The Thermal Wind

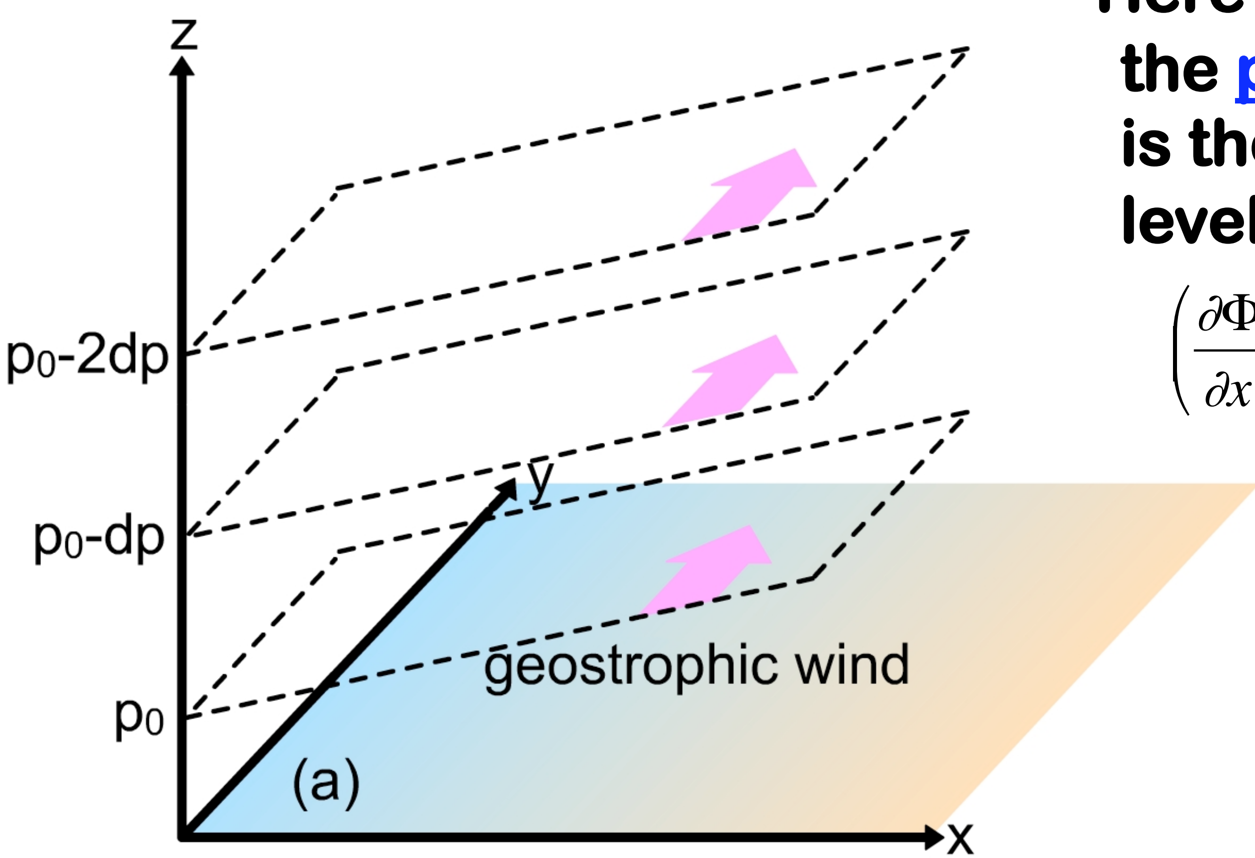
- As the name implies, the thermal wind is an expression for the relationship between the temperature and wind fields in the atmosphere.

- Formally, the thermal "wind" is a bit of a misnomer as it is actually a vertical wind shear (the difference in wind speed and direction between two pressure levels), and more specifically, it refers to vertical shear of the geostrophic wind:

$$\vec{V}_T \equiv \vec{V}_g(p_1) - \vec{V}_g(p_0) \quad \text{where } p_1 < p_0$$

- Before deriving the equations that define the thermal wind, let's examine physically how vertical changes in the geostrophic wind arise.

- Consider the figure below, in which the surfaces of constant pressure are tilted such that they are higher above the ground (colored surface) to the east (+x direction) and lower to the ground to the west.
- Thus, $\frac{\partial \Phi}{\partial x} > 0$ and since $v_g = \frac{1}{f}\frac{\partial \Phi}{\partial x}$, there are southerly winds.

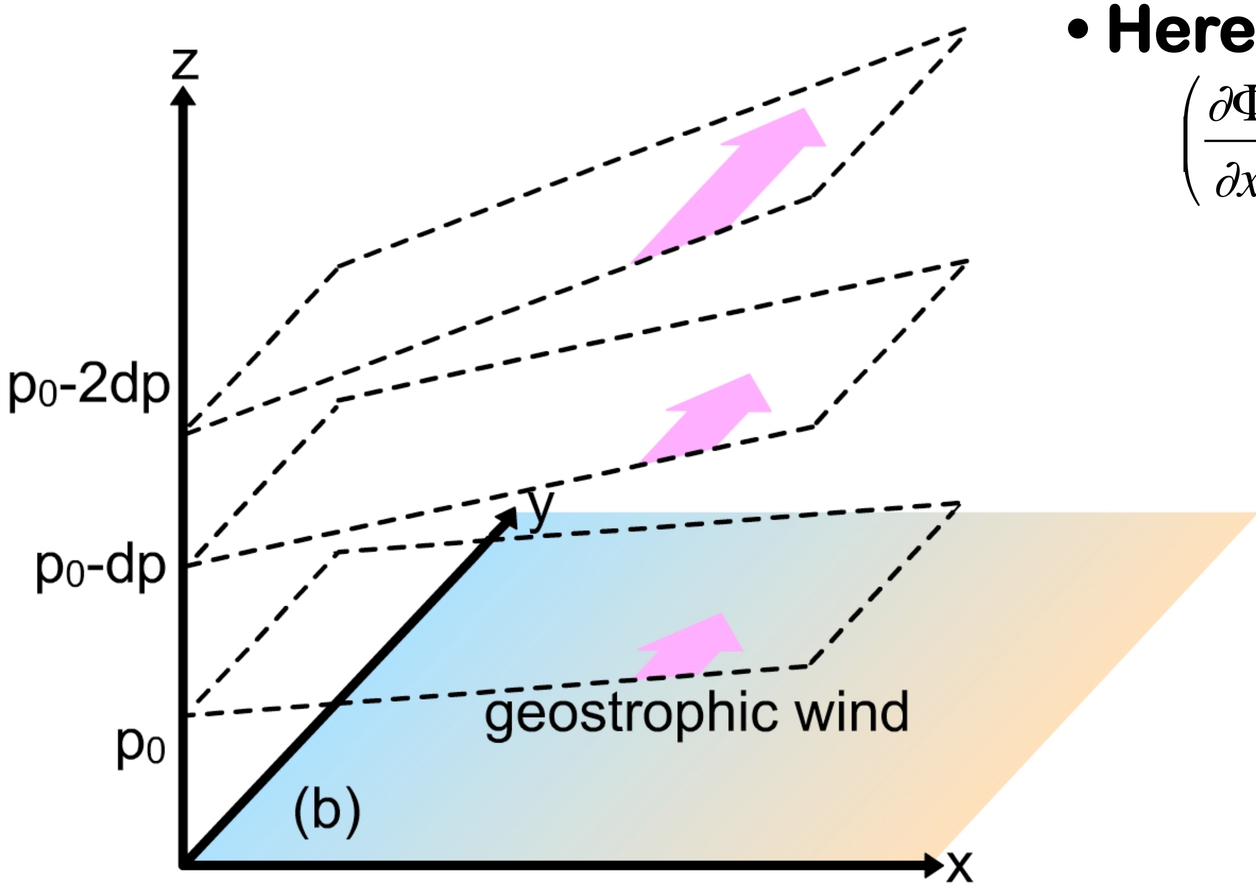


- Here the slope of the pressure surfaces is the same at each level, so

$$\left(\frac{\partial \Phi}{\partial x}\right)_{p_o} = \left(\frac{\partial \Phi}{\partial x}\right)_{p_o - dp} = \left(\frac{\partial \Phi}{\partial x}\right)_{p_o - 2dp}$$

and the geostrophic wind is the same at each height and there is no shear.

- Now consider the same situation where the pressure surfaces are tilted such that they are higher above the ground to the east and lower to the ground to the west and we still have $\frac{\partial \Phi}{\partial x} > 0$ and $v_g > 0$, but the slant of the pressure surfaces now increases with height.

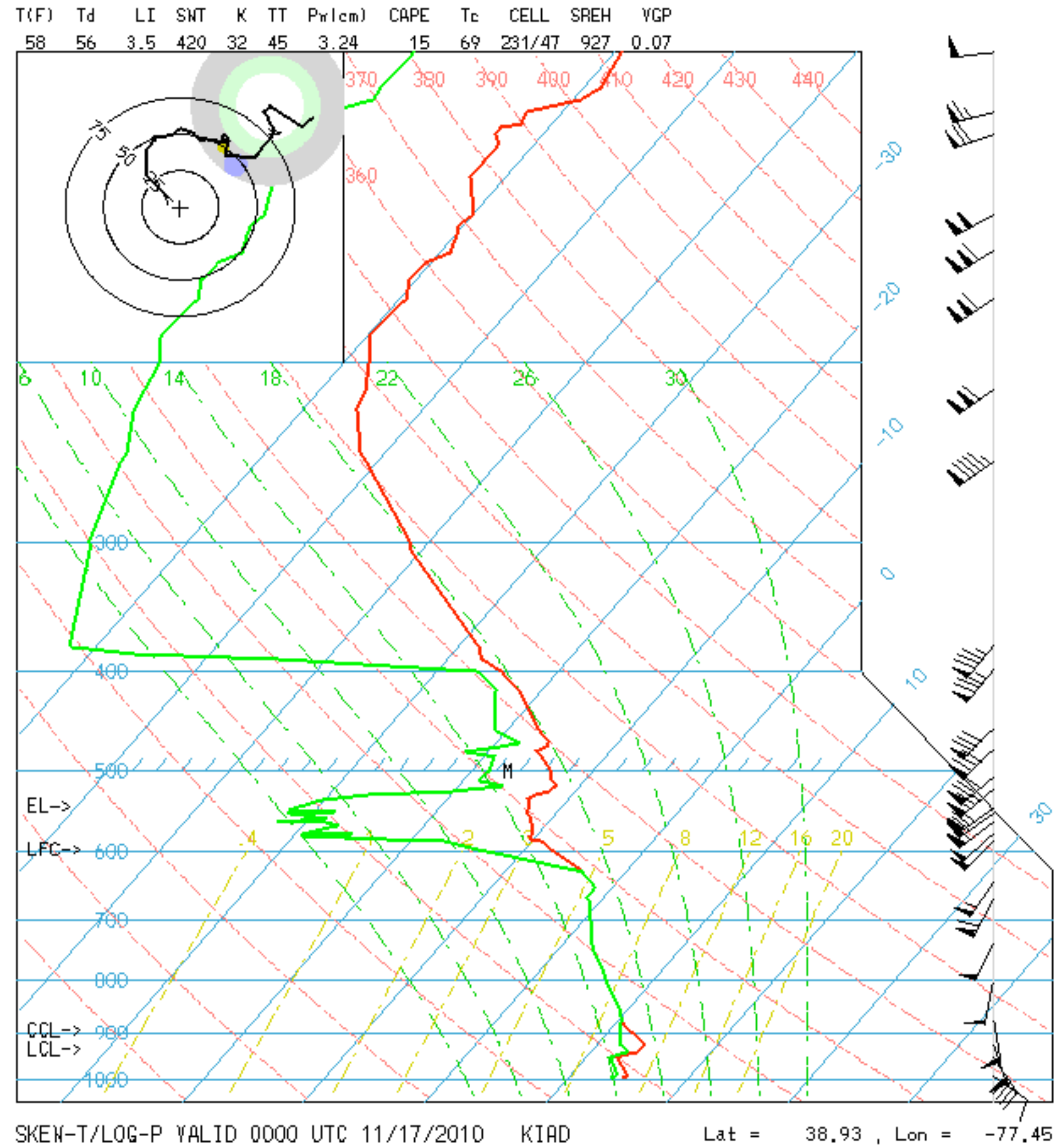


- Here

$$\left(\frac{\partial \Phi}{\partial x}\right)_{p_o} < \left(\frac{\partial \Phi}{\partial x}\right)_{p_o - dp} < \left(\frac{\partial \Phi}{\partial x}\right)_{p_o - 2dp}$$

so that the southerly geostrophic winds increase with height, producing a change in speed with height (a wind shear) as shown in the figure.

- In the previous simple example, only the geostrophic wind speed changed with height, but in the real atmosphere both the wind speed and direction can change with height as we can clearly see from the sounding to the right that was taken at Dulles Airport (Washington, DC) last night.



- Let's now derive the mathematical expressions for the thermal wind!

- Beginning with the component equations for the geostrophic wind:

$$v_g = \frac{1}{f}\frac{\partial \Phi}{\partial x} \quad \text{and} \quad u_g = -\frac{1}{f}\frac{\partial \Phi}{\partial y}$$

- We recall the hydrostatic relationship $\frac{\partial p}{\partial z} = -\rho g$, and the definition of the geopotential $\partial \Phi = g\,\partial z$, and substitute in for ∂z from the later into the former to find $\frac{\partial \Phi}{\partial p} = -\frac{1}{\rho}$.

- Eliminating ρ using the Equation of State and rearranging we have:

$$\frac{\partial \Phi}{\partial p} = -\frac{R_d T}{p}$$

- We now differentiate the geostrophic wind components with respect to pressure and interchange the order of differentiation to find:

$$\frac{\partial v_g}{\partial p} = \frac{1}{f}\frac{\partial}{\partial p}\left(\frac{\partial \Phi}{\partial x}\right) = \frac{1}{f}\frac{\partial}{\partial x}\left(\frac{\partial \Phi}{\partial p}\right)$$

$$\frac{\partial u_g}{\partial p} = -\frac{1}{f}\frac{\partial}{\partial p}\left(\frac{\partial \Phi}{\partial y}\right) = -\frac{1}{f}\frac{\partial}{\partial y}\left(\frac{\partial \Phi}{\partial p}\right)$$

- **Substituting in for** $\frac{\partial \Phi}{\partial p}$ **from the hydrostatic relationship:**

$$\frac{\partial v_g}{\partial p} = \frac{1}{f}\frac{\partial}{\partial x}\left(-\frac{R_d T}{p}\right)$$

$$p\frac{\partial v_g}{\partial p} = \frac{\partial v_g}{\partial \ln p} = -\frac{R_d}{f}\left(\frac{\partial T}{\partial x}\right)_p$$

and similarly

$$p\frac{\partial u_g}{\partial p} = \frac{\partial u_g}{\partial \ln p} = \frac{R_d}{f}\left(\frac{\partial T}{\partial y}\right)_p$$

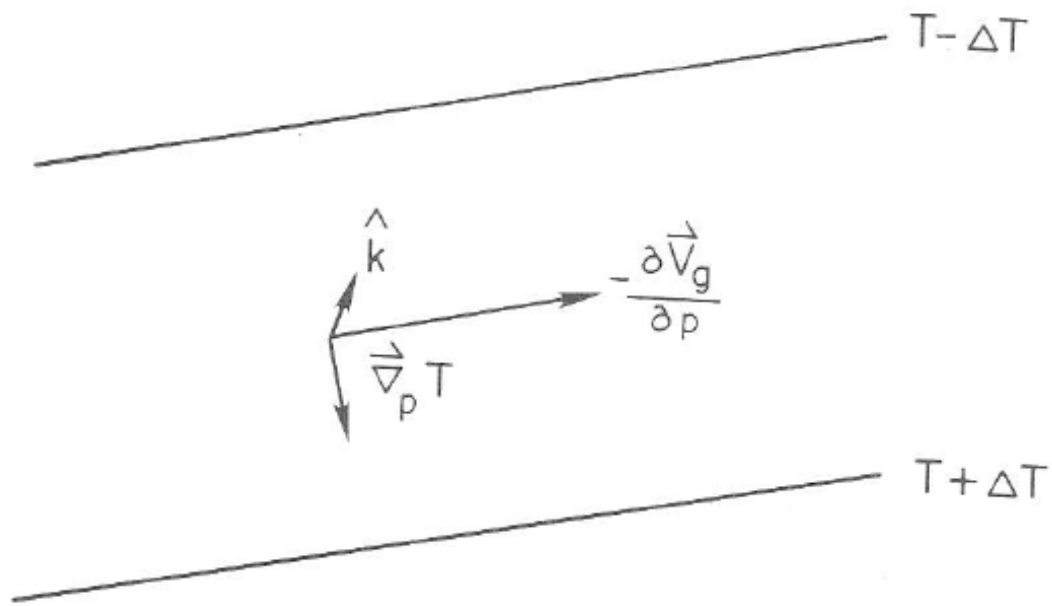
which in vector form gives us one form of the thermal wind equation:

$$\vec{V}_T = \vec{V}_g(upper) - \vec{V}_g(lower) = \frac{\partial \vec{V}_g}{\partial \ln p} = -\frac{R_d}{f}\hat{k} \times \vec{\nabla}_p T$$

$$\vec{V}_T = \vec{V}_g(upper) - \vec{V}_g(lower) = -\frac{\partial \vec{V}_g}{\partial \ln p} = \frac{R_d}{f}\hat{k} \times \vec{\nabla}_p T$$

- This form of the thermal wind equation says that the thermal wind is the vector difference between the geostrophic wind at a given pressure and the geostrophic wind at some higher pressure, and that for an atmosphere in geostrophic and hydrostatic balance, any horizontal gradient in temperature must be associated with a vertical shear of the geostrophic wind.

- The equation also tells us that the thermal wind "blows" parallel to isotherms with cold air to the left in the Northern Hemisphere.



- We may also write the thermal wind equation in terms of geopotential height as our first example of the day demonstrated.

- Integrating the thermal wind equation between two pressure levels, p_0 and p_1, such that $p_1 < p_0$, we find

$$\vec{V}_T = \int_{p_0}^{p_1} \partial \vec{V}_g = \vec{V}_g(p_1) - \vec{V}_g(p_0) = -\frac{R_d}{f} \int_{p_0}^{p_1} \left(\hat{k} \times \vec{\nabla}_p T \right) \partial \ln p$$

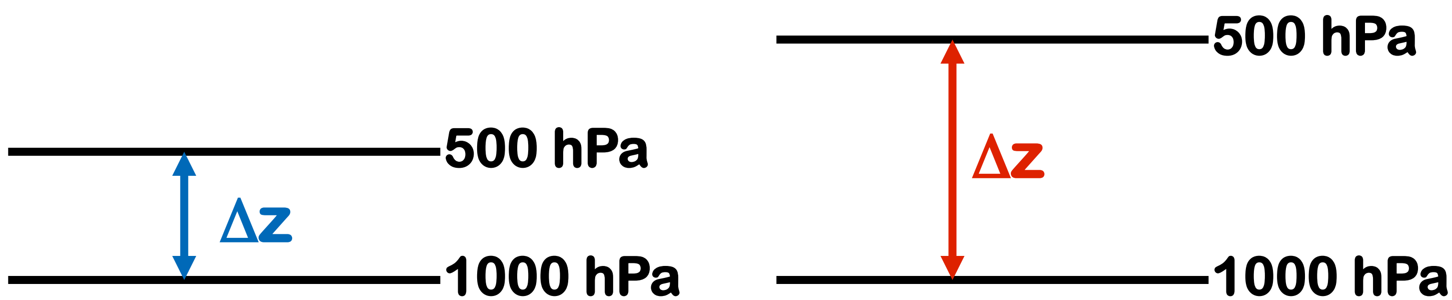
- Since we know temperature varies with pressure, we define a mean temperature in the layer p_0 to p_1 as $\langle T \rangle$, and we may write the components of the thermal wind as

$$u_T = -\frac{R_d}{f} \left(\frac{\partial \langle T \rangle}{\partial y} \right) \ln\left(\frac{p_0}{p_1} \right) \quad \text{and} \quad v_T = \frac{R_d}{f} \left(\frac{\partial \langle T \rangle}{\partial x} \right) \ln\left(\frac{p_0}{p_1} \right)$$

- To see how these equations relate back to geopotential height, we recall the hypsometric equation that we derived previously as:

$$\partial\Phi = \Phi_1 - \Phi_0 = -R_d \int_{p_0}^{p_1} T \partial \ln p = R_d \langle T \rangle \ln\left(\frac{p_0}{p_1}\right)$$

- **We recall $\partial\Phi \equiv Z_T$, the thickness (depth) of the atmosphere between two pressure levels p_0 and p_1 measured in units of geopotential height, and that the thickness is proportional to the mean temperature in the layer, as we see below.**



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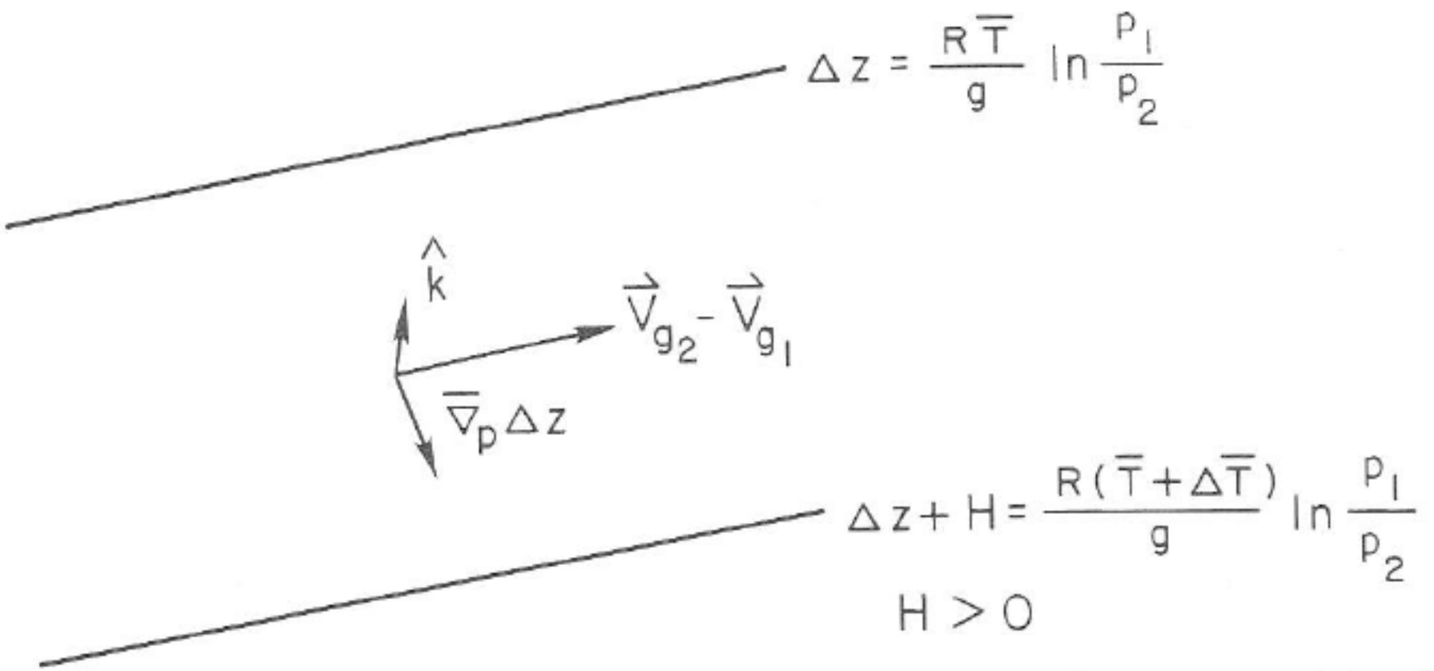
- **We now use the relationship above to rewrite the thermal wind components as:**

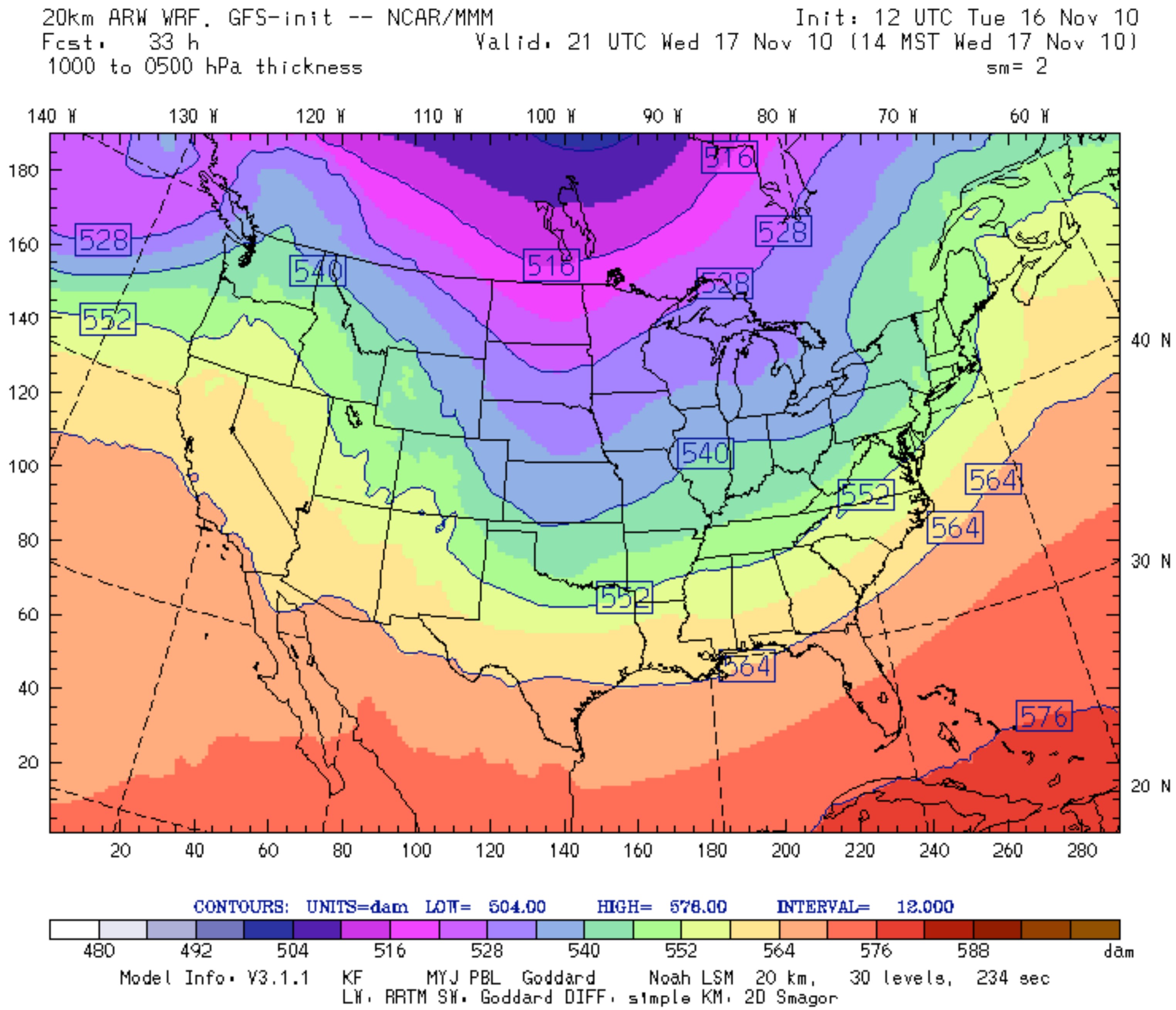
$$u_T = -\frac{1}{f}\frac{\partial}{\partial y}(\Phi_1 - \Phi_0) \quad \text{and} \quad v_T = \frac{1}{f}\frac{\partial}{\partial x}(\Phi_1 - \Phi_0)$$

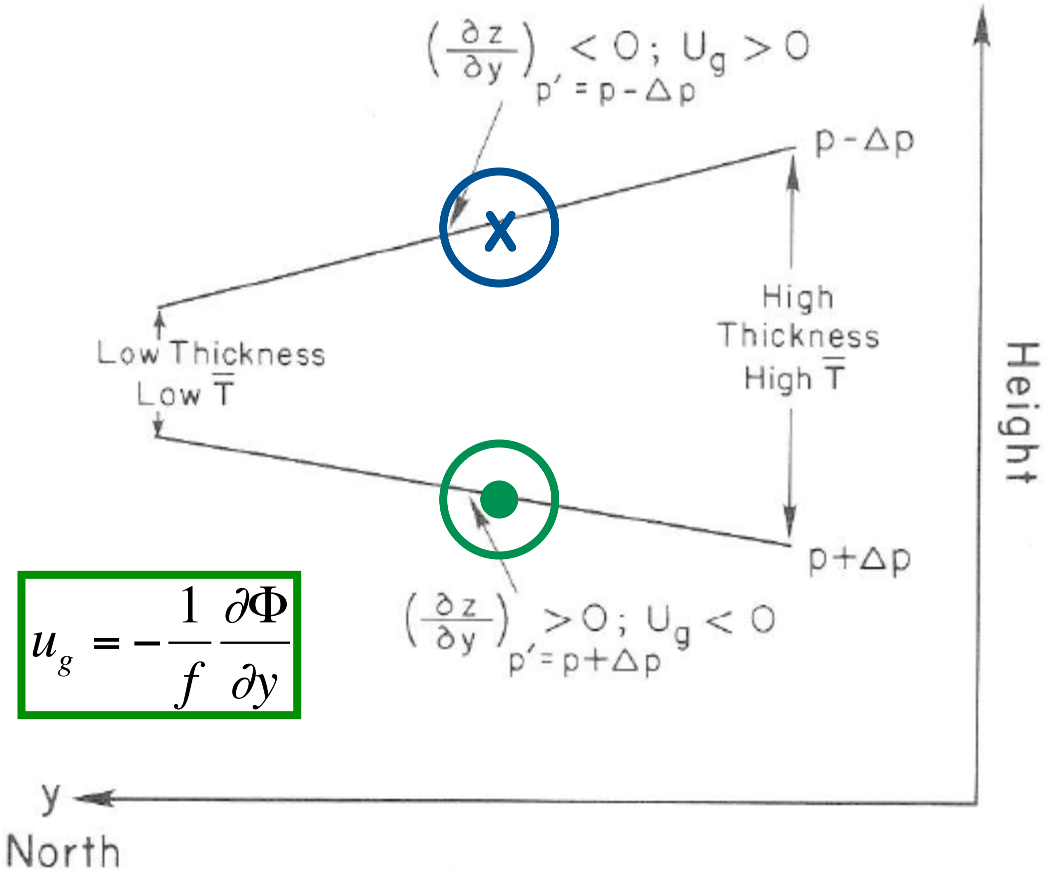
- **Or in vector form:**

$$\vec{V}_T = \frac{1}{f}\hat{k} \times \vec{\nabla}\underbrace{\left(\Phi_1 - \Phi_0\right)}_{\text{Thickness}}$$

which says that the <u>thermal wind</u> "<u>blows</u>" <u>parallel to lines of constant</u> <u>thickness</u> with <u>lower</u> <u>thickness</u> values (i.e., <u>colder air</u>) <u>to the left</u>, as seen in the figure below.

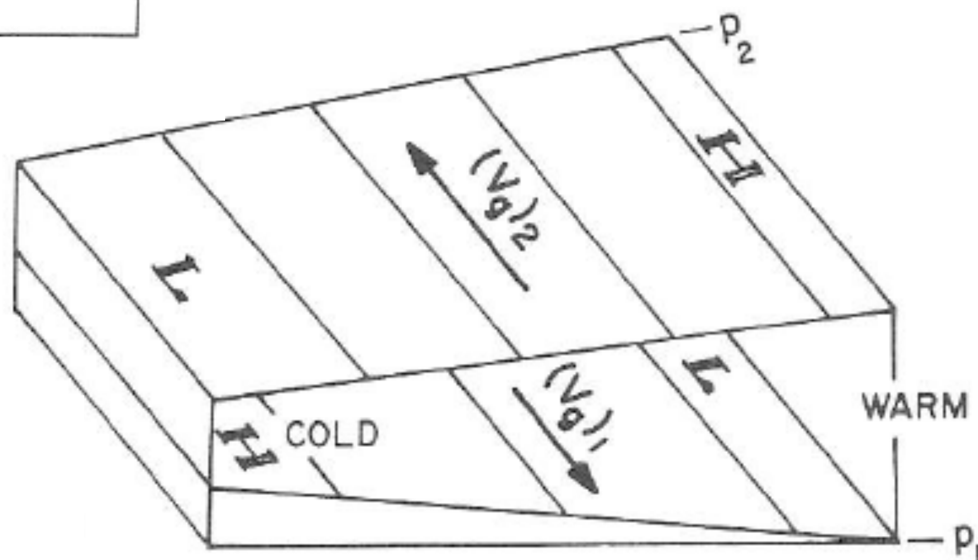



20km ARW WRF, GFS-init -- NCAR/MMM
Init: 12 UTC Tue 16 Nov 10
Fcst: 33 h
Valid: 21 UTC Wed 17 Nov 10 (14 MST Wed 17 Nov 10)
1000 to 0500 hPa thickness
sm= 2
140 W
130 W
120 W
110 W
100 W
90 W
80 W
70 W
60 W
40 N
30 N
20 N
516
528
540
552
564
576
CONTOURS: UNITS=dam LOW= 504.00 HIGH= 576.00 INTERVAL= 12.000
480
492
504
516
528
540
552
564
576
588
dam
Model Info: V3.1.1 KF MYJ PBL Goddard Noah LSM 20 km, 30 levels, 234 sec
LW: RRTM SW: Goddard DIFF: simple KM: 2D Smagor



$$u_g = -\frac{1}{f}\frac{\partial \Phi}{\partial y}$$

$$v_g = \frac{1}{f}\frac{\partial \Phi}{\partial x}$$

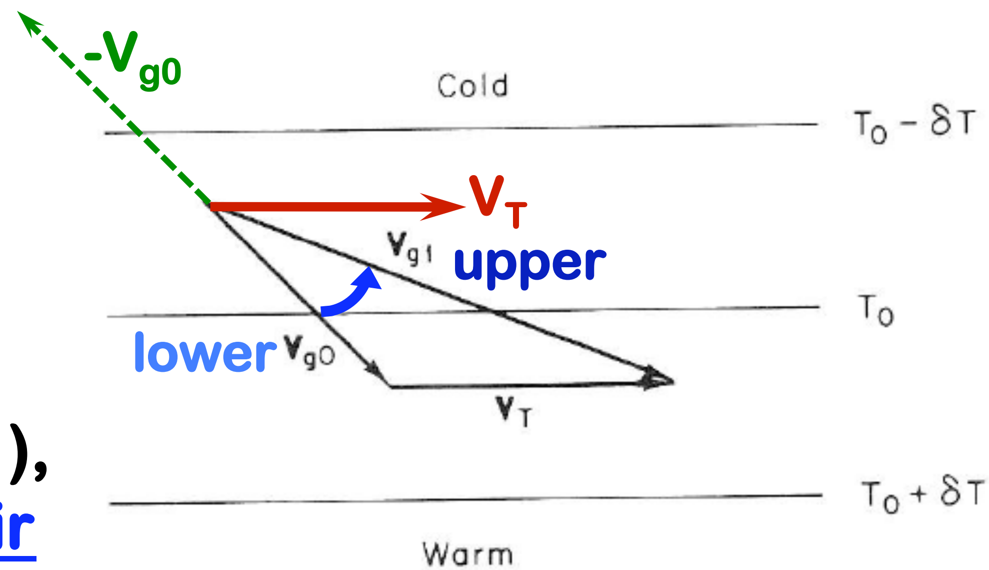


- **Two more examples of how the thermal wind relates to a thickness gradients in the atmosphere.**

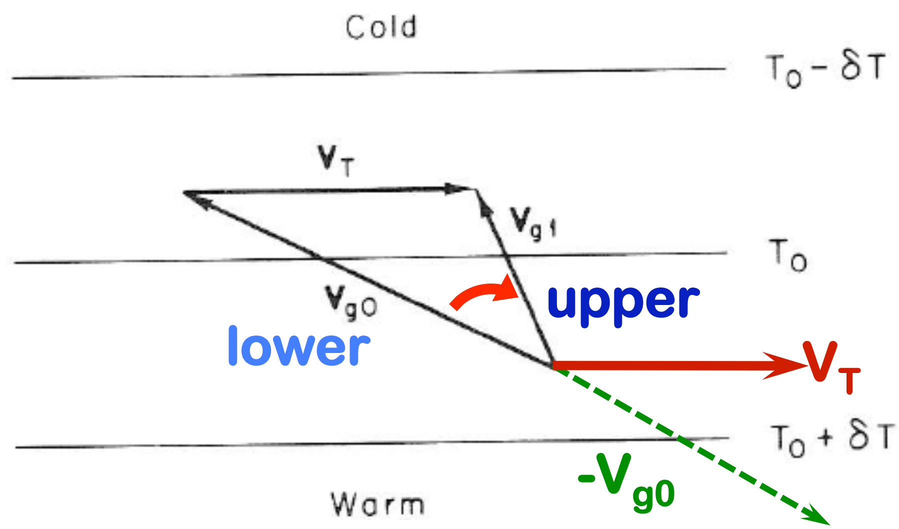
- The thermal wind is a very useful tool which can help weather forecasters and numerical modelers alike as a check for consistency between the wind, height and temperature fields.

- It also allows us to estimate the temperature advection in a layer if we know the geostrophic winds at the top and bottom of the layer.

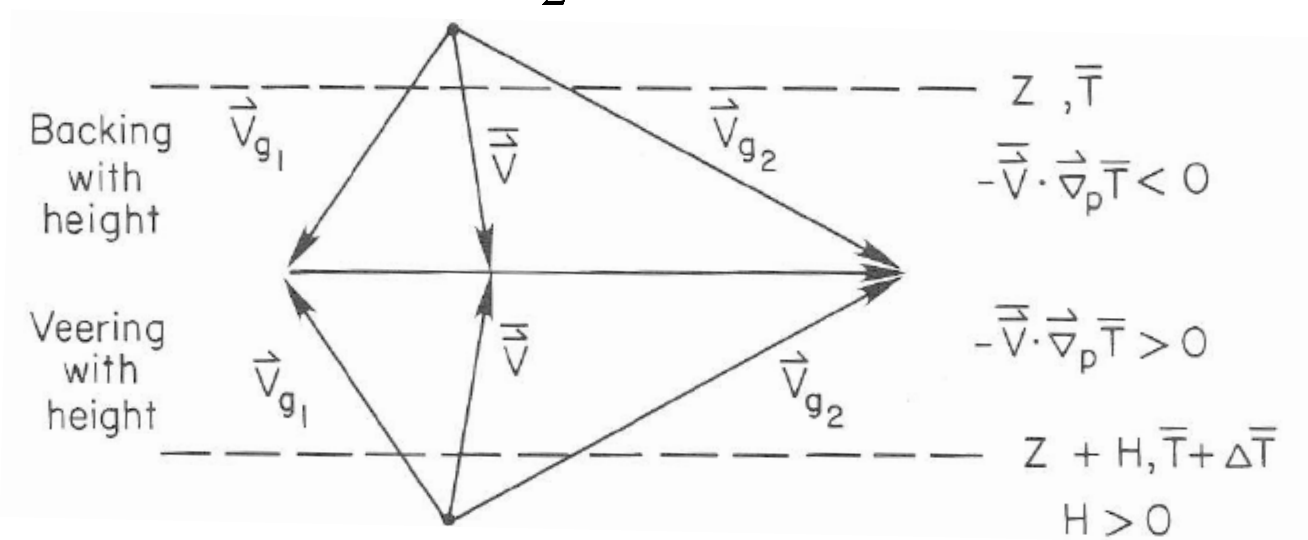
- For example, consider the figure to the right, where $V_T = V_{g1} - V_{g0} = V_{g1} + (-V_{g0})$. The geostrophic wind turns counterclockwise, or backs, with height (0→1), and we see there is cold air advection in the layer.



- The geostrophic wind turns clockwise, or veers, with height (0→1), and there is warm air advection in the layer.

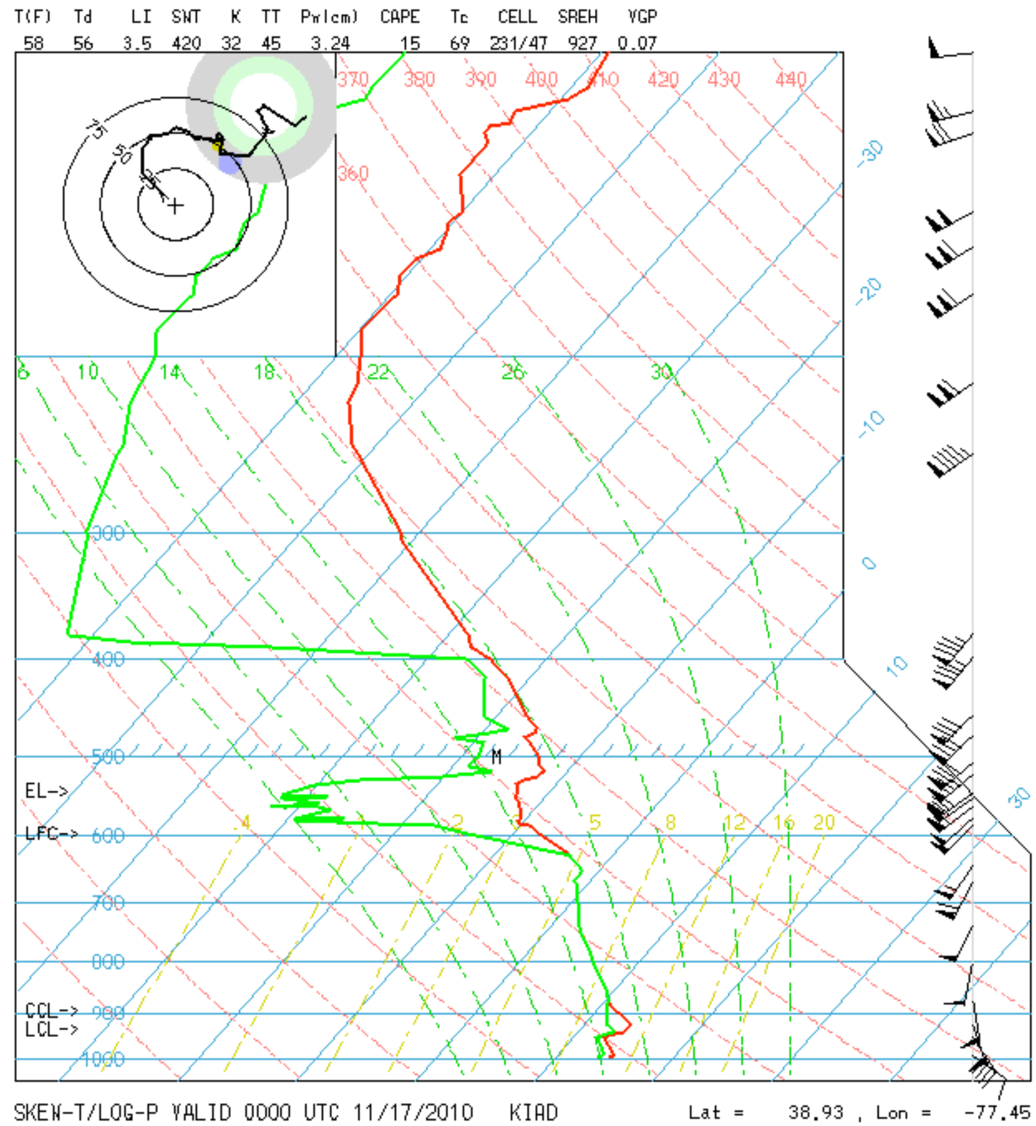


- Looking at this method of determining the sense of advection another way, we consult the figure below and define a mean geostrophic wind in the layer as $\vec{\bar{V}} = \frac{\vec{V}_{g1} + \vec{V}_{g2}}{2}$

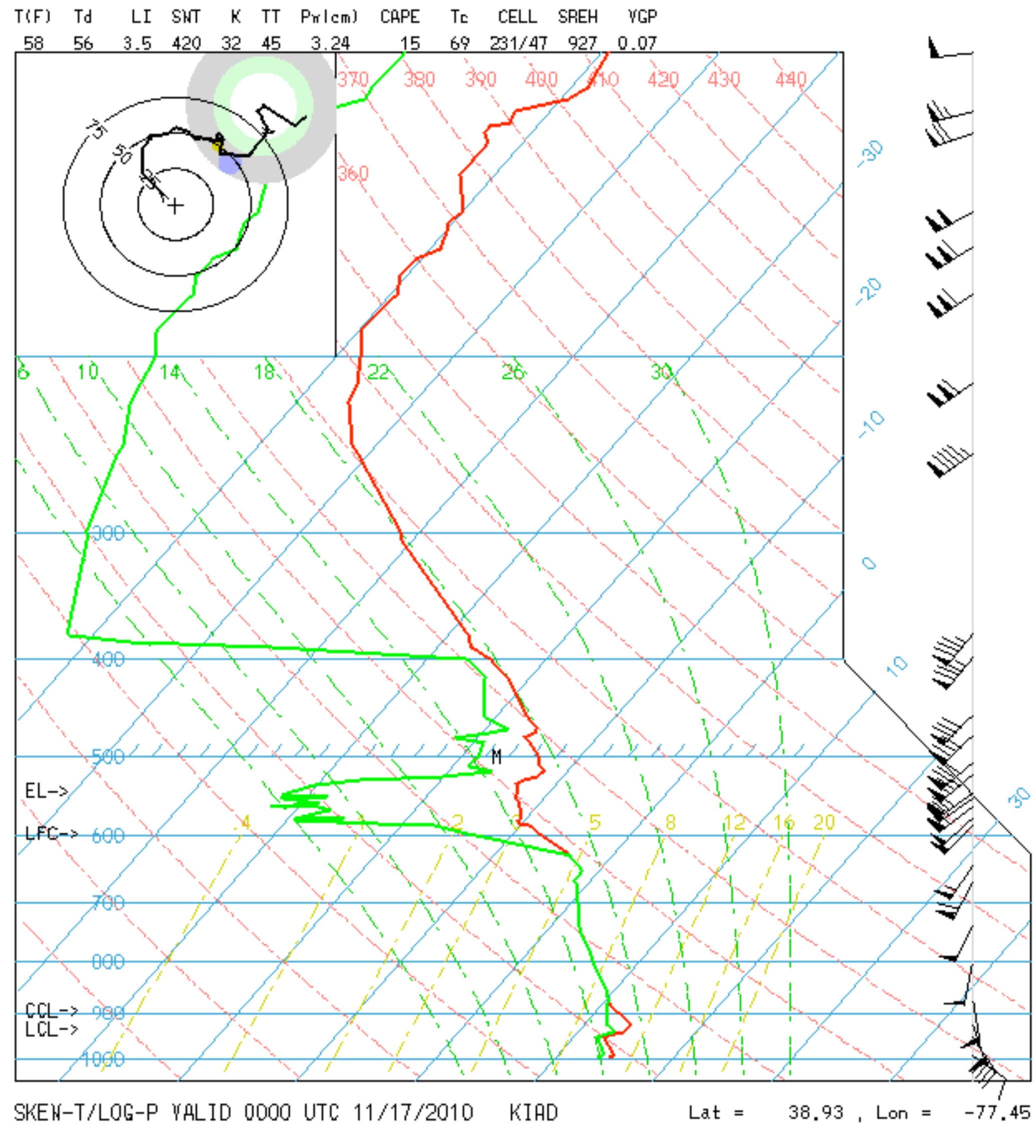


- **Based upon the preceding, it is possible to determine the sense of advection and its vertical variation in the atmosphere from a single sounding!**

- **Returning to our Dulles Airport example from earlier, where the winds shift from southeasterly near the surface to southwesterly aloft, what sense of advection do we have?**

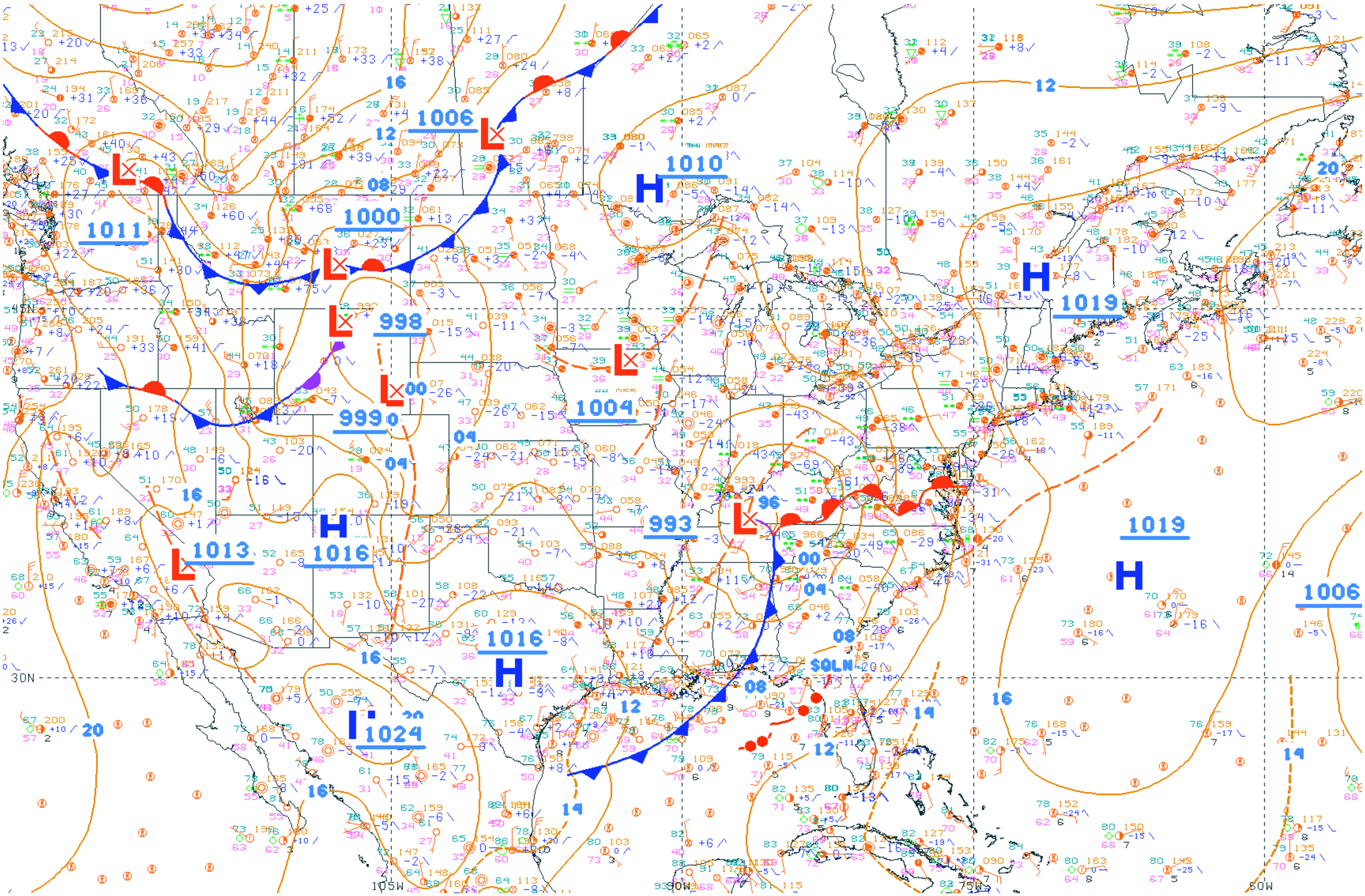


- **Based upon the preceding, <u>it is possible to determine the sense of</u> <u>advection</u> and its vertical variation in the atmosphere <u>from a single</u> <u>sounding</u>!**
- **Returning to our <u>Dulles</u> Airport <u>example</u> from earlier, where the <u>winds</u> <u>shift from</u> <u>southeasterly</u> near the surface <u>to</u> <u>southwesterly</u> aloft, <u>what sense of</u> <u>advection</u> <u>do we have</u>?**



Winds veering with height!

<u>WARM AIR ADVECTION</u>



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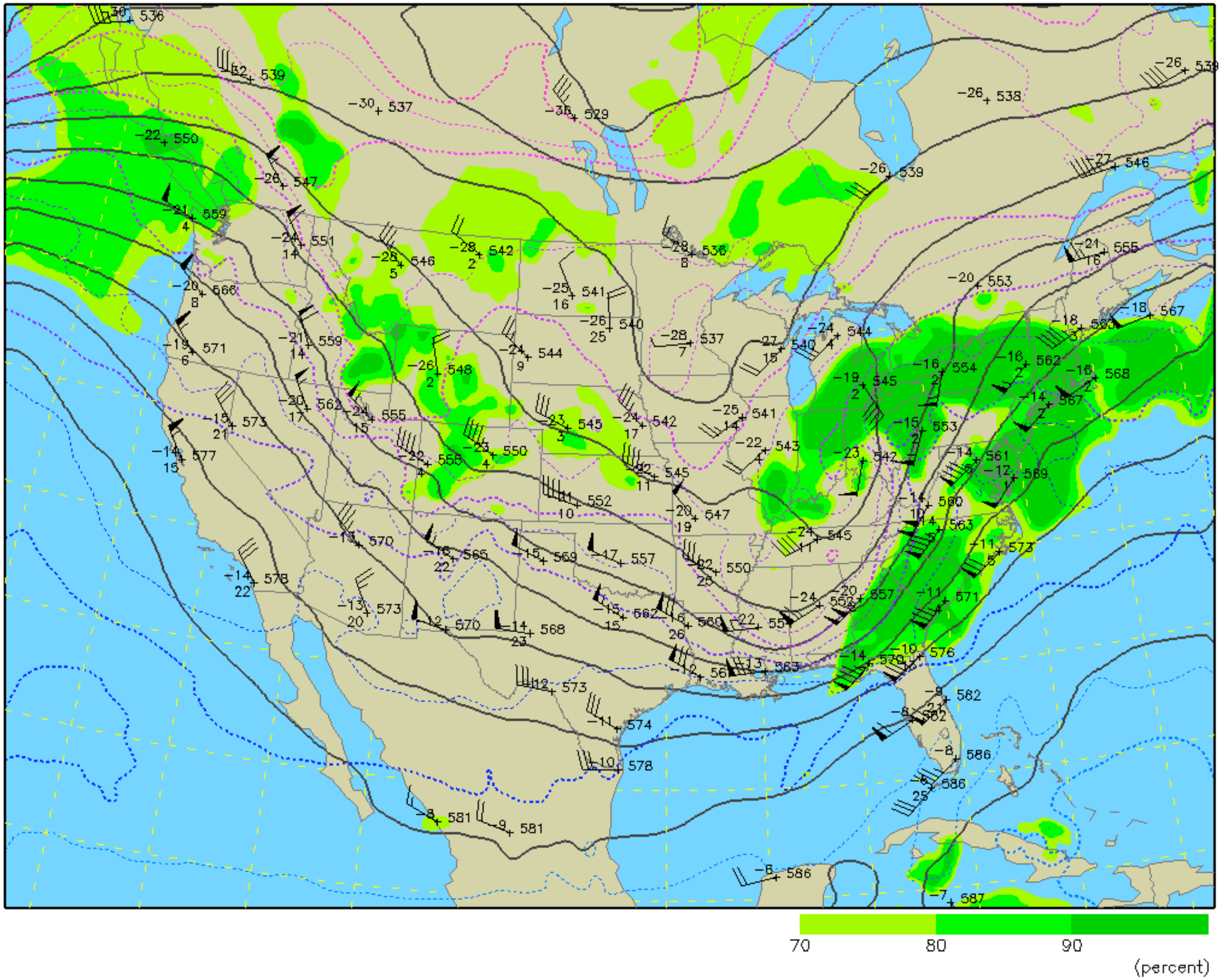
- Finally, we may use our knowledge of the thermal wind to define three different vertical states of the atmosphere.

- An atmosphere like that we have assumed for the preceding derivations, one in which we have both temperature and height gradients on isobaric surfaces, is called baroclinic (from the Greek "baro-" meaning pressure and "-klines" meaning inclining or intersecting) and describes most of the midlatitudes, as seen on the 500 hPa chart on the following slide.

- To put it another way, a baroclinic atmosphere is one in which the pressure at any given point is dependent on both the density and temperature of the air, i.e., $p = \rho R_d T$.

500 mb Heights (dm) / Temperature (^{o}C) / Humidity (%)

0–hour analysis valid 0000 UTC Wed 17 Nov 2010 RUC (00z 17 Nov)

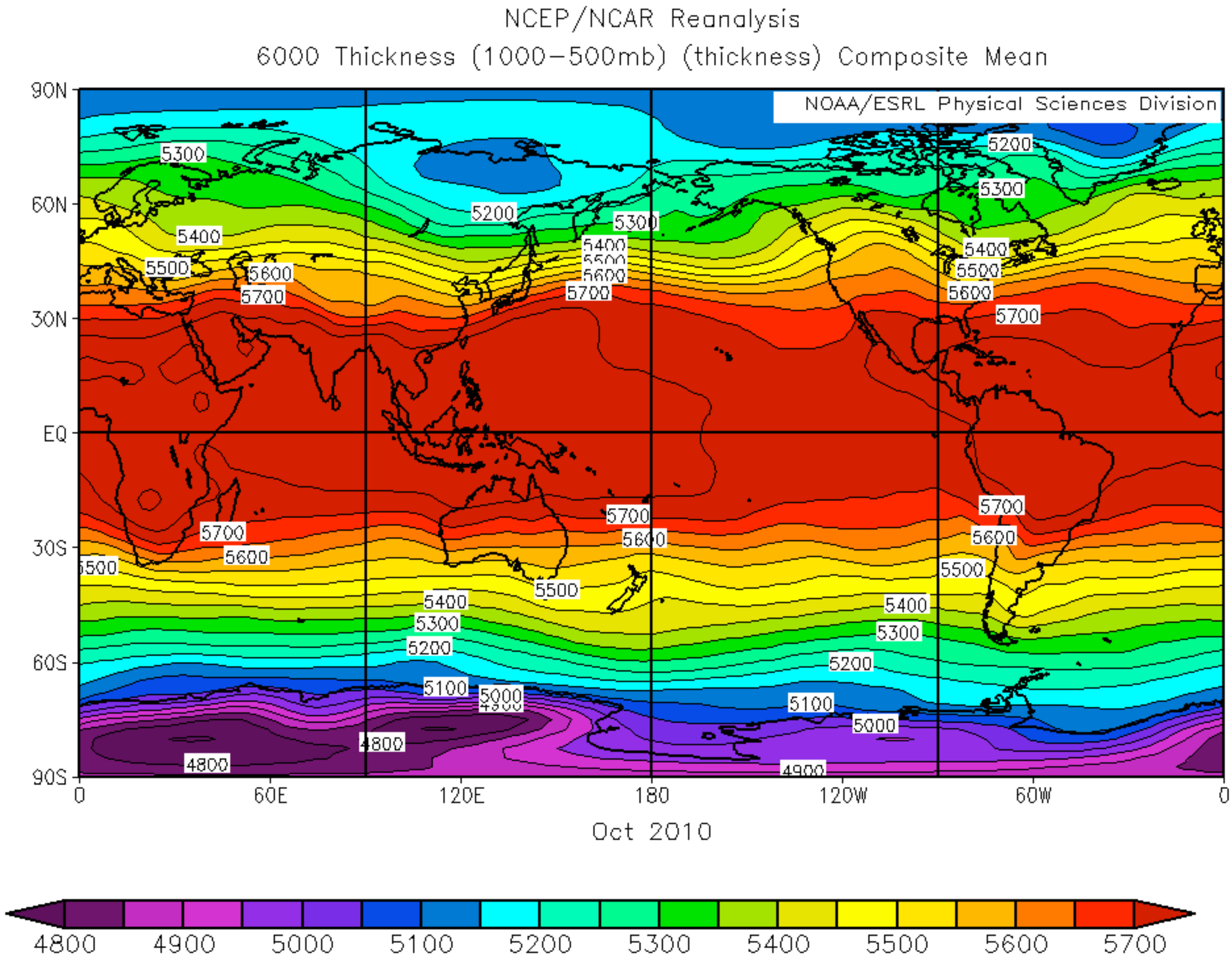


- If instead, there were no gradients of temperature on isobaric surfaces (everywhere on that pressure level it's the same temperature) and thus the pressure only depends on density, the atmosphere is called barotropic.

- The thermal wind equation tells us that since there are no temperature gradients, there is no vertical shear and the geostrophic wind is independent of height.

$$\vec{V}_T = \frac{\partial \vec{V}_g}{\partial \ln p} = -\frac{R_d}{f}\hat{k} \times \vec{\nabla}_p T$$

- And since lines of constant thickness are equivalent to isotherms in a hydrostatic and geostrophically balanced atmosphere, the 1000-500 hPa thickness chart on the following slide shows that in the deep tropics, the atmosphere is approximately barotropic.


NCEP/NCAR Reanalysis
6000 Thickness (1000-500mb) (thickness) Composite Mean
NOAA/ESRL Physical Sciences Division
90N
60N
30N
EQ
30S
60S
90S
0
60E
120E
180
120W
60W
0
Oct 2010
4800
4900
5000
5100
5200
5300
5400
5500
5600
5700

- Returning once again to the 500 hPa geopotential height and temperature map three slides back, we see that there are several places where the isotherms and geopotential height lines are parallel.

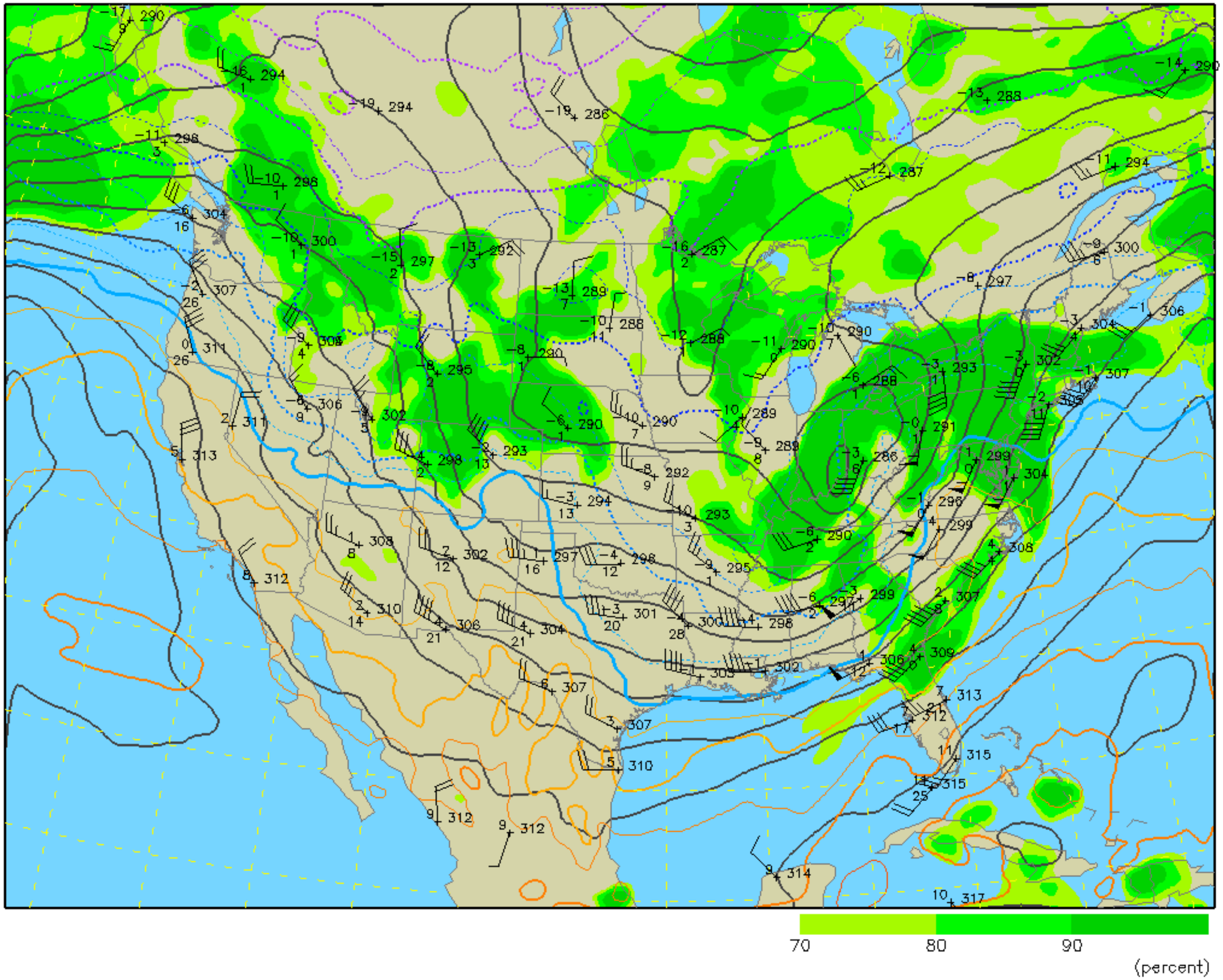
- In this case, we call the atmosphere equivalent barotropic as any line of constant height is also a line of constant temperature, or thickness, so the shape of the height contours is fixed with height and thus the direction of the geopotential wind does not change with height.

- However, because there are temperature gradients on isobaric surfaces, the geostrophic wind speed must vary from pressure level to pressure level and there is vertical speed shear of the geostrophic wind.

$$\vec{V}_T = \frac{\partial \vec{V}_g}{\partial \ln p} = -\frac{R_d}{f}\hat{k} \times \vec{\nabla}_p T$$

700 mb Heights (dm) / Temperature (°C) / Humidity (%)

0-hour analysis valid 0000 UTC Wed 17 Nov 2010 RUC (00z 17 Nov)



300 mb Heights (dm) / Isotachs (knots)

0–hour analysis valid 0000 UTC Wed 17 Nov 2010 RUC (00z 17 Nov)