The Geostrophic Wind

- On Monday, after we derived the component expressions for the momentum equation, we performed a scale analysis on the horizontal equations choosing to look at the synoptic scale of motion at 45° N, as summarized in the following table:

Table 2.1 *Scale Analysis of the Horizontal Momentum Equations*

	A	B	C	D	E	F	G
x-Eq.	$\frac{Du}{Dt}$	$-2\Omega v \sin\phi$	$+2\Omega w \cos\phi$	$+\frac{uw}{a}$	$-\frac{uv \tan\phi}{a}$	$= -\frac{1}{\rho}\frac{\partial p}{\partial x}$	$+F_{rx}$
y-Eq.	$\frac{Dv}{Dt}$	$+2\Omega u \sin\phi$		$+\frac{vw}{a}$	$+\frac{u^2 \tan\phi}{a}$	$= -\frac{1}{\rho}\frac{\partial p}{\partial y}$	$+F_{ry}$
Scales	U^2/L	$f_0 U$	$f_0 W$	$\frac{UW}{a}$	$\frac{U^2}{a}$	$\frac{\delta P}{\rho L}$	$\frac{\nu U}{H^2}$
($m\ s^{-2}$)	10^{-4}	10^{-3}	10^{-6}	10^{-8}	10^{-5}	10^{-3}	10^{-12}

- **From the table, it is apparent that for typical synoptic scale, midlatitude motions, the pressure gradient and Coriolis (associated with horizontal flow) forces are of the same order and approximately balance each other; i.e.,**

$$-2\Omega v \sin\phi = -\frac{1}{\rho}\frac{\partial p}{\partial x} \quad \text{in the x - direction}$$

$$2\Omega u \sin\phi = -\frac{1}{\rho}\frac{\partial p}{\partial y} \quad \text{in the y - direction}$$

- **These two expressions represent geostrophic ("geo" for Earth and "strophic" for turning) balance relating the horizontal pressure and wind fields.**

- **Letting $f = 2\Omega \sin\phi$ and defining u_g and v_g as the zonal and meridional components of the geostrophic wind such that $\vec{V}_g = u_g\hat{i} + v_g\hat{j}$, we may write**

$$-fv_g = -\frac{1}{\rho}\frac{\partial p}{\partial x} \longrightarrow v_g = \frac{1}{f\rho}\frac{\partial p}{\partial x} \quad \text{and} \quad fu_g = -\frac{1}{\rho}\frac{\partial p}{\partial y} \longrightarrow u_g = -\frac{1}{f\rho}\frac{\partial p}{\partial y}$$

- We can **simplify** this **further** by **switching** from **height to geopotential height** coordinates. Recall $\Phi = gz$, the **hydrostatic equation** $\partial z = -\partial p / g\rho$ and the **height to pressure transform**:

$$-\frac{1}{\rho}\left(\frac{\partial p}{\partial x}\right)_z = -g\left(\frac{\partial z}{\partial x}\right)_p = -\left(\frac{\partial \Phi}{\partial x}\right)_p$$

we may **rewrite the geostrophic wind components** as

$$v_g = \frac{1}{f}\frac{\partial \Phi}{\partial x} \quad \text{and} \quad u_g = -\frac{1}{f}\frac{\partial \Phi}{\partial y}$$

and see the true **beauty of** the **geopotential height** in that the **density does not appear in the equations**!

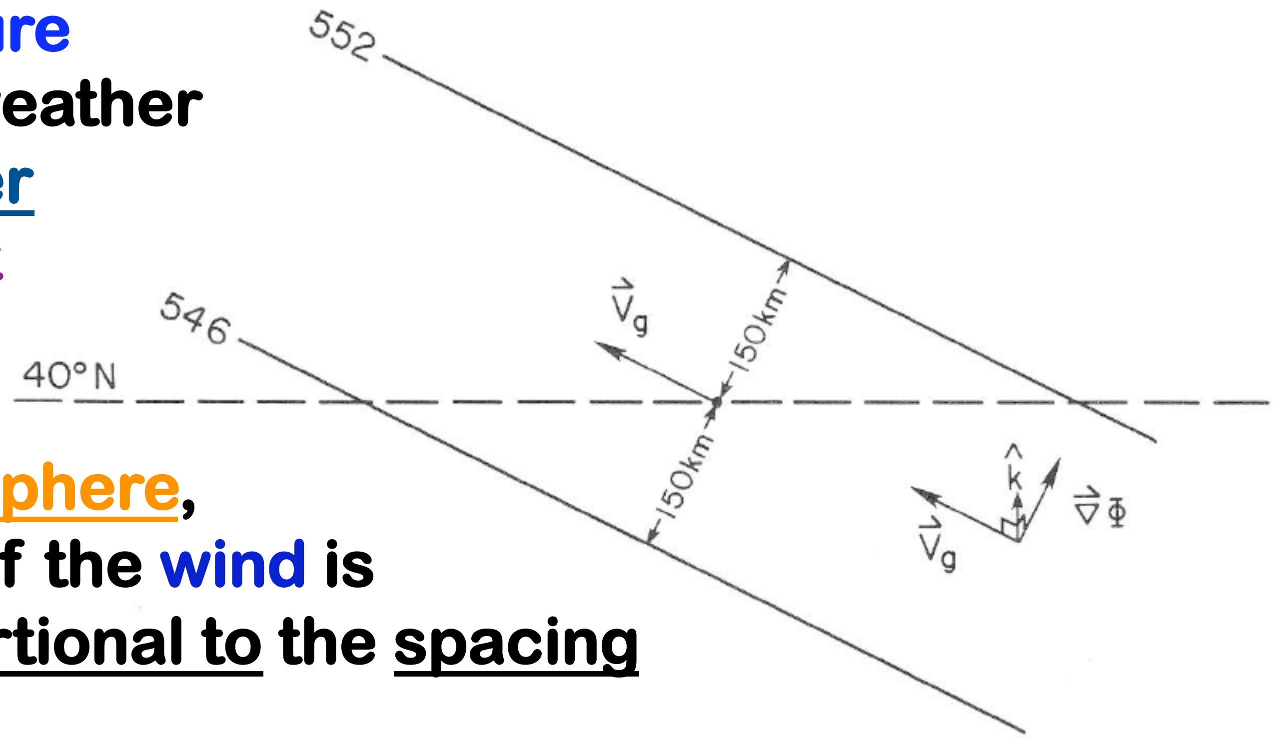
- Thus, in **vector form** we may write

$$\vec{V}_g = u_g\hat{i} + v_g\hat{j} = -\frac{1}{f}\frac{\partial \Phi}{\partial y}\hat{i} + \frac{1}{f}\frac{\partial \Phi}{\partial x}\hat{j}$$

$$\vec{V}_g = \frac{1}{f}\left(-\frac{\partial \Phi}{\partial y}\hat{i} + \frac{\partial \Phi}{\partial x}\hat{j}\right)$$

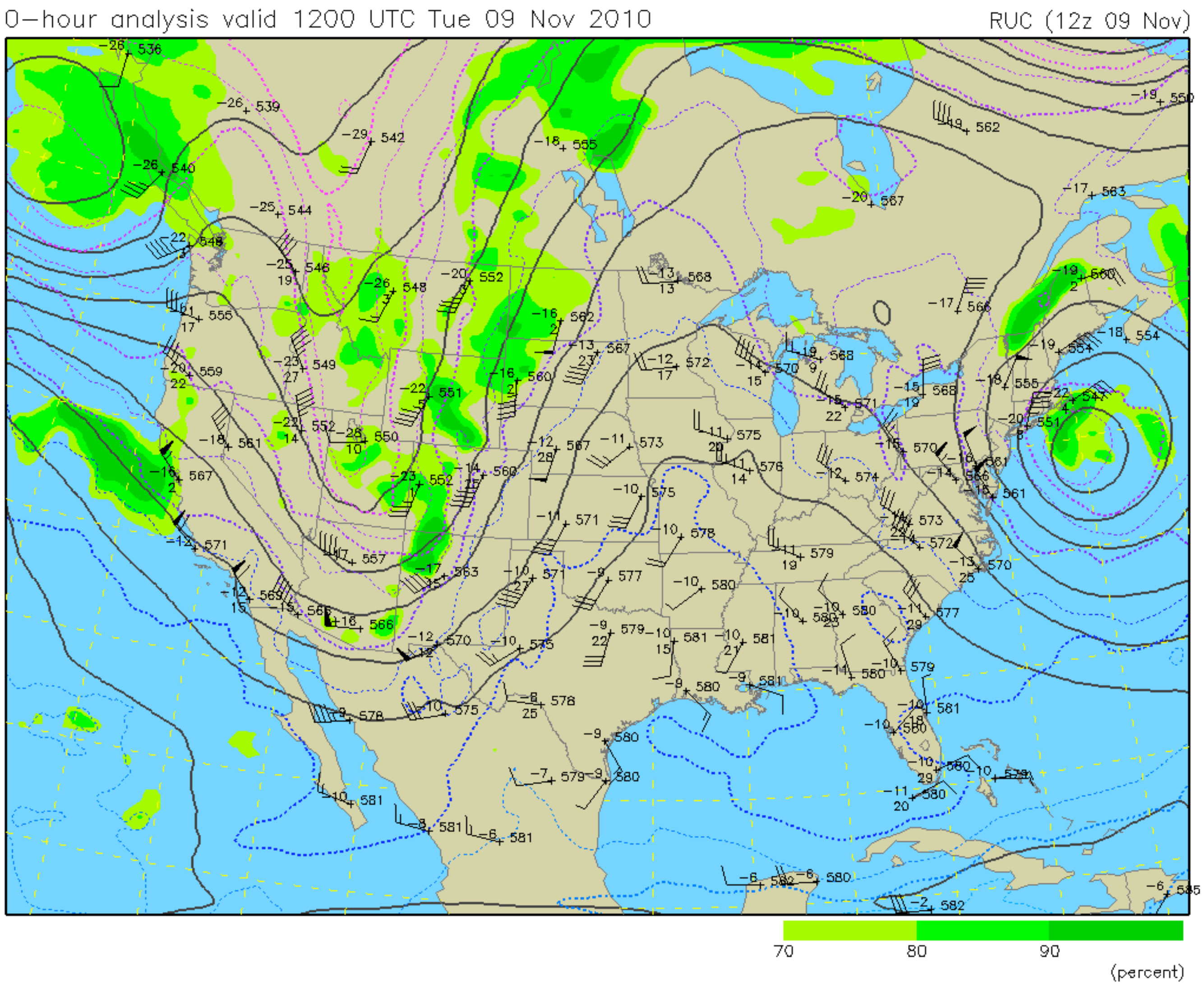
$$\vec{V}_g = \frac{1}{f}\hat{k} \times \vec{\nabla}_p \Phi \quad \longrightarrow \quad \vec{V}_g = \frac{1}{f}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 1 \\ \frac{\partial \Phi}{\partial x} & \frac{\partial \Phi}{\partial y} & \frac{\partial \Phi}{\partial p} \end{vmatrix}$$

- As seen in the figure below, the geostrophic wind blows parallel to the geopotential height contours on a constant pressure surface (most weather maps) with lower values of height to the left of the wind in the Northern Hemisphere, and the speed of the wind is inversely proportional to the spacing of the contours.



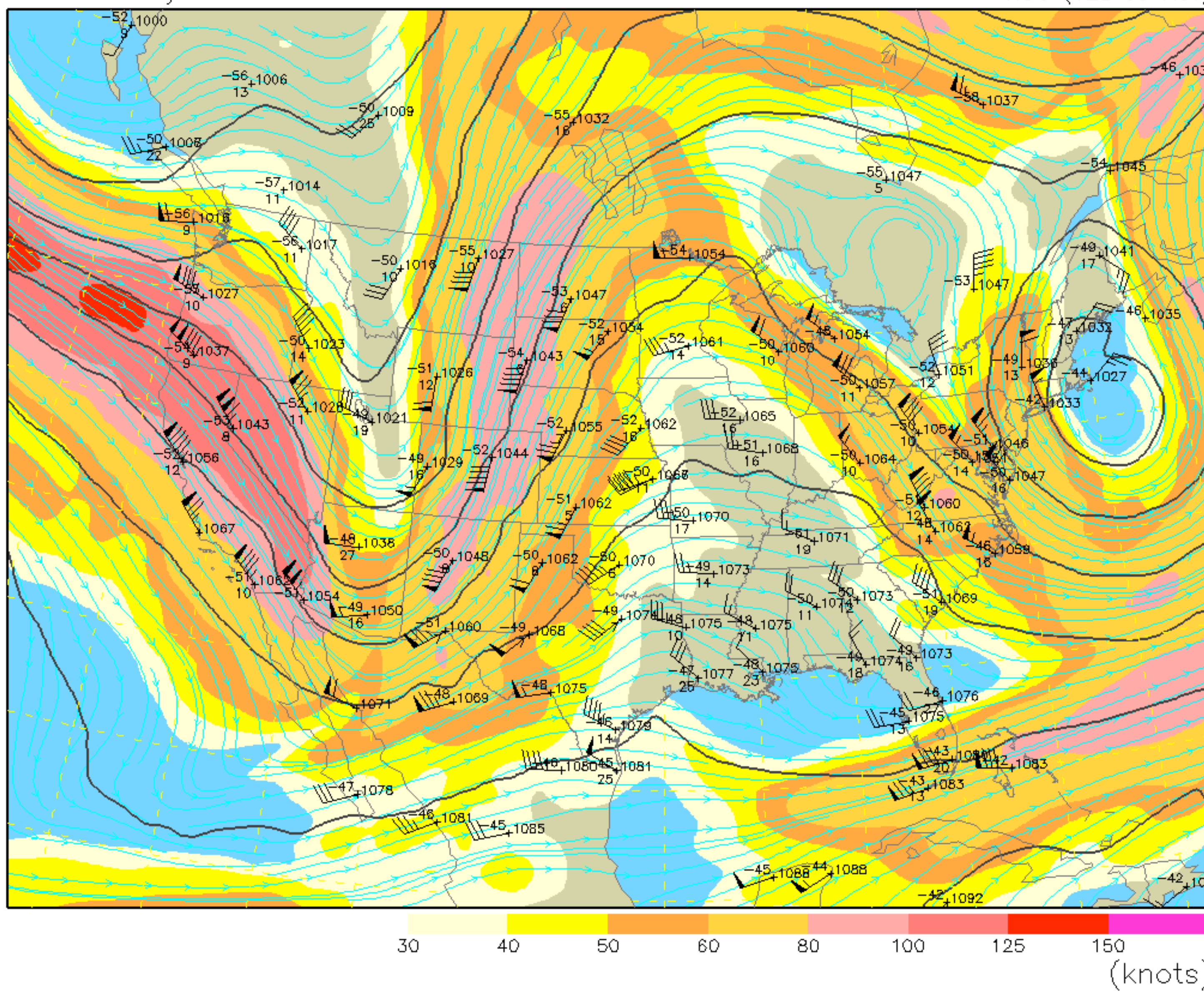
- **How well <u>does this work in the</u> <u>atmosphere</u>?**
- **Let's look at some <u>weather maps</u>!**

500 mb Heights (dm) / Temperature (°C) / Humidity (%)



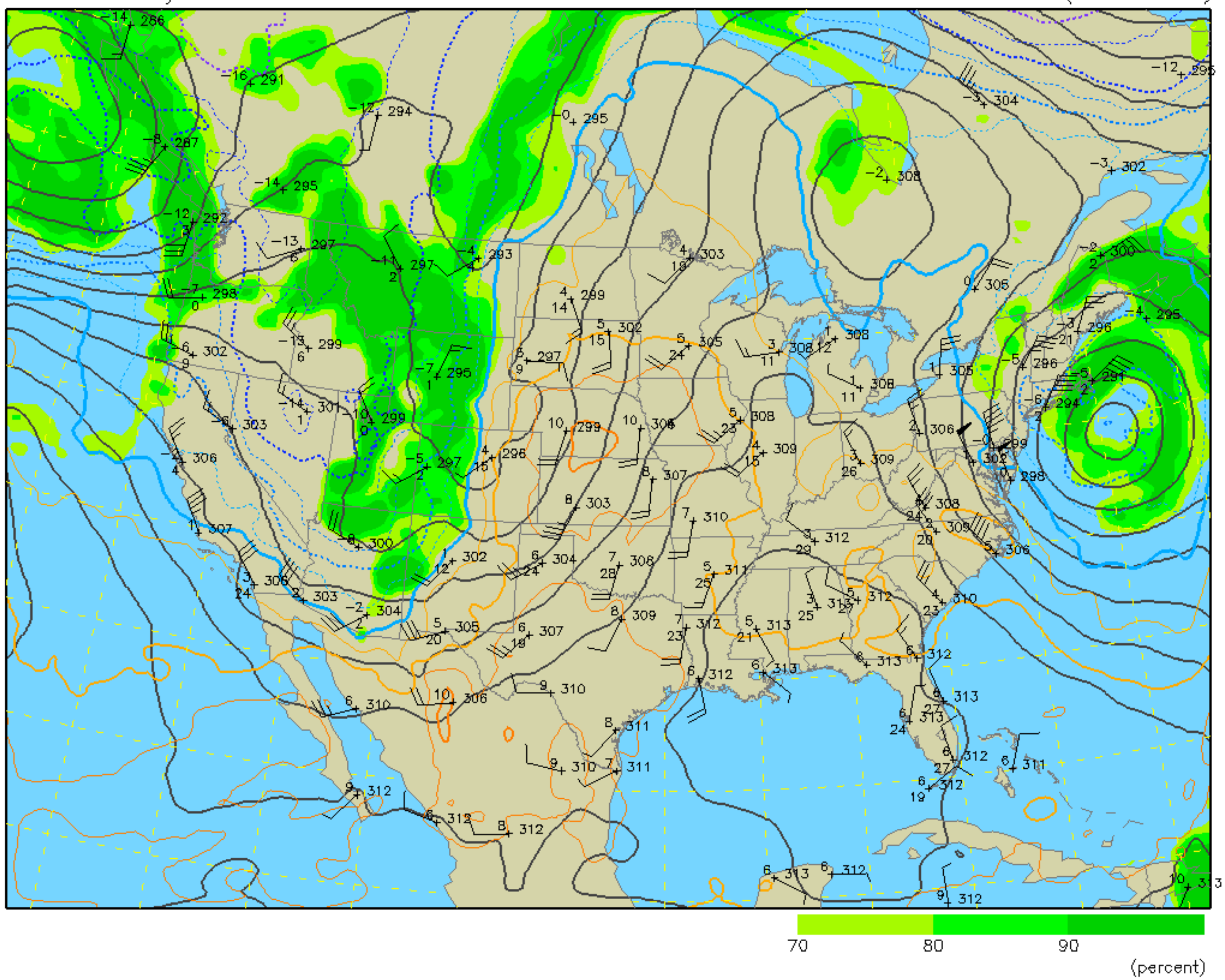
250 mb Heights (dm) / Isotachs (knots)

0–hour analysis valid 1200 UTC Tue 09 Nov 2010 RUC (12z 09 Nov)



700 mb Heights (dm) / Temperature (^oC) / Humidity (%)

0–hour analysis valid 1200 UTC Tue 09 Nov 2010 RUC (12z 09 Nov)



- We see that geostrophic balance approximates the wind pretty darn well away from the surface of the Earth (where we have friction; we'll cover this in a little bit) and not at the equator where $f = 0$.
- Thus, the geostrophic wind is frequently used in place of the real wind for midlatitude applications and is accurate for most situations to within 10-15 %.
- Alas, making the assumption of geostrophy does have some unfortunate consequences that we shall now examine.
- The first consequence is that there is no reference to time in the geostrophic wind equation; i.e., the equation is diagnostic, not prognostic.
- This means that with knowledge of the height field, we can diagnose the wind field, but we can not predict how it will evolve in time because there is no D / Dt term.

- To obtain predictive equations for the wind field we must retain the (D / Dt) terms. If we do this, the resulting momentum equations are

$$\frac{Du}{Dt} = fv - \frac{\partial \Phi}{\partial x} \quad \text{and} \quad \frac{Dv}{Dt} = -fu - \frac{\partial \Phi}{\partial y}$$

- Substituting in for the height gradients using the definition of the geostrophic wind components we have

$$\boxed{\frac{Du}{Dt} = f\left(v - v_g\right) \quad \text{and} \quad \frac{Dv}{Dt} = -f\left(u - u_g\right)}$$

  which says that the acceleration of the horizontal wind is proportional to the difference between the actual wind and the geostrophic wind times the Coriolis parameter.

- This confirms that the acceleration of the wind is an order of magnitude smaller than the Coriolis force (e.g., $f\ (u - u_g) < f\ u$, with $u = 10\ \text{m s}^{-1}$ and $u_g = 9\ \text{m s}^{-1}$).

- A convenient measure of how geostrophically the flow is behaving is the Rossby number, defined as the ratio of the acceleration to the Coriolis force:

$$Ro = \frac{\left|\frac{D\vec{V}}{Dt}\right|}{\left|-2\vec{\Omega}\times\vec{V}\right|} = \frac{U^2/L}{f_0 U} = \frac{U}{f_0 L}$$

- Following the argument from the previous slide, when Ro is small, $\vec{V} \sim \vec{V}_g$ and the flow is approximately geostrophic.

- Going back to our 250 hPa geopotential height map, where might Ro not be small and the flow deviating significantly from geostrophy?

- A more serious issue with using the geostrophic approximation is that there is no w equation, and if the atmosphere were indeed purely geostrophic, there would be no vertical motion.
- Let's prove it!
- Last lecture, we derived the continuity equation: $\vec{\nabla} \bullet \vec{V}_h = -\frac{\partial \omega}{\partial p}$
- If the atmosphere were in geostrophic balance:

$$\vec{\nabla} \bullet \vec{V}_g = -\frac{\partial \omega}{\partial p}$$

$$\frac{\partial u_g}{\partial x} + \frac{\partial v_g}{\partial y} = -\frac{\partial \omega}{\partial p}$$

- Substituting in the expressions for u_g and v_g we have:

$$-\frac{\partial \omega}{\partial p} = \frac{\partial}{\partial x}\left(-\frac{1}{f_0}\frac{\partial \Phi}{\partial y}\right) + \frac{\partial}{\partial y}\left(\frac{1}{f_0}\frac{\partial \Phi}{\partial x}\right)$$

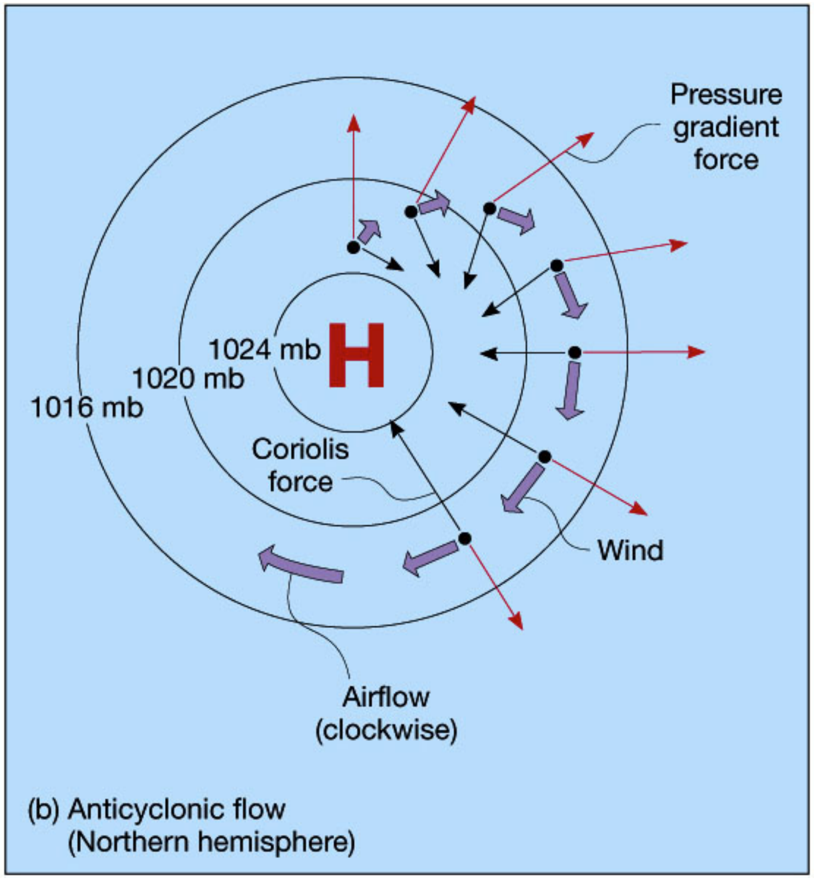
- Interchanging the order of differentiation we see

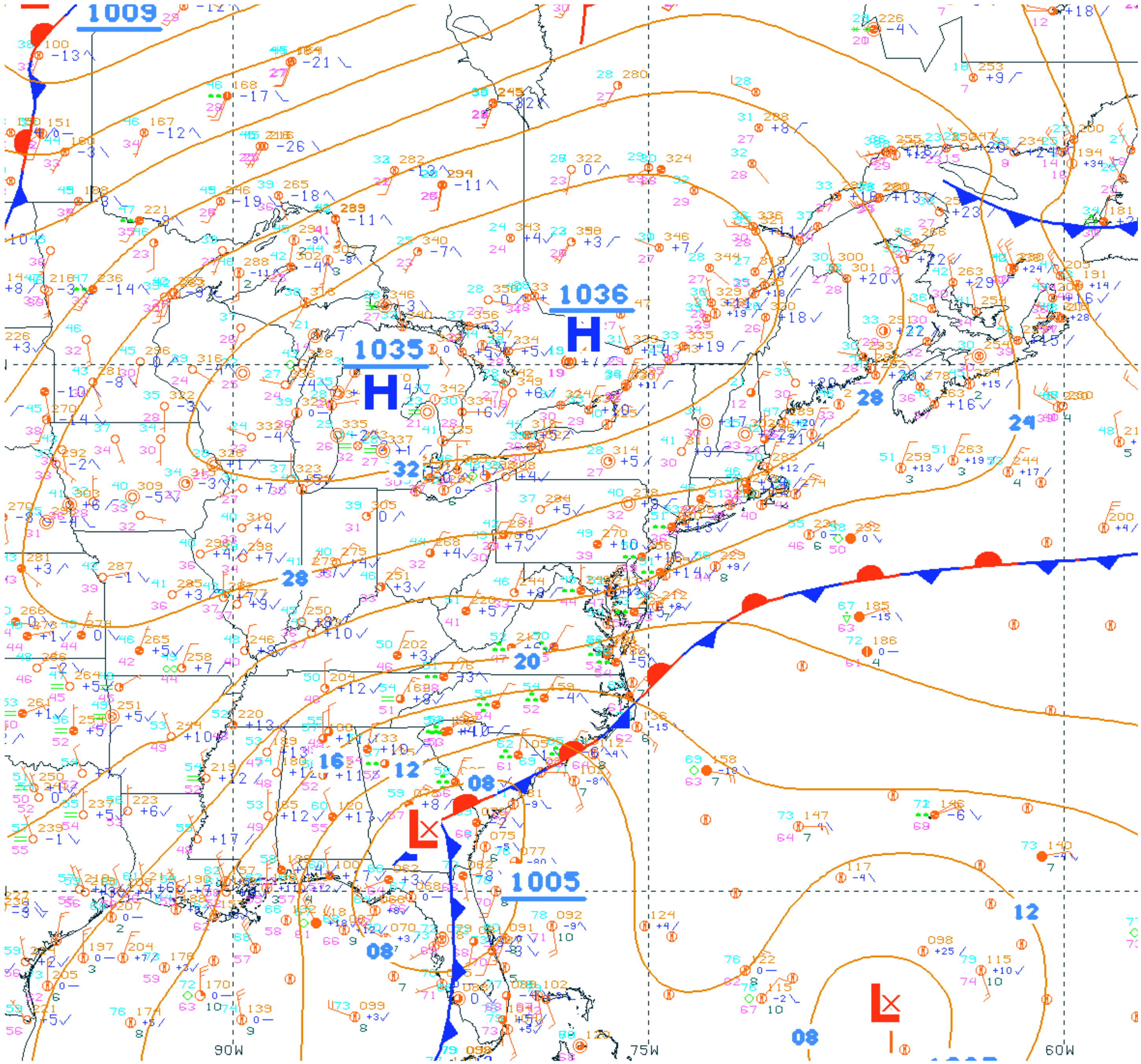
$$-\frac{\partial \omega}{\partial p} = \frac{1}{f_0}\left(-\frac{\partial^2 \Phi}{\partial x \partial y} + \frac{\partial^2 \Phi}{\partial x \partial y}\right) = 0$$

- So the geostrophic wind is non-divergent and if w = 0 at the ground and tropopause, w = 0 everywhere in between!

- Therefore, the small (but significant in some areas) difference between the true and geostrophic winds is intimately tied to both the acceleration of the horizontal wind and the synoptic scale vertical motion.

- Finally, we have seen that away from jet streaks, the equator and the surface of the Earth, the geostrophic wind is a good approximation for the real wind.
- Well, what about at the surface?
- If there was no friction, the flow around a high pressure system would look like the figure to the right.
- Air would flow in a clockwise direction around high pressure systems and counterclockwise around low pressure systems in the Northern Hemisphere.
- The real weather map on the next slide reveals a slightly different picture.





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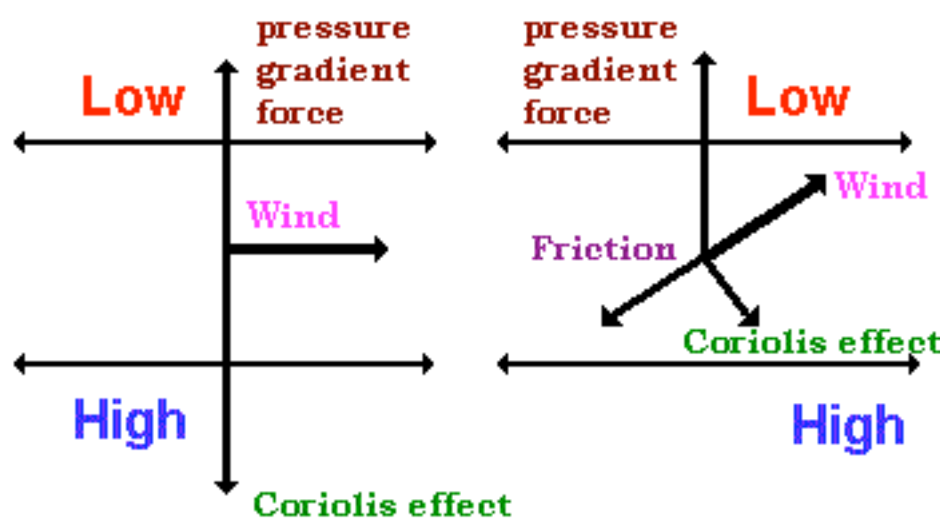
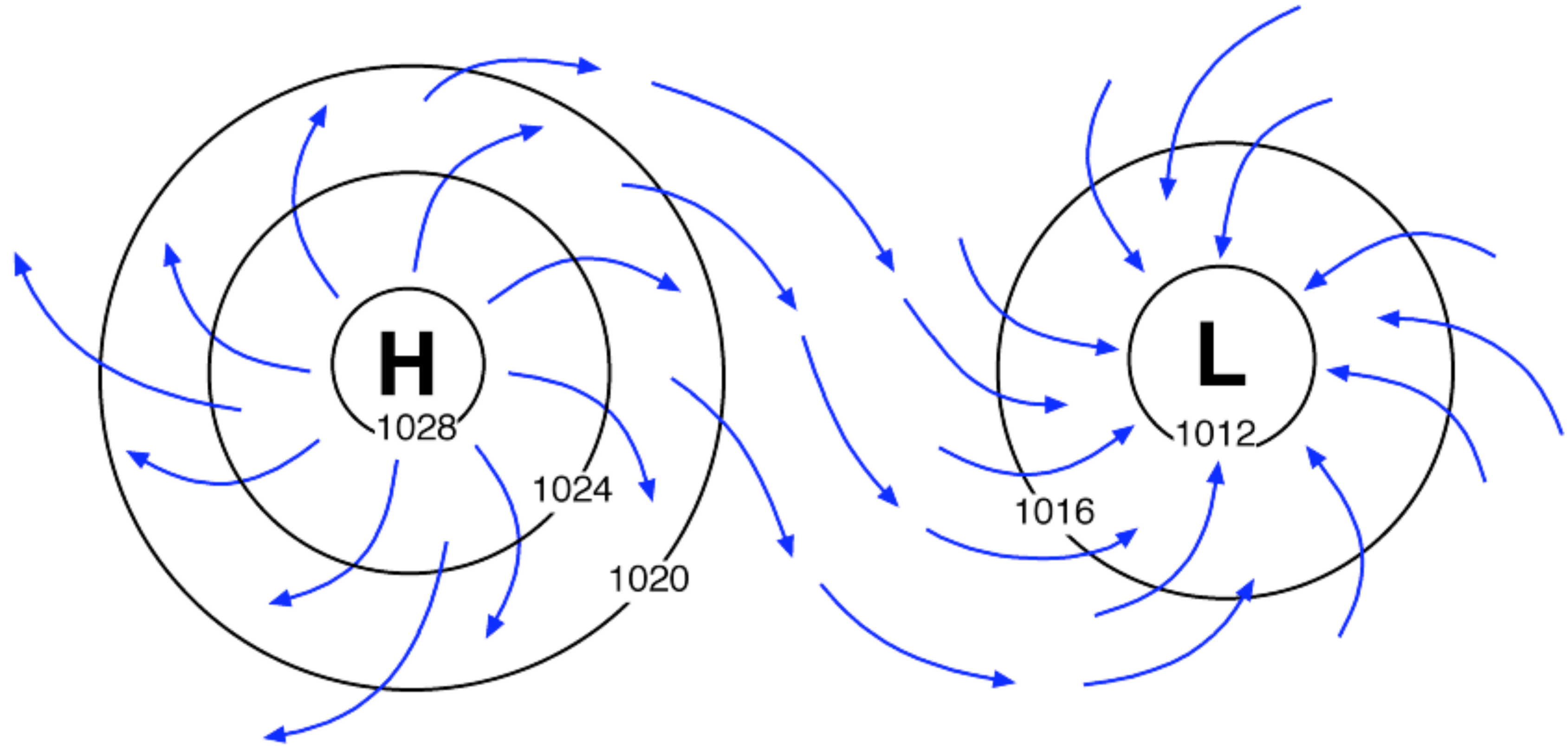
- Examining the surface weather map, we see that instead of blowing parallel to the isobars, the real surface winds cross the isobars directed out from the area of high pressure and in towards the area of low pressure.
- This cross isobaric flow is the result of the frictional force slowing down the wind, making the flow subgeostrophic.
- This reduces the strength of the Coriolis force (which is directly proportional to the wind speed).
- Thus, the Coriolis force isn't quite strong enough to balance the pressure gradient force and the wind crosses the isobars in the direction of the pressure gradient force from higher to lower pressure.





A high pressure and a low pressure center with wind directions (northern hemisphere).
Circles are isobars with pressure in mb.

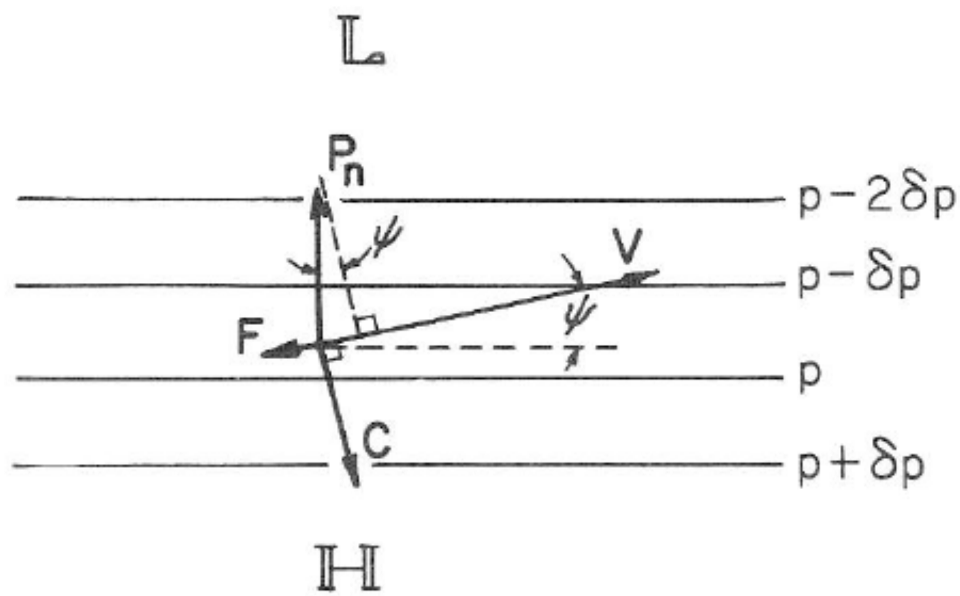
~ Cross isobaric flow ~

- Mathematically, ignoring the acceleration (DV / Dt = 0), we have a three way balance at the surface between the pressure gradient, Coriolis and frictional forces:

$$0 = -\frac{1}{\rho}\vec{\nabla}p - 2\vec{\Omega}\times\vec{V} + \nu\vec{\nabla}^2\vec{V}$$

$$\boxed{\frac{1}{\rho}\vec{\nabla}p = -2\vec{\Omega}\times\vec{V} + \nu\vec{\nabla}^2\vec{V}}$$

- Written this way, we can see that the pressure gradient force (directed perpendicular to the isobars), must balance both the Coriolis (perpendicular to the wind), and frictional (directed opposite to the velocity) forces, as indicated on the figure.



- The wind speed is determined by the requirement that the Coriolis force be just large enough to balance the component of the pressure gradient force in the direction perpendicular to the velocity.
- The angle between V and V_g (ψ on the figure) is determined by the requirement that the component of the pressure gradient force in the direction of the velocity be equal and opposite to the frictional force.