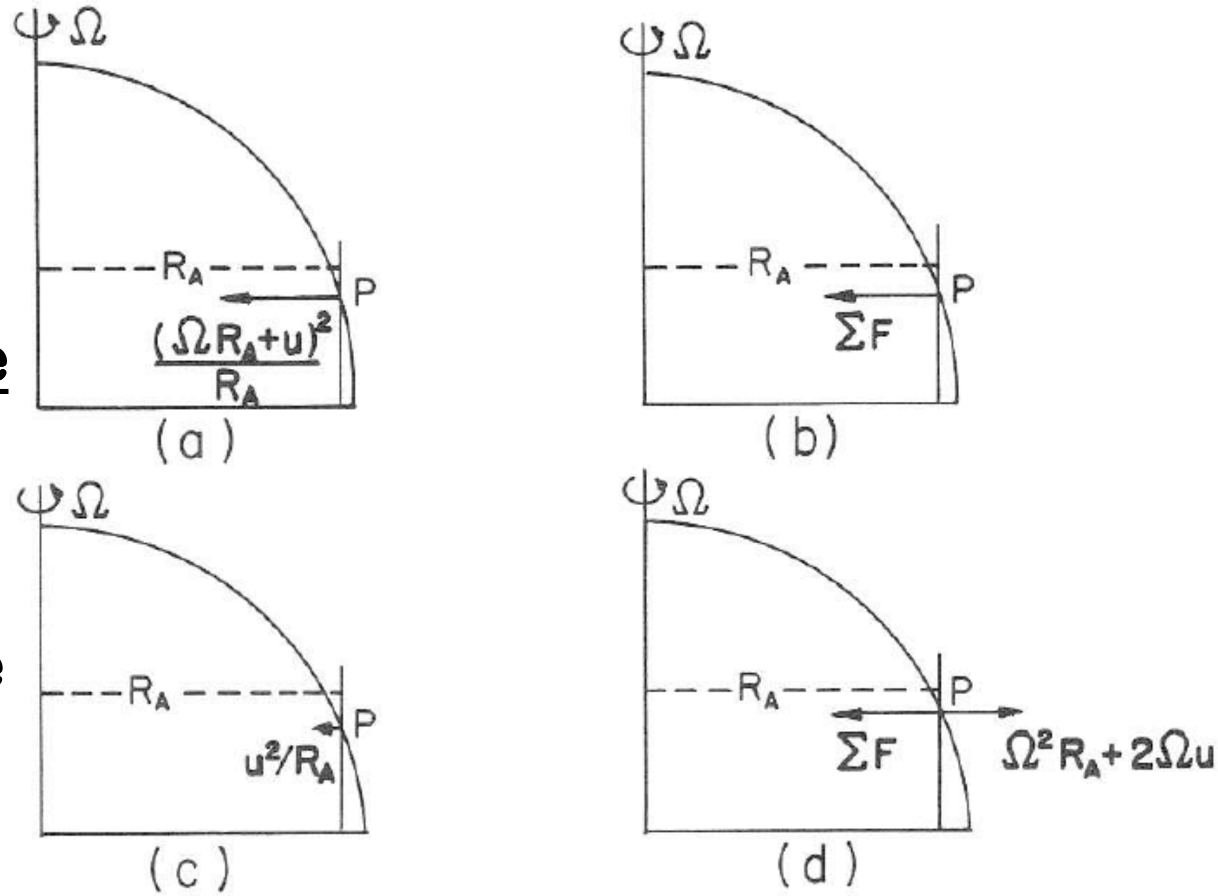
The Coriolis Force

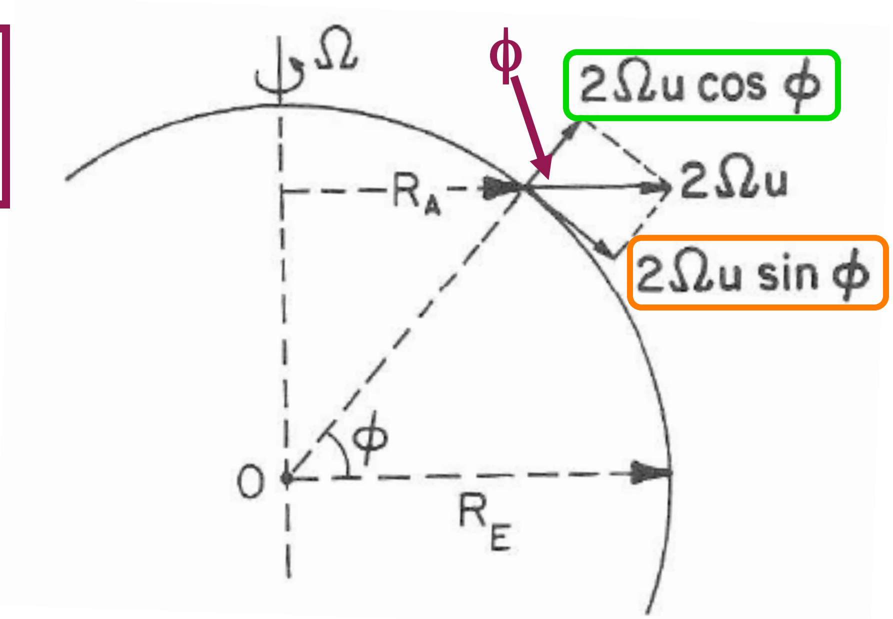
- The second part of the two apparent forces that arise due to the use of a non-inertial, geocentric coordinate system is the $2\Omega u$ term.
- This term is called the Coriolis force and is directed radially outward (along R_A) from the axis of rotation if $u > 0$ (westerly flow) and radially inward, toward the axis of rotation, if $u < 0$ (easterly).



- The figure below shows that the Coriolis force, $2\Omega u$, associated with an eastward ($u > 0$) velocity (into the page) can be resolved into a horizontal component in the negative y-direction (towards the south), $2\Omega u \sin\phi$, and a vertical component in the positive z-direction (up), $2\Omega u \cos\phi$.

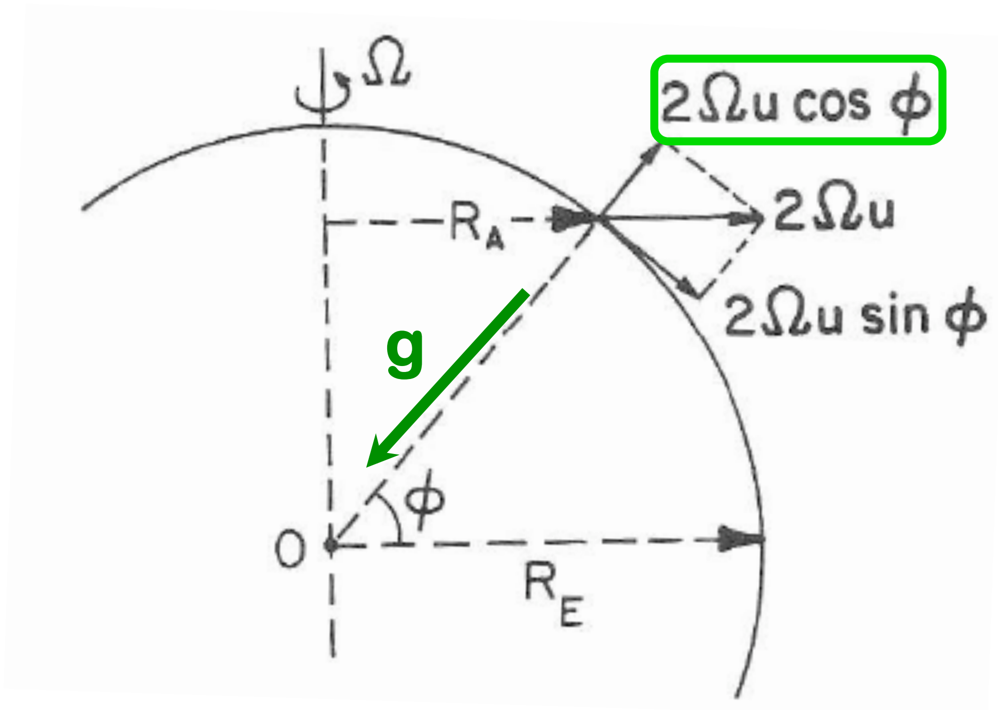
- Since the parcel had no meridional or vertical velocity to start with, the accelerations are:

$$\frac{dv}{dt} = -2\Omega u \sin\phi \quad \text{and} \quad \frac{dw}{dt} = 2\Omega u \cos\phi$$



- A parcel moving eastward is deflected to the south and a parcel moving west is deflected to the north in the Northern Hemisphere. Both to the right of the initial motion.

- The vertical component of the Coriolis force is generally much smaller than the gravitational force in the same direction (and smaller than the other terms we will find to be important in the vertical equation of motion), so the small effect it has in changing the apparent weight of an object is usually ignored.

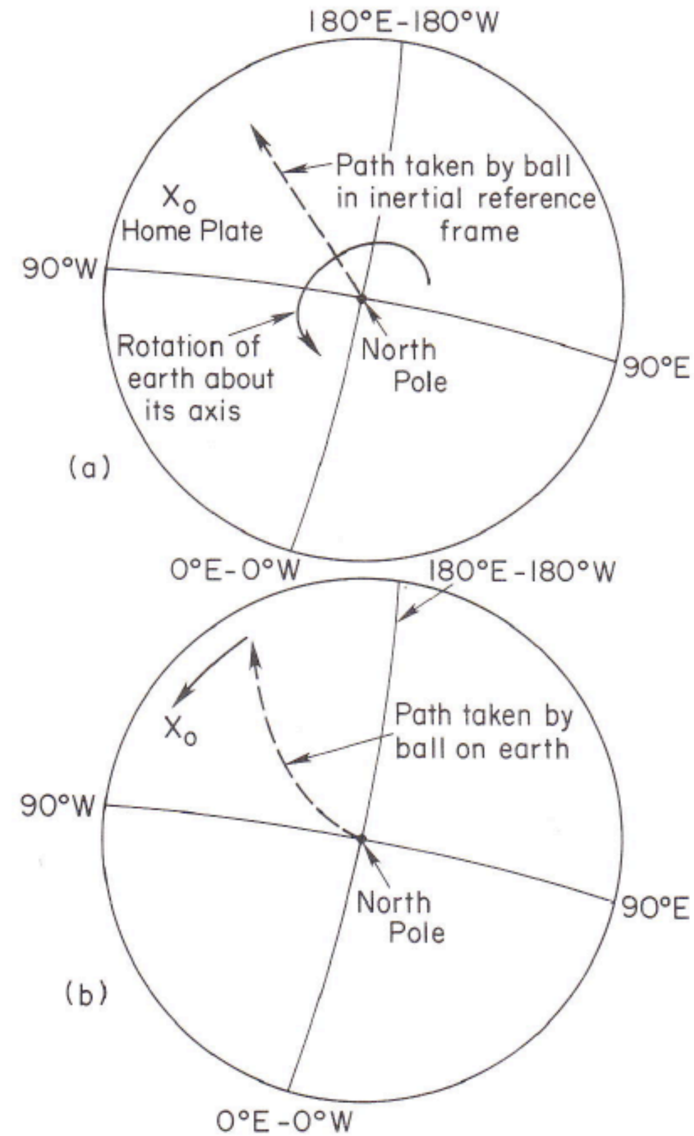


- So far we have only considered the Coriolis force associated with zonal motion, u. Can flow in the meridional direction be influenced by the Coriolis force as well?

- Taking a step back, consider the image to the right. A pitcher located at the North Pole is trying to throw a ball to the catcher located at home plate, X_0.

- By the time the ball arrives at the catcher, she will have rotated to the pitcher's left as the Earth rotates.

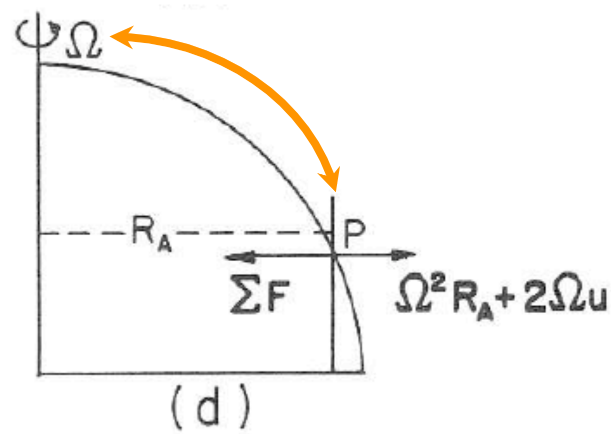
- Thus, it appears to the pitcher as if the ball was deflected to the right (to the west).



- So, we have learned that northerly ($v < 0$) motion induced a westward drift, $du/dt < 0$ (since there was no zonal flow to begin with).
- Can we understand this from the equations we have already derived for a geocentric coordinate system?

★ **Yes, we can!** ★

- Formally, we can understand the existence of the Coriolis force in association with meridional flow from the conservation of angular momentum.
- As a parcel with unit mass moves in the meridional direction, its distance from the axis of rotation, R_A, changes.



- Thus, as the parcel moves toward or away from the axis of rotation, in the absence of any real forces acting upon it, angular momentum is conserved. That is:

$$\frac{d\omega}{dt} = \frac{d}{dt}\left[R_A\left(\Omega R_A + u\right)\right] = 0$$

- Carrying out the differentiation, we find:

$$2\Omega R_A \frac{dR_A}{dt} + u\frac{dR_A}{dt} + R_A\frac{du}{dt} = 0$$

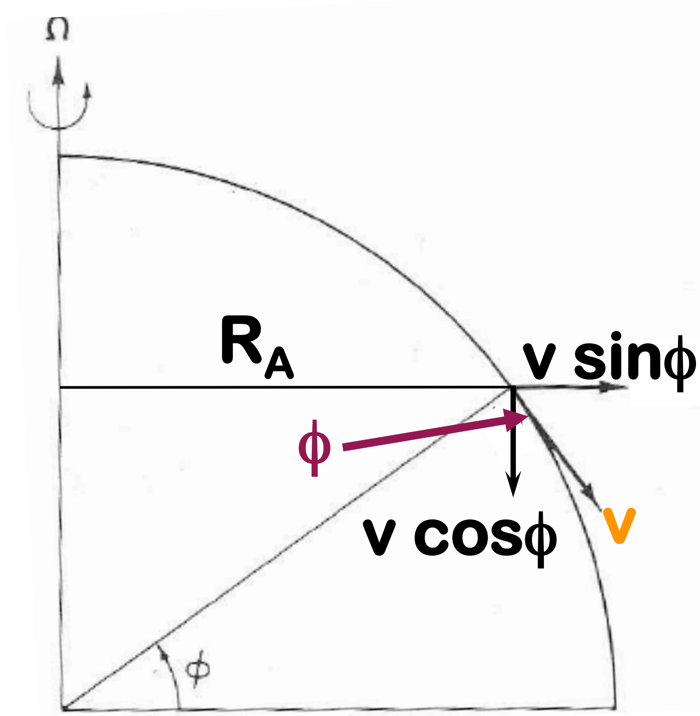
- If the zonal velocity, u, is set equal to 0 (no initial zonal flow), the expression reduces to:

$$\boxed{\frac{du}{dt} = -2\Omega\frac{dR_A}{dt}}$$

- Therefore, if a parcel is moving radially outward from the Earth's axis of rotation, $dR_A/dt > 0$ (as it would for motion towards the equator, $v < 0$), $du/dt < 0$ and the parcel will experience an easterly acceleration, to the west, just like our pitcher and ball example.

- Similarly, if a parcel is moving toward the axis of rotation, $dR_A/dt < 0$ (such as motion toward the pole, $v > 0$), then $du/dt > 0$ and the parcel experiences a westerly acceleration.

- Furthermore, to define the components of the Coriolis force that arise from meridional flow, we can resolve the y-direction wind (v) into a component, $v \cos\phi$, parallel to the axis to rotation, and a component, $v \sin\phi$, in the plane perpendicular to the axis of rotation (radially outward along R_A).

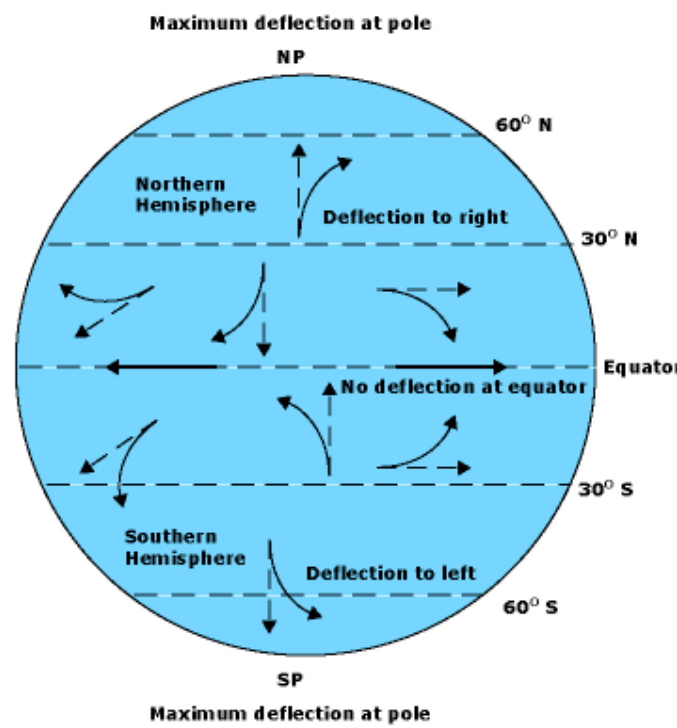


- The first component is directed into the ground and generally small, while the second induces a force $2\Omega v \sin\phi$ in the zonal direction.

- Thus, the two primary horizontal components of the Coriolis force we are concerned with are

$$\frac{dv}{dt} = -2\Omega u \sin\phi \quad \text{and} \quad \frac{du}{dt} = 2\Omega v \sin\phi$$

- Since the Coriolis force appears so frequently in the equations of motion, the factor 2Ω sinϕ is given the symbol f and called the Coriolis parameter. By definition, $f > 0$ in the Northern Hemisphere and $f < 0$ in the Southern Hemisphere.



- We should also note that vertical motions do give rise to horizontal Coriolis forces, but vertical velocities on the large scale are generally so small (< 1 m s^{-1}) that these effects are ignored.

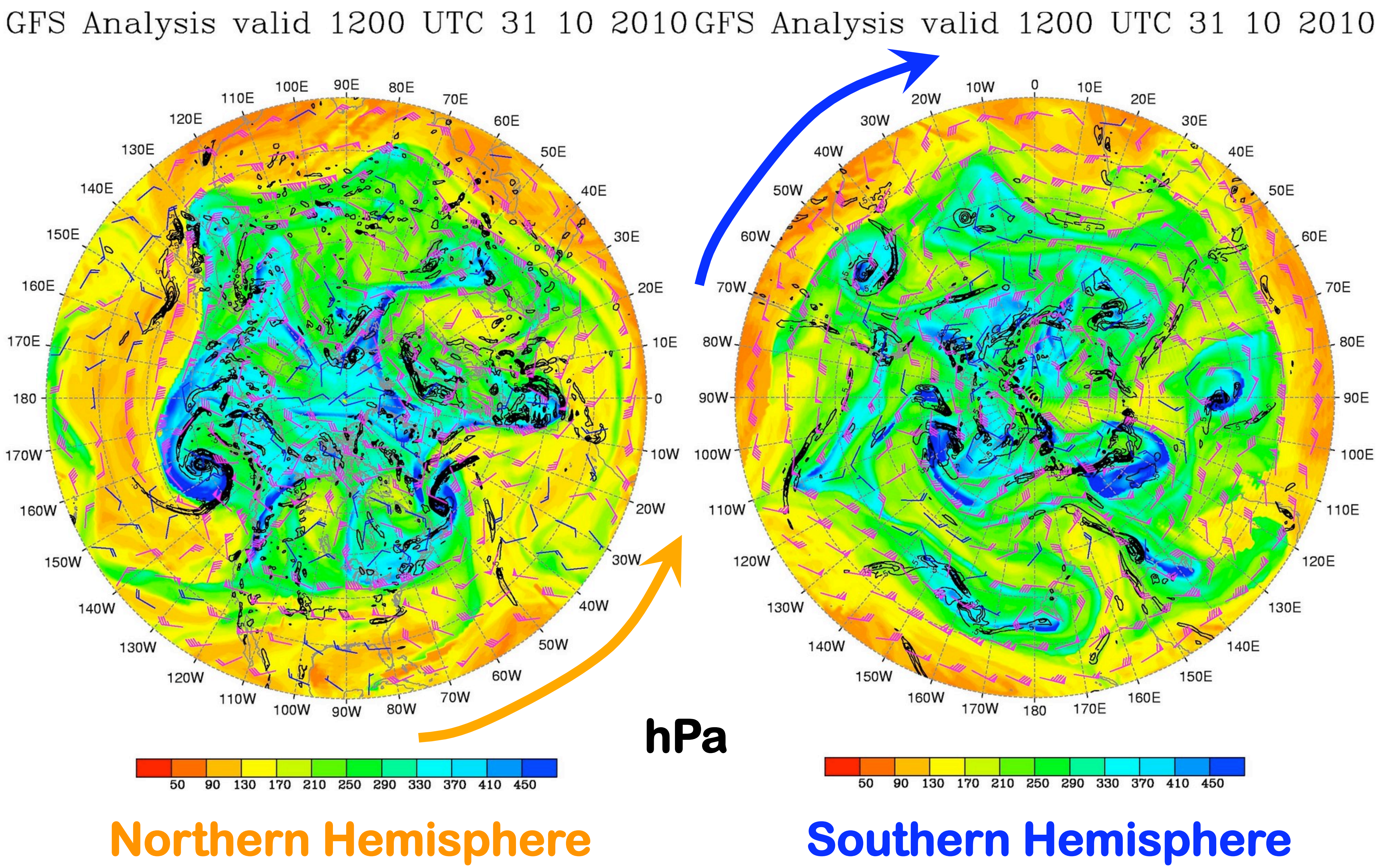
- The above results describing the horizontal Coriolis forces induced by horizontal motions in the geocentric coordinate system can be summarized:

   1) A parcel with horizontal velocity $\vec{V}_h$ experiences a Coriolis force whose horizontal component has magnitude $|2\Omega V_h \sin\phi|$.

   2) The horizontal component of the Coriolis force is directed perpendicular to the horizontal velocity vector; toward the right in the Northern Hemisphere where the planetary rotation is counterclockwise (as viewed from above), and toward the left in the Southern Hemisphere where the planetary rotation is clockwise.

- Wait a minute…the sense rotation of the Earth is different in the Northern and Southern Hemispheres? Yes, when viewed from aloft looking down on the poles!

Dynamic tropopause pressure and vertical wind shear



- The deflection by the Coriolis force is negligible for atmospheric motions with time scales that are short compared to the period of the Earth's rotation, 1 day.
- So the Coriolis force is negligible for individual cumulus clouds or a tornado, but fundamental to our understanding of mid-latitude weather systems and hurricanes.