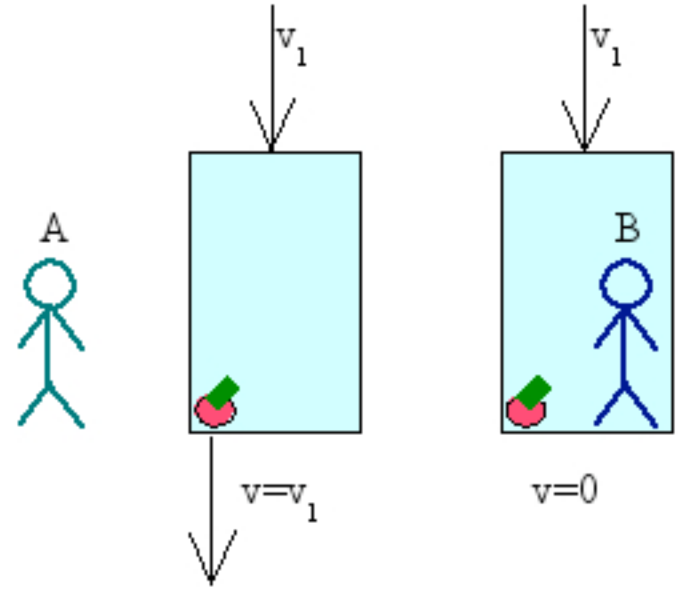
Frames of reference

- Last Friday, I stressed the italicized, middle section of the statement of Newton's Second Law of Motion:

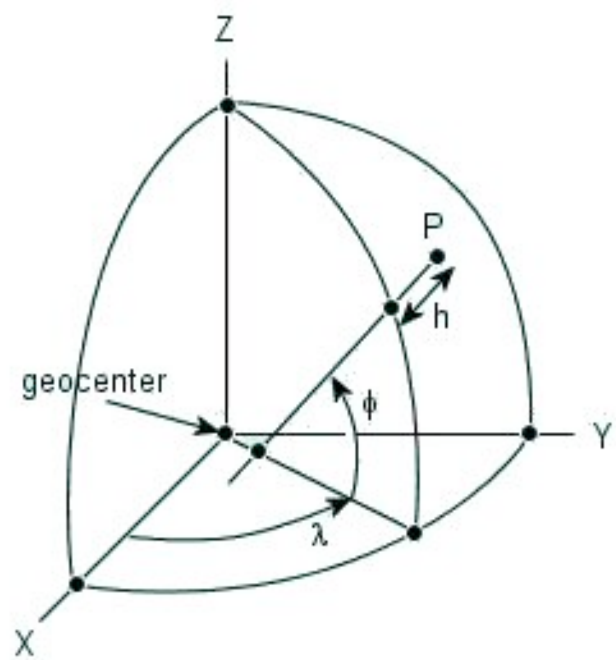
  → A mass in uniform motion relative to a *coordinate system fixed in space* will remain in uniform motion in the absence of forces acting upon it.

- This uniform motion is called inertial motion and the fixed reference frame is an inertial, or absolute frame.
- There are, however, many instances where it is more convenient to observe and measure a system from a non-inertial frame of reference; i.e., one in which the orientation of the coordinate system is changing with time.

- The example I showed on Friday was someone (A) outside an elevator watching the car, with an apple inside it, descend (inertial frame), versus someone (B) inside the elevator observing the motion of the apple (non-inertial).



- In formulating the equations that govern atmospheric dynamics, it is most convenient to use a non-inertial frame of reference: a geocentric frame, which is fixed with respect to the rotating Earth.

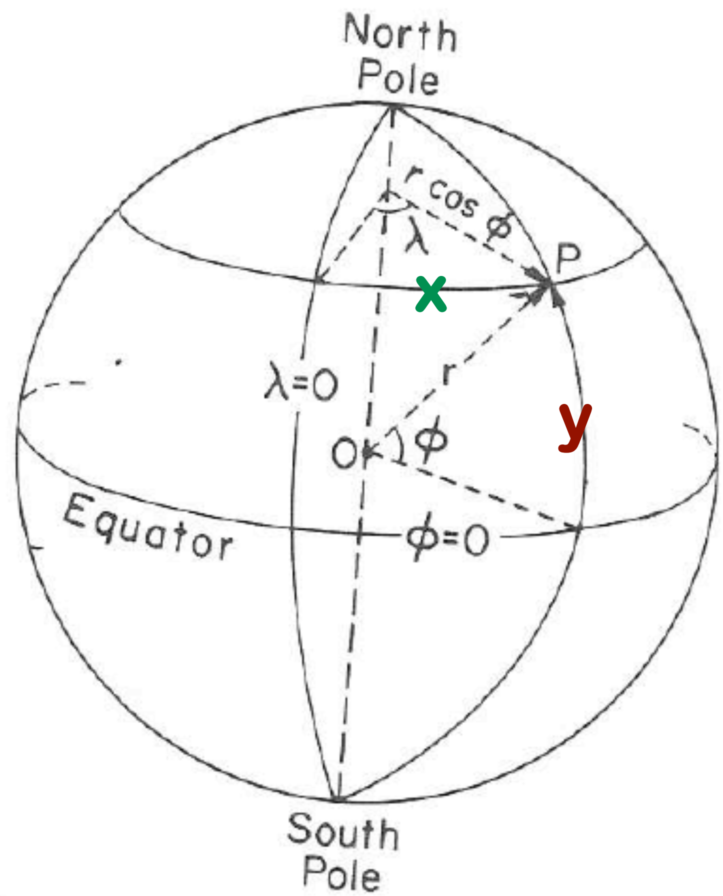


- The horizontal coordinates are latitude (ϕ) and longitude (λ).

- These angles are often replaced by the geocentric coordinates:

$$dx \equiv r\, d\lambda \cos\phi \qquad dy \equiv r\, d\phi$$

  where x is the arc distance east of the Greenwich meridian ($\lambda = 0°$), y is the distance north of the equator ($\phi = 0°$), and r is the distance from the center of the Earth.



- In dealing with atmospheric motions below 50 km, r is replaced by a, the mean radius of the Earth, which introduces an error of <1% but simplifies things tremendously.

- In this coordinate system:

$u \equiv dx / dt = a \cos\phi \, d\lambda/dt$ (zonal velocity; $u > 0$ to the east; westerly)

$v \equiv dy / dt = a \, d\phi/dt$ (meridional velocity; $v > 0$ to the north; southerly)

$w \equiv dZ / dt$ (vertical velocity; $w > 0$ up)

with Z = geopotential height = Φ / g_0.

- Using this coordinate system, that rotates with the Earth, causes a violation of Newton's Second Law of Motion because an object at rest or in uniform motion with respect to the rotating Earth is NOT at rest or in uniform motion relative to a coordinate system that is fixed in space (i.e., what an observer in outer space would see).

- Therefore, motion that appears inertial to an observer on the rotating Earth is actually non-inertial, or accelerated, motion.

- Newton's Second Law of Motion can still be applied in non-inertial frames of reference, but the accelerated motion of the coordinate system must be accounted for by the introduction of "apparent" forces because accelerated motions can only be caused forces.

- Thus, we shall add the "apparent" centrifugal and Coriolis forces to our real forces of gravity, pressure gradient and friction.

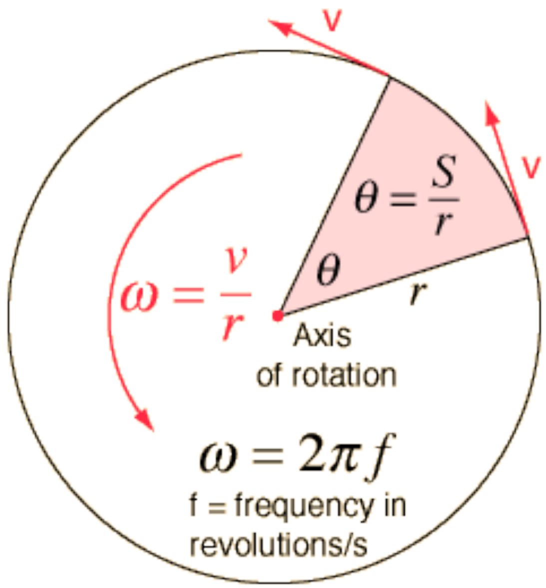
- It should be noted that these apparent forces are not unique to the Earth, but apply to any coordinate system in uniform motion.

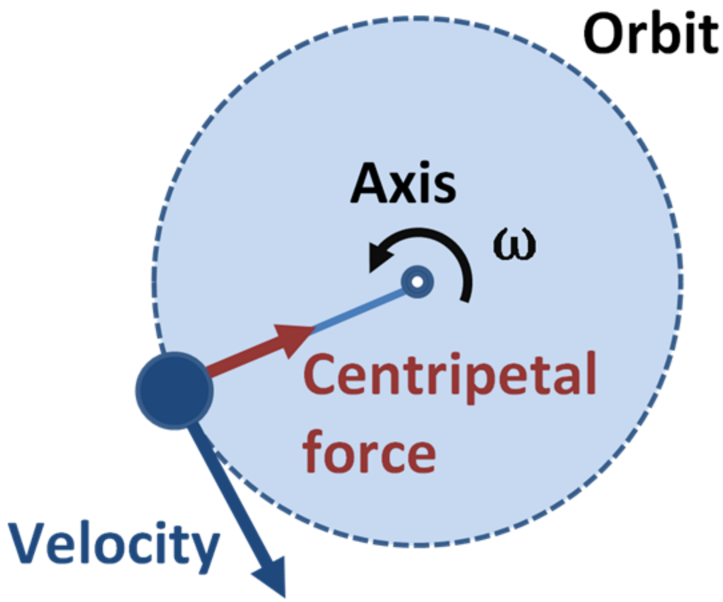
Apparent forces in a rotating coordinate frame

- Consider an air parcel moving from west to east relative to the Earth, which is rotating with angular velocity:

$$\Omega \text{ (or } \omega) = 2\pi \text{ / day} = 7.292 \times 10^{-5} \text{ s}^{-1}$$

- From the viewpoint of an omniscient observer, external to the Earth's rotation, the parcel is moving along a circular trajectory (a latitude circle) with a velocity $\Omega R_A + u$, where R_A is the distance from the Earth's axis of rotation, and ΩR_A is the tangential velocity of the coordinate system at the location of the parcel.





The centripetal acceleration is the external force required to make a body follow a curved path.

- Because the parcel is traveling in a circle with radius R_A and velocity ΩR_A + u, it has an acceleration, the centripetal acceleration, towards the center of the circle given by:

$$\frac{\left(\vec{\Omega} R_A + u\right)^2}{R_A}$$

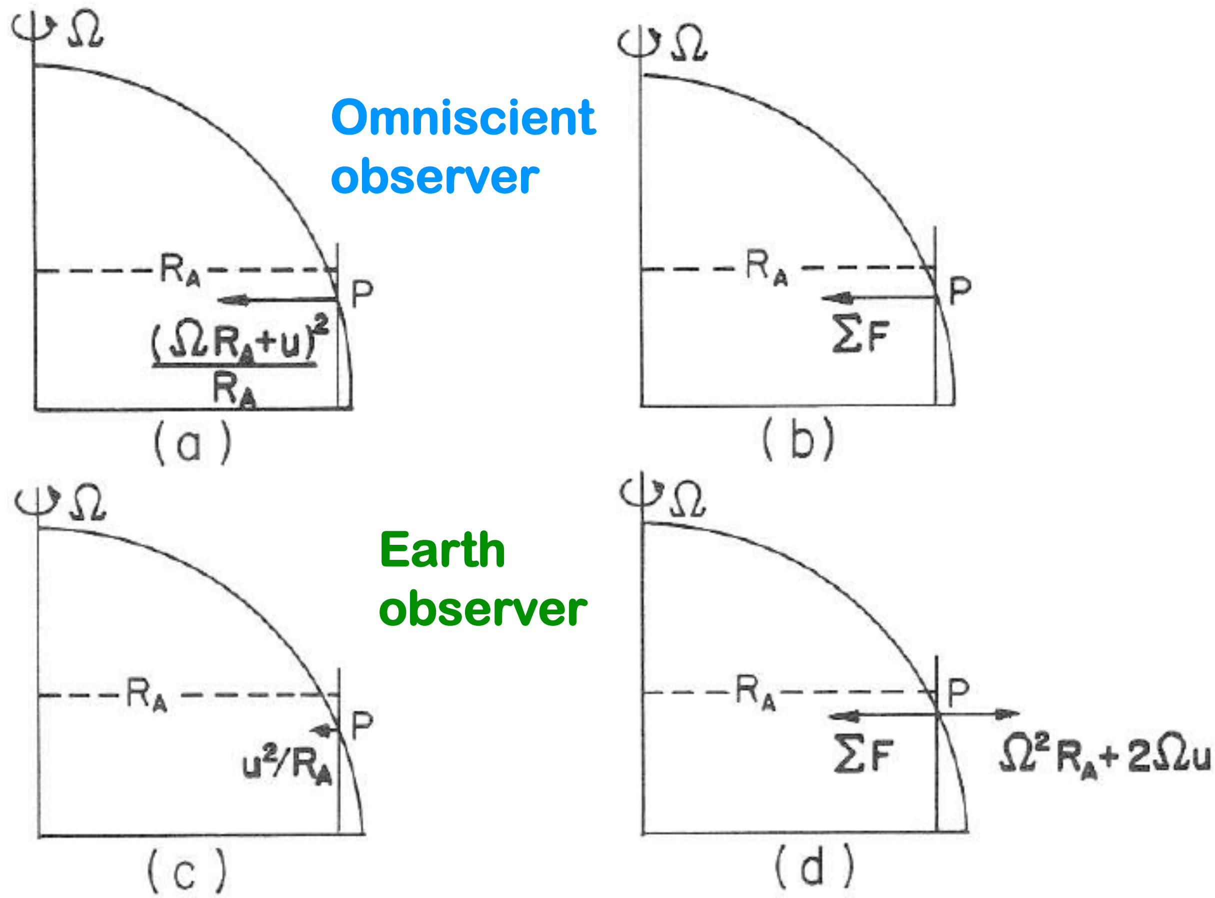
as shown in panel (a) on the next slide.

- According to Newton's Second Law of Motion, the vector sum of the forces per unit mass, ΣF, acting upon the parcel must be identical to the above acceleration, as indicated in panel (b).

- From the viewpoint of an observer on the rotating Earth this appears to violate Newton's Second Law because the velocity of the parcel is simply u and the centripetal acceleration is u^2 / R_A, as indicated in panel (c).

- Expanding out the centripetal force expression from the previous page:

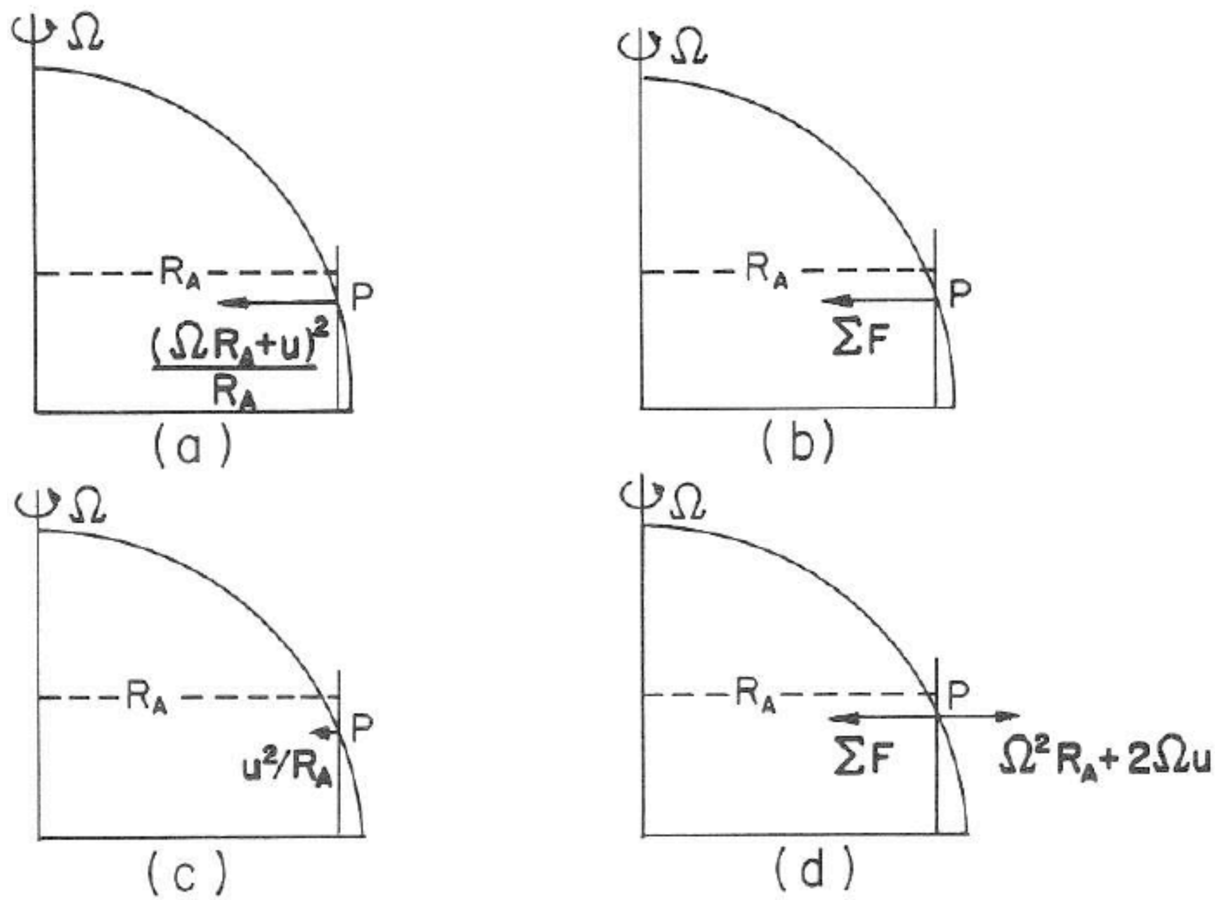
$$\sum F = \frac{\left(\vec{\Omega} R_A + u\right)^2}{R_A} = \vec{\Omega}^2 R_A + 2\vec{\Omega} u + \frac{u^2}{R_A}$$



- So the difference in forces between the two frames of reference are the $\Omega^2 R_A$ and $2\Omega u$ terms.

- Thus, the violation of Newton's Law can be eliminated by introducing these two "apparent" forces for the rotating, non-inertial coordinate system.

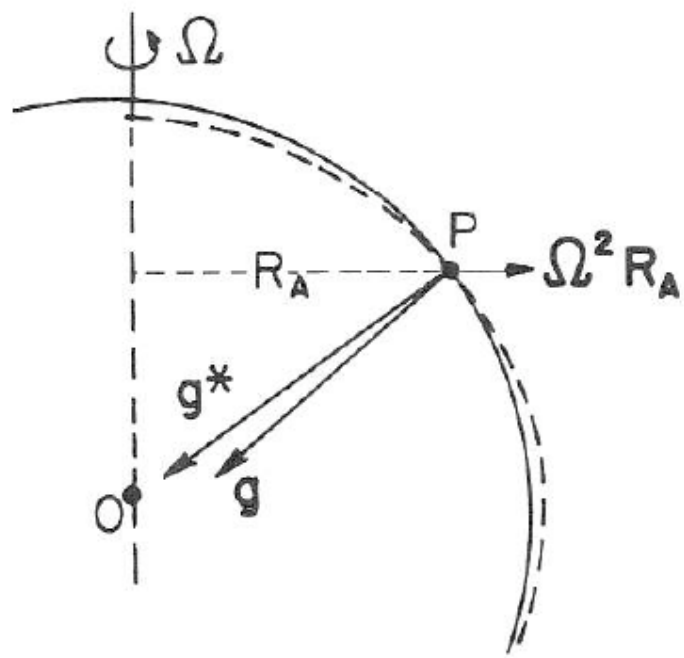
- These two, outwardly directed, apparent forces that are added to the geocentric coordinate system are the centrifugal force, $\Omega^2 R_A$, and the Coriolis force, $2\Omega u$, as indicated in panel (d).



Effective gravity

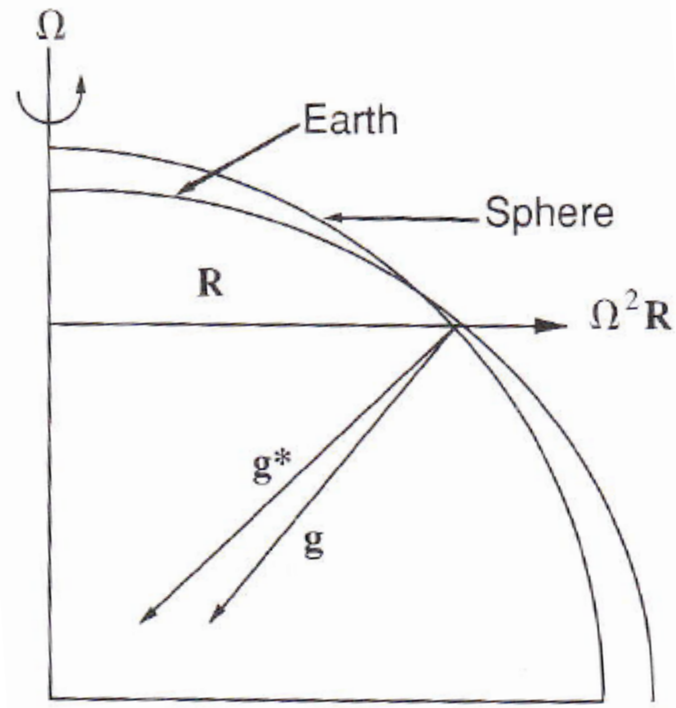
- The force per unit mass called gravity or effective gravity, g, is actually the vector sum of the true gravitational attraction, g*, that draws all objects towards the center of mass (O) of the Earth (discussed last class) and the much smaller apparent centrifugal force, $\Omega^2 R_A$, that pulls objects outward from the axis of planetary rotation as shown below.

- Since the gravitational force (g*) is directed toward the center of the Earth (along a) and the centrifugal force is directed outward from the axis of rotation (along R_A), the effective gravity (g) is NOT directed towards the center of the Earth, except at the equator and poles!



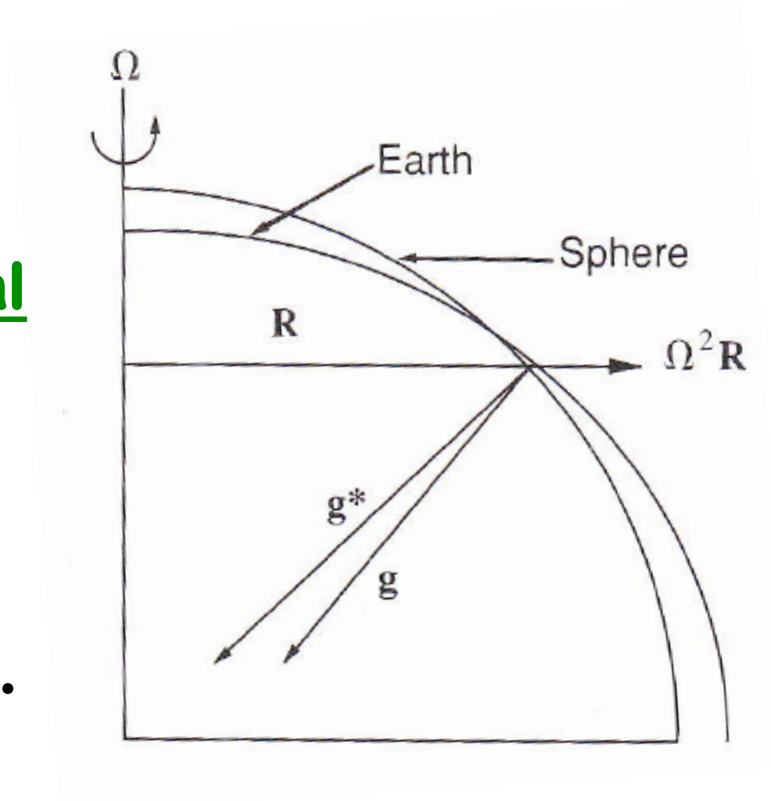
- This is another reason we defined the geopotential, Φ.

- As indicated on the figure, the effective gravity (g) is normal to surfaces of constant geopotential (labeled "Earth" to the left, or the dashed line on the previous slide) which the solid Earth and sea level, under the strong pull of gravity, have assumed the shape of.



- This shape is an oblate spheroid, which is a sphere, but with flattened poles and an equatorial bulge that makes the radius of the Earth 21 km larger at the equator than the poles.

- Since a body (like an air parcel) rotating with the Earth has no way of separately sensing the distinct forces of the gravitational (g^*) and centrifugal components of the effective gravity, there is nothing to be gained by expressing the two as separate forces in the equations of motion.



- Therefore, the $\Omega^2 R_A$ term does not explicitly appear in the equations of motion; it is included implicitly as part of g.