A Method for Calculating the Effects of Deep Cumulus Convection in Numerical Models

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ABSTRACT

A closure is proposed for the b parameter of Kuo (1974), using the framework developed by Krishnamurti *et al.* (1976). Emphasis is placed on the time-dependent behavior of the solutions. The proposed closure is found to be the only one of several tested to produce an approach to moist neutrality in both temperature and moisture under strong external forcing. The sensitivity of the grid-scale evolution to the partitioning of moisture defined by the b parameter suggests that such partitioning must be carefully dealt with in any method for computing the effects of cumulus convection, whether or not b is explicitly present.

By including entrainment in the cloud lapse rate, the observed large-scale behavior of the vertical profile of moist static energy under disturbed conditions is simulated. The approach is shown to be easily invertible when precipitation rate is specified, thus insuring internal consistency in a model when such a procedure is used as part of a dynamic initialization.

Because it is relatively simple and general, and reproduces observed large-scale θ_e variations under strong forcing, the approach may be particularly suitable for large-scale models. An economical way to extend the procedure to mesoscale models is proposed.

1. Introduction

A major source of difficulty in numerical modeling is the calculation of vertical transports of sensible and latent heat by cumulus clouds. Rosenthal (1978; 1979) has suggested that cumulus effects can be computed explicitly on the grid scale, and that such an approach is superior to an implicit method, for which somewhat arbitrary parameters allow fine-tuning to obtain satisfactory results, without contributing to understanding of the interaction between cumulus and larger scales. As grid size increases, however, the explicit approach must become inadequate at some resolution because, as noted by Rosenthal (1978), transports by clouds smaller than the grid scale are neglected. At this or any larger scale, the role of cumulus convection must be determined implicitly as a function of grid-scale variables. In this paper, an approach is developed, based on the work of Kuo (1974), which is relatively simple to apply, but minimally dependent on arbitrary constants whose actual values may vary in time and space. This simplicity and generality may be particularly appropriate for operational models, which are constrained by computer time limits, and usually hemispheric or global in extent.

Kuo (1965) proposed a method for implicitly including the tendency for deep convection to force the larger scale toward moist neutrality. Latent heating and moistening were proportional to the temperature and specific humidity differences between the grid scale and a moist adiabatic cloud element. Kanamitsu (1975) noted the tendency of Kuo's approach to moisten too rapidly, sometimes causing saturation simultaneously with large moist instability, which is rarely observed in nature. Because too little of the available moisture condensed prior to saturation, rainfall rates were considerably underestimated (Krishnamurti *et al.*, 1976; Carr and Bosart, 1978). To correct this, Kuo (1974) introduced a parameter b, the fraction of available moisture which increases storage, with fraction $1 - b$ condensing. Kuo suggested from observations that $b \ll 1$, but proposed no formal expression for b to close the system. Kanamitsu (1975) and Krishnamurti *et al.* (1976) further developed the approach. An important aspect of their procedure was that it allowed a smooth transition between condensation in a moist stable environment and that in a conditionally unstable environment. Krishnamurti *et al.* (1980) tested five methods of convective parameterization on GATE data, and found that a semi-prognostic version of their approach produced the most accurate rainfall estimates.

The scheme proposed here is an extension of that of Krishnamurti *et al.* (1976). A simple closure will be presented for the b parameter which produces a more accurate time variation of stability and approach toward moist neutrality than previous closures. The importance of including entrainment in the cloud lapse rate will be noted. In addition, the approach will be shown to be easily invertible when the precipitation rate is known, thus allowing an internally consistent approach to dynamic initialization using observed precipitation rates (Molinari, 1982).

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TABLE 1. List of symbols.

Symbol	Description
a_θ, a_q	parameters for convective heating and moistening
b	parameter for partitioning of moisture
c_p	specific heat at constant pressure
E	entrainment parameter
g	gravitational acceleration
H	cloud thickness
I	rate of moisture supply to a column
J	vertically integrated adiabatic cooling
L	latent heat of condensation
p	pressure
p_b	cloud base pressure
p_t	cloud top pressure
P	precipitation rate
q	specific humidity
q_s	saturation specific humidity
Q_θ	rate of moisture supply required to raise temperature to cloud temperature
Q_q	rate of moisture supply required to saturate the column at cloud temperature
R	cloud radius
$\overline{\mathrm{RH}}$	mean relative humidity in the cloud layer
RH_c	critical relative humidity
t	time
T	temperature
α	entrainment constant
β	vertical heating distribution parameter
γ	ratio of cloud radius to depth
ω	vertical velocity [$=dp/dt$]
$\Delta\tau$	cloud time scale
θ	potential temperature
θ_e	equivalent potential temperature
θ_{es}	saturated equivalent potential temperature
$(\ )_c$	value in cloud
$(\partial/\partial t)_{\mathrm{CON}}$	local change due to convective scale motions plus grid-scale vertical advection
$\int_c$	integral over cloud layer from p_b to p_t.

2. Description of procedure

a. Without entrainment

In the simplest case of zero entrainment, the equation for the moist adiabat, $-\partial\theta_{es}/\partial p = 0$, can be written with some approximation as

$$-\frac{\partial\theta_c}{\partial p} = \frac{L}{c_p}\left(\frac{\theta}{T}\frac{\partial q_s}{\partial p}\right)_c. \quad (1)$$

Symbols are defined in Table 1. Unsubscripted variables are grid-scale variables, while the subscript c refers to "cloud" temperature, i.e., temperature of a rising parcel.

Eq. (1) is solved iteratively for θ_c and q_c assuming $\theta_c = \theta$ and $q_c = q_s(\theta)$ at cloud base. Vertical structure and finite differencing follow Kanamitsu (1975). Cloud base pressure is that of the base of the lowest unstable layer for which relative humidity exceeds a critical value, chosen as 0.8. Under the above conditions, a sufficient grid-scale condition for buoyancy of the rising parcel is simply

$$-\frac{\partial\theta}{\partial p} < \frac{L\theta}{c_pT}\frac{\partial q_s}{\partial p}. \quad (2)$$

The grid-scale moisture supply is defined by

$$I = -\frac{1}{g}\int_{p_t}^{p_b} \omega\frac{\partial q}{\partial p}dp, \quad (3)$$

where p_t represents the cloud top pressure, taken as the pressure at the level below which the sounding becomes warmer than the moist adiabat. When an unstable layer exists and $I > 0$, the convective scheme is invoked.

The procedure will be tested in a one-dimensional model which includes only convective heating and large-scale vertical motion. Following Krishnamurti *et al.* (1976), the temperature and moisture equations can be written as

$$\left(\frac{\partial\theta}{\partial t}\right)_{\mathrm{CON}} = -\omega\frac{\partial\theta}{\partial p} + a_\theta\left(\frac{\theta_c - \theta}{\Delta\tau} + \omega\frac{\partial\theta}{\partial p}\right), \quad (4)$$

$$\left(\frac{\partial q}{\partial t}\right)_{\mathrm{CON}} = a_q\left(\frac{q_c - q}{\Delta\tau}\right), \quad (5)$$

where

$$a_\theta = \frac{(1-b)I}{Q_\theta}, \quad (6)$$

$$a_q = \frac{bI}{Q_q}, \quad (7)$$

$$Q_\theta = \frac{1}{g}\int_c \frac{c_pT}{L\theta}\frac{\theta_c - \theta}{\Delta\tau}dp + \frac{1}{g}\int_c \omega\frac{c_pT}{L\theta}\frac{\partial\theta}{\partial p}dp, \quad (8)$$

$$Q_q = \frac{1}{g}\int_c \frac{q_c - q}{\Delta\tau}dp, \quad (9)$$

$$J = -\frac{1}{g}\int_c \frac{c_pT}{L\theta}\omega\frac{\partial\theta}{\partial p}dp. \quad (10)$$

The integral over c represents a vertical integration from p_b to p_t. Q_θ is the rate of condensation required to raise the grid-scale temperature to cloud temperature in time $\Delta\tau$; Q_q is an analogous rate for saturating the column at the cloud temperature; and J is the net adiabatic temperature change in the column. The formulation is insensitive to the value of the time scale $\Delta\tau$, as will be shown in the results.

The numerical solution of (4)–(5) utilizes a relative humidity equation, and includes stable heating if moist neutrality is reached. To close the system, the parameter b must be defined. It has the following properties:

(a) bI	goes directly to storage, thus increasing the grid-scale q.
(b) $(1-b)I$	condenses, thus increasing the grid scale θ.
(c) $b = 0$	produces zero moistening; all available energy goes to condensation.

(d) $b = (J + I)/I$ produces zero *net* heating, i.e., the sum of adiabatic cooling and condensation heating in the column is zero [shown by vertically integrating Eq. (4)].

(e) $0 < b < (J + I)/I$ allows both warming and moistening by the available latent energy supply.

(f) $b > (J + I)/I$ produces net cooling of the column.

(g) $b < 0$ condenses more moisture than is supplied, i.e., depletes moisture storage.

The last property is physically realistic, as noted by Cho (1976), who found that precipitation exceeded moisture supply on the east side of the composite easterly wave of Reed and Recker (1971).

Kanamitsu (1975), in order to avoid the too rapid moistening of the Kuo (1965) scheme, proposed the closure

$$\frac{1}{g}\int_c c_p \frac{\partial T}{\partial t}dp = \frac{1}{g}\int_c L\frac{\partial q}{\partial t}dp. \qquad (11)$$

Eq. (11) partitions available energy equally into net warming and moistening. Using (4)–(10) and the definition of θ, this leads to

$$b = (J + I)/2I. \qquad (12)$$

In the present work, an alternate closure is proposed which more explicitly requires temperature and moisture to approach their limiting values. Noting that $(T_c - T)/(\partial T/\partial t)_{\text{CON}}$ is the time it would take for T to reach T_c, the closure is written as

$$\frac{\int_c (T_c - T)dp}{\int_c \left(\frac{\partial T}{\partial t}\right)_{\text{CON}} dp} = \frac{\int_c (q_c - q)dp}{\int_c \left(\frac{\partial q}{\partial t}\right)_{\text{CON}} dp}, \qquad (13)$$

which, again using (4)–(10), gives

$$b = \frac{J + I}{I}\left(\frac{Q_q}{Q_q + Q_\theta + J}\right) \qquad (14a)$$

or, alternately,

$$b = \frac{J + I}{I}\left[\frac{\int_c (q_c - q)dp}{\int_c (q_c - q)dp + \int_c \frac{c_p T}{L\theta}(\theta_c - \theta)dp}\right]. \qquad (14b)$$

This forces the instantaneous time tendencies of θ and q to be such that they would reach their limiting values on the moist adiabat simultaneously. This formulation reduces to that of Kanamitsu when $Q_q = Q_\theta + J$, moistens more for a drier sounding $(Q_q < Q_\theta + J)$, and less for a moister sounding. Although the closure expression (13) appears to be based on numerical and not physical reasoning, it will be shown to produce the most realistic time evolution of stratification of various definitions of b. In (13), the subscripts on the time derivatives indicate that even in a three-dimensional model, $\partial\theta/\partial t$ and $\partial q/\partial t$ are represented by (4) and (5). Such processes as horizontal advection would act to change the grid scale θ at a level, but would not directly influence the computation of b. In addition, it should be noted that the limiting moist adiabat is itself time-dependent as cloud base conditions change.

b. *Addition of entrainment*

Following Stommel (1947), (1) is rewritten to include entrainment effects:

$$-\frac{\partial\theta_c}{\partial p} = \frac{L}{c_p}\left(\frac{\theta}{T}\frac{\partial q_s}{\partial p}\right)_c + E\left(\frac{\theta}{c_p T}\right)_c \times [c_p(T_c - T) + L(q_c - q)], \qquad (15)$$

where E is the entrainment (mb^{-1}). Many investigators (Kurihara, 1973; Anthes, 1977a) have used a cloud radius-dependent definition of entrainment ($E = 0.183/R$) following Simpson and Wiggert (1969). Cloud radius, however, has little meaning for a numerical model in which the grid distance is many times the largest cloud radius. Recently, Kuo and Qian (1981) also used an entraining cloud lapse rate, but did not specify the manner in which it was calculated. In this work, entrainment will be assumed to be depth-dependent, i.e.,

$$E = \frac{\alpha}{H}, \qquad (16)$$

where H is the cloud depth (mb). The constant α contains Simpson and Wiggert's formulation plus the ratio of cloud radius to depth γ:

$$\alpha = \frac{0.183}{\gamma}. \qquad (17)$$

The value of γ is taken as 0.25, which lies within the range of values observed or computed from cloud models (Plank, 1969; Betts, 1973; Fraedrich, 1976), and thus $\alpha = 0.772$. In terms of radius, the entrainment rate for the thickest possible model cloud corresponds to $R = 2000$ m in Simpson and Wiggert's formula. The overall procedure removes some of the arbitrariness of the radius-dependent formulation, because instead of being externally specified and constant, entrainment is determined using the cloud

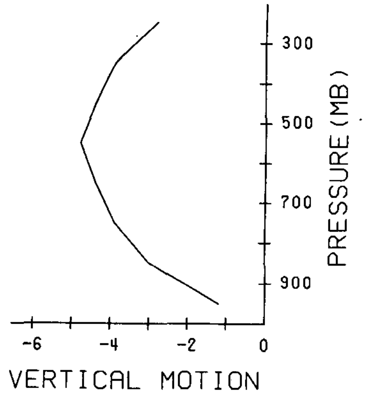

FIG. 1. Vertical motion (p-system) specified for one-dimensional model integrations (units: 10^{-2} mb s^{-1}).

depth, which is computed within the model and is space- and time-dependent.

It should be emphasized that inclusion of entrainment does not alter the fundamental properties of the approach, only the vertical distribution of heating and moistening. As $T_c \rightarrow T$ and $q_c \rightarrow q$, the entrainment term in (15) vanishes; thus the limiting stratification remains moist neutral, and the smooth transition to and from heating in a stable environment is retained. Details of the entrainment calculation are presented in the Appendix.

c. Inversion for a known rainfall rate

Carr (1977), Molinari (1982) and Fiorino and Warner (1981) have externally specified a rainfall rate determined from observations in a primitive equation model. In the latter two studies, the procedure was used as part of a dynamic initialization of the model. The vertical distribution of heating and moistening was specified and constant in all three cases, rather than determined by a formal approach to convective heating. Such a procedure causes some problems at the end of the initialization period, when the known heating is turned off and an internal calculation replaces it (Fiorino and Warner, 1981), because a discontinuity occurs in the heating rate. It will be shown below that the method proposed in this work can easily be inverted when the precipitation rate is known, thereby allowing it to be used both during and after the period of specified heating.

The precipitation rate is

$$P = (1 - b)I. \tag{18}$$

When P is known, Eqs. (18) and (14) form two equations in the two unknowns (b and I) and

$$I = \frac{JX + P}{1 - X}, \tag{19}$$

where

$$X = \frac{Q_q}{Q_q + Q_\theta + J},$$

and b is determined from (14). The calculation then proceeds exactly as before, producing an internally compatible approach between model initialization and forecast.

3. Results

To provide a severe test for the approach, a large upward motion is specified for the one-dimensional model integrations (Fig. 1). Each integration is started with subtropical standard atmosphere temperature and moisture (Jordan, 1958), and the model is integrated for 12 h, by which time a quasi-balance has been achieved between condensation heating and adiabatic cooling. In one dimension, of course, the vertical velocity cannot respond to the heating, and the results are limited in this regard. Much information can be gained, however, by examining the limiting behavior of an approach under strong external forcing.

a. Without entrainment

Four formulations for the b parameter are tested:

1) $b = \overline{(J + I)/2I}$ (Kanamitsu, 1975)

2) $b = (1 - \overline{RH})/(1 - RH_c)$; $b = 1$ for $\overline{RH} < RH_c$ (Anthes, 1977a)

3) b = constant = initial value

4) $b = [(J + I)/I][Q_q/(Q_q + Q_\theta + J)]$ (present study).

In case 2, the overbar indicates a vertical average over the cloud layer, and critical relative humidity RH_c is chosen to be 0.5, following Anthes (1977b).

Of interest is the time variation of temperature and moisture stratification. Fig. 2 shows the initial θ_e and θ_{es} profiles. Fig. 3 shows the same variables at hour 12 for case 1. Near moist neutrality is present in the temperature profile, but the column remains highly unsaturated. Such a combination is rarely if ever observed. In this case, temperature but not moisture has reached its limiting value. This formulation succeeds in removing the rainfall deficit present in the Kuo (1965) approach, but when starting from a dry initial

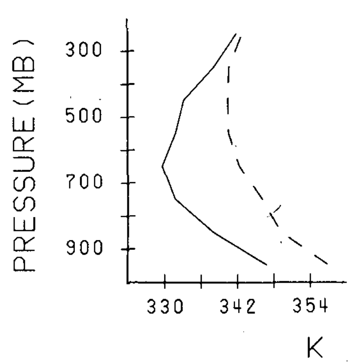

FIG. 2. Initial profiles of equivalent potential temperature (θ_e) and saturated equivalent potential temperature (θ_{es}).

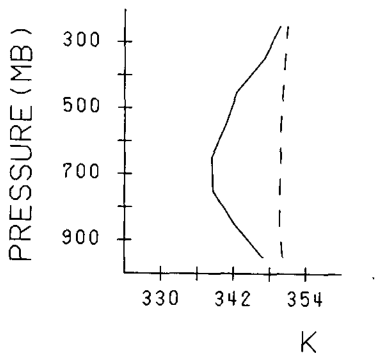

FIG. 3. θ_e and θ_{es} profiles at hour 12 for $b = (J + I)/2I$.

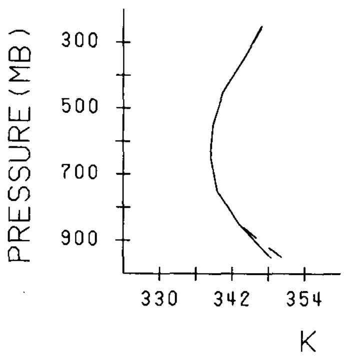

FIG. 5. θ_e and θ_{es} profiles at hour 5 for the integration with constant b.

state fails to moisten sufficiently as a moist neutral lapse rate is approached.

Fig. 4 shows the same two variables at hour 3 for Anthes' (1977b) form of b. Rapid moistening occurs early in the integration and the atmosphere is nearly saturated while considerable moist instability is present, again a combination which is rarely observed. Strong cooling occurs early in the integration when $b > (J + I)/I$ ($b = 0.85$ initially). Better results might be obtained by adjusting the critical relative humidity, but it is unlikely that a single value could be appropriate for all circumstances. This does not reflect on the overall approach of Anthes (1977a), which is not being examined here, only on his form for b, which appears to be inappropriate for the framework represented by Eqs. (4)–(5).

As a third experiment, the initial value of b is computed for the new approach, then held constant for the entire integration. Fig. 5 shows that saturation is again reached with a significantly unstable profile. In contrast, Fig. 6 shows the same two variables for the new formulation of b after 12 hours, during which b varies freely in time following Eq. (14). In this case, warming and moistening of the column produce a smooth approach toward moist neutrality in both temperature and moisture. It appears that a constant value of b, even if determined carefully from an ensemble of observations, is insufficient for a range of grid-scale conditions, all of which require different rates of moistening. Instead the value of b must be time-dependent and a function of the evolving stratification.

In all of the integrations, $\Delta\tau$ was chosen as 20 min. This value plays no role in the column-mean temperature and moisture variation, as shown by vertically integrating (4)–(5) and using (6)–(9). It enters only in the vertical heating distribution function in Eq. (4), i.e.,

$$\beta(p) = \frac{\theta_c - \theta}{\Delta\tau} + \omega\frac{\partial\theta}{\partial p}. \qquad (20)$$

The value of $\Delta\tau$ determines the relative importance of the two terms. The first term is generally much larger than the second, except when moist neutrality is approached, when the second term assumes a dominant role in the process of shifting to heating in a stable sounding. In this sense, $\Delta\tau$ determines to what extent the $\theta_c - \theta$ vertical distribution is retained as moist neutrality is being approached. One integration was repeated with $\Delta\tau = 10$, 5 and 1 min. After 12 h in the most extreme case, the largest temperature and relative humidity differences were 0.04°C and

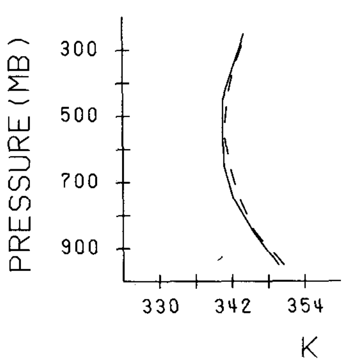

FIG. 4. θ_e and θ_{es} profiles at hour 3 for Anthes' (1977b) form of b.

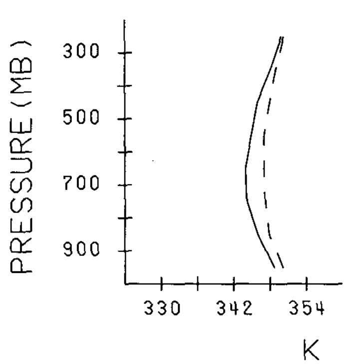

FIG. 6. θ_e and θ_{es} profiles at hour 12 for the integration with b defined by (14).

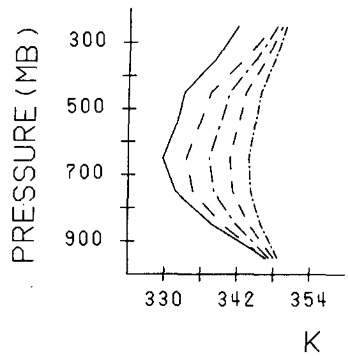

FIG. 7. Evolution of the vertical profile of θ_e at (from left to right) hours 0, 3, 6, 9 and 12, with b defined by (14).

.02%, respectively, indicating that the results are not dependent upon the value of $\Delta\tau$.

The four integrations were repeated using vertical velocity profiles with lower and upper tropospheric maxima, and all integrations were repeated using moist and dry initial soundings. The improvement shown earlier using the proposed form of b appeared in every case, independent of vertical velocity and relative humidity variations.

In the formulation of Kanamitsu (1975), the initial b value is 0.096, compared to 0.129 for the new formulation and the constant b integration. Because rainfall is proportional to $1 - b$, the initial rainfall rates in the three cases vary by less than 4%. As a result, each formulation provides comparable rainfall rates in a diagnostic or semi-prognostic sense, yet three entirely different temperature and moisture profiles evolve over several hours of integration. This emphasizes the fact that diagnostic testing of a method alone is inadequate to determine its validity in the time-dependent framework of a numerical model.

Aspliden (1976) and Betts (1974) show the variation in mean θ_e profiles for various undisturbed and disturbed conditions. Their soundings indicate that as conditions become more disturbed, i.e., as external forcing becomes stronger, θ_e tends to increase most near the level of its minimum. Fig. 7 shows the time evolution of θ_e at 3 h intervals for the new b formulation without entrainment (corresponding to Fig. 6). By far the largest initial increases in θ_e occur at the uppermost level, reflecting the large cloud–environment temperature differences present at upper levels with pure moist adiabatic ascent. This unrealistic aspect to the simulation prompted the addition of entrainment effects to the cloud parcel path.

b. With entrainment

Fig. 8 shows the evolution of θ_e for the new b formulation with (15) used for the cloud lapse rate. Entrainment tends to reduce the heating rate, increases the rate of moistening, and shifts the maximum heating to lower levels. As a result, θ_e increases most rapidly at middle levels and upper level warming is considerably reduced. The evolution of the profile is much closer to observations when entrainment is included.

Several of the earlier integrations for the various forms of b were repeated with entrainment, and the same limiting behavior was observed. Thus addition of entrainment does not alter in any way the findings of the previous section.

4. Summary and conclusions

A new closure for the b parameter of Kuo (1974) was compared to previous formulations in a one-dimensional model. Emphasis was placed on the time-dependent behavior of the solutions under strong external forcing. The quasi-steady-state solutions were found to be extremely sensitive to the form of b, which determines the partitioning of available latent energy into heating and moistening. Although initial rainfall rates were comparable for several methods, only the proposed form for b produced a smooth approach to moist neutral equilibrium in its limiting state. Other approaches led to saturation simultaneously with large instability, or to a highly unsaturated profile and moist neutrality, neither of which is generally observed. The sensitivity of the evolution of grid-scale temperature and humidity to partitioning of moisture for convection suggests that such partitioning must be carefully dealt with in any method for calculating cumulus effects, whether or not b is explicitly present.

The effects of entrainment were incorporated into the cloud lapse rate while retaining the smooth transition to heating in a moist stable environment. The new closure with entrainment reproduced the observed large-scale variation of equivalent potential temperature under disturbed conditions. The approach was shown to be easily invertible for use in a dynamic initialization using known rainfall rates.

The general approach of Kuo has received support recently in work by Krishnamurti *et al.* (1980) who

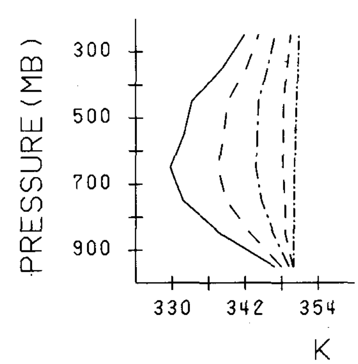

FIG. 8. As in Fig. 7, but with effects of entrainment included.

examined semi-prognostic rainfall rates in GATE, and by Zawadski *et al.* (1981) who found that the vertically integrated moist adiabat-environment temperature difference was more correlated with area-averaged precipitation than any other thermodynamic parameter tested. Kuo and Raymond (1980), using a cumulus cloud model, showed that a $T_c - T$ vertical distribution provides a reasonable measure of the profile of heating for deep cumulus convection. The Kuo-type approach presented here has the advantage of simplicity but does not contain arbitrarily defined parameters which critically influence the results, and still reproduces the bulk effects of cumulus clouds on the large scale.

In principle, the method can be made more sophisticated while retaining its economy, by keeping the same framework and closure for b, but adding complexity to the cloud parcel temperature. It may be desirable, particularly for mesoscale models, to replace the simple entraining updraft with a more complex profile based upon the net effect of updrafts and downdrafts. The nature of this profile and its inclusion into the model are the subject of further research.

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APPENDIX

Calculation of Entraining Profile

The equation for the entraining cloud parcel (15) can be written in finite difference form in terms of temperature alone, i.e.,

$$F(T_k) = 0 = T_k(\pi_k + \Delta p_k E \pi_k) + f_1(T_k) f_2(T_k) \times \frac{(0.622)(6.11) L \pi_k (1 + E\Delta p_k)}{c_p p_k} - \pi_{k-1} T_{k-1} - \frac{L\pi_k}{c_p} q_{s_{k-1}} - \Delta p_k E \pi_k \left(\bar{T} + \frac{L\bar{q}}{c_p} \right), \quad \text{(A1)}$$

where k is a vertical index, and T and q are grid-scale values,

$$f_1(T_k) = \exp[25.22(1 - 273.15/T_k)], \quad \text{(A2)}$$

$$f_2(T_k) = \left(\frac{273.15}{T_k} \right)^{5.31}, \quad \text{(A3)}$$

$$\pi_k = \left(\frac{p_0}{p_k} \right)^{R/c_p}. \quad \text{(A4)}$$

The numerical constants and the expression for saturation vapor pressure in (A1)–(A3) follow Kuo (1965). Eq. (A1) is solved by the Newton-Raphson technique:

$$T_k^{n+1} = T_k^n - \frac{F(T_k^n)}{F'(T_k^n)}, \quad \text{(A5)}$$

where n is an iteration count and

$$F'(T_k) = \pi_k(1 + E\Delta p_k) + \frac{f_1 f_2}{T_k} \times \left[\frac{(25.22)(273.16)}{T_k} - 5.31 \right] \times \frac{(0.622)(6.11) L \pi_k (1 + E\Delta p_k)}{c_p p_k}. \quad \text{(A6)}$$

Eq. (A5) converges rapidly to a solution, usually within two or three iterations, to a tolerance of 10^{-2} K.

A difficulty occurs in practice because E depends upon cloud depth and thus θ_c, yet E must be known to obtain θ_c. An iteration is developed in which E is defined initially using a guess value of the cloud depth equal to the thickest possible cloud. The value of θ_c is calculated using (A5) and cloud depth re-determined. If it does not match the guess value, E is defined using the new depth and the procedure is repeated. Rarely are more than two iterations required.

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