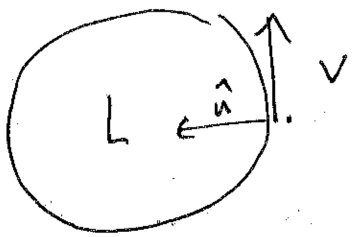


Regular low in the SH

The centrifugal force acts in the same direction as Coriolis and the gradient wind is just slightly weaker than the geostrophic wind for a given PGF.

Third case: $\frac{\partial \Phi}{\partial n} < 0$ and $R > 0$

Counter-clockwise flow with heights increasing to the right of the flow direction



Thus this case is counter-clockwise flow around a Low.

Again, there is no restriction on the shape of the geopotential field, and

$$\sqrt{\frac{f^2R^2}{4} - R\frac{\partial \Phi}{\partial n}} > \left| \frac{fR}{2} \right|$$

In this case, $-\frac{fR}{2} > 0$

So if we take the negative root, we get $V < 0$ which is unphysical

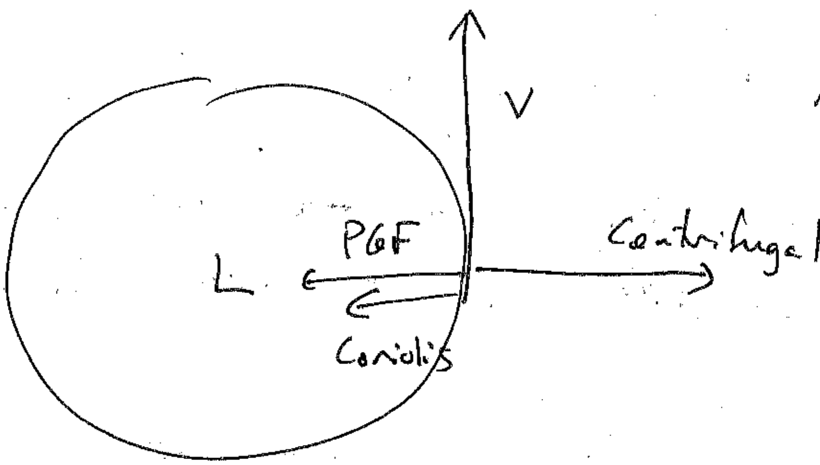
The only possible solution is the positive root

$$V = -\frac{fR}{2} + \sqrt{\frac{f^2R^2}{4} - R\frac{\partial\Phi}{\partial n}}$$

which gives $V > 0$

So this is a physically possible solution in which the flow is going counter-clockwise around a low in the SH — the "wrong" way!

The force balance looks like this:



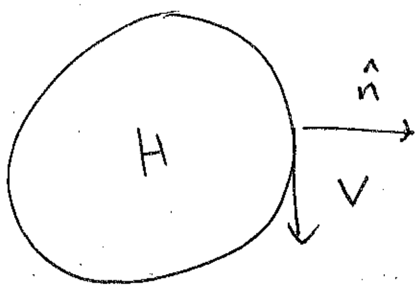
Anomalous Low in the SH

Notice that the Coriolis force acts in the same direction as the PGF — So the wind is actually in the opposite direction of the geostrophic wind for this PGF. Coriolis and PGF are balanced by a large centrifugal force due to fast wind speed.

This type of balanced flow is physically possible, but rarely observed because it would require very unusual initial conditions to set up this type of motion.

Fourth case: $\frac{\partial \Phi}{\partial n} < 0$ and $R < 0$

Clockwise flow with heights increasing to the right of the flow direction



This is clockwise flow around a high.

First, like all cases involving flow around a high, there is a condition on the slope of the geopotential field. Since $-R\frac{\partial \Phi}{\partial n} < 0$

then we need $\left|\frac{\partial \Phi}{\partial n}\right| < \frac{f^2|R|}{4}$

so that $\frac{f^2R^2}{4} - R\frac{\partial \Phi}{\partial n} > 0$ and we can have real solutions.

But now we have a problem: since $-\frac{fR}{2} < 0$

the only way to have a physical solution with $V > 0$ is if we take the positive root, and if $\sqrt{\frac{f^2R^2}{4} - R\frac{\partial \Phi}{\partial n}} > \left|\frac{fR}{2}\right|$

but that is not true in this case!

Both the positive and negative roots give $V < 0$ and are therefore unphysical.

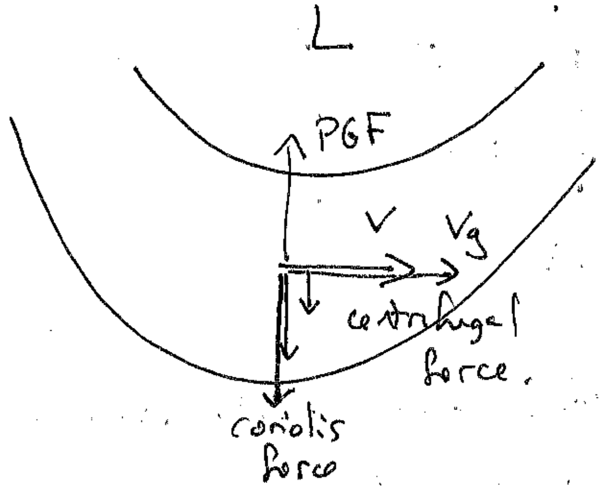
I summarize these results in a table like table 3.1 in Holton:

Classification of the roots of the Gradient Wind Equation in the Southern Hemisphere ($f < 0$)

Sign of $\frac{\partial \Phi}{\partial n}$	$R > 0$	$R < 0$
positive	positive root: anomalous high $V > -\frac{fR}{2}$	positive root: regular low
	negative root: regular high $V < -\frac{fR}{2}$	negative root: unphysical
negative	positive root: anomalous low	positive root: unphysical
	negative root: unphysical	negative root: unphysical

Problem 7

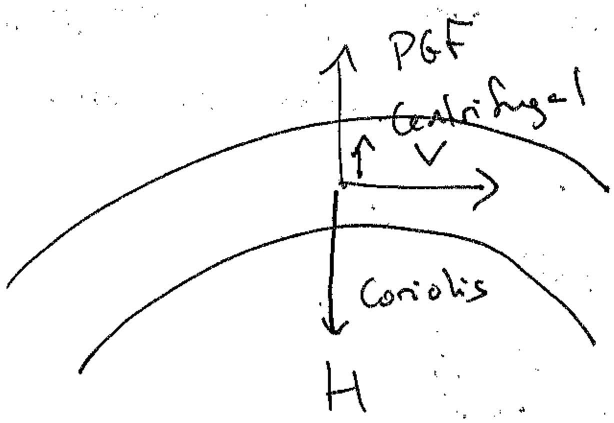
a) why is the balanced wind speed sub-geostrophic in a trough? sketch the force balance:



~~the~~ Because the coriolis force is proportional to wind speed, stronger wind speed = stronger Coriolis force.

For a given PGF (which is dictated by the height field) the geostrophic wind is the wind whose Coriolis force exactly balances the PGF. In a trough, as sketched above, there is an additional centrifugal force acting radially outward. In this case it acts in the same direction as the Coriolis force. They must add up to a force that balances PGF. Thus the Coriolis force needs to be weaker than the purely geostrophic case, and the wind ~~is~~ speed is therefore weaker than ~~the~~ geostrophic.

In a ridge:



The Coriolis force acts radially inward, so in the opposite direction of the centrifugal force. Again, these two must add up to a net force that balances the PGF. In this case the Coriolis force must be larger than the purely geostrophic case since we need some additional Coriolis force to offset the centrifugal force. This requires the wind speed to be greater than geostrophic in a ridge.

b) The sketches above were for the Northern Hemisphere, but the same reasoning applies in the SH.

The $\hat{n}$ equation of motion is simply our gradient wind balance

$$\frac{V^2}{R} + fV = -\frac{\partial \Phi}{\partial n}$$

or using the definition of the geostrophic wind

$$V_g = \frac{-1}{f}\frac{\partial \Phi}{\partial n}$$

we can write the gradient wind balance as

$$\frac{V^2}{R} + fV = f V_g$$

or

$$V^2 + fRV = fR\,V_g$$

This equation depends only on the product fR.
~~The difference between V and V_g depends on the~~
whether $V > V_g$ or $V < V_g$ will depend on the sign of the product fR.

~~In NH~~ For a ridge in NH: $f > 0$, $R < 0$ (clockwise) and so $fR < 0$
which, as we sketched in part a, means that V must be greater than V_g.

For a ridge in SH: $f < 0$, $R > 0$ (counter clockwise) and so $fR < 0$ again.
The result is the same: $V > V_g$

For a trough: $f > 0$, $R > 0$ in NH $\Rightarrow$ $fR > 0$
in SH: $f < 0$, $R < 0$ $\Rightarrow$ $fR > 0$
Result is the same either way: the gradient wind is subgeostrophic, $V < V_g$.

See question 5 for explicit examples in the Northern Hemisphere, and question 6 for sketches of the force balances in the S.H.