Keep in mind that wind is defined by the direction the air is coming from. A wind direction of 0° is northerly, 90° is easterly, 180° is southerly, and 270° is westerly. Also, keep in mind that meteorological wind directions are defined as clockwise from north.

1. Temperature is decreasing to the north at 5°C per 100 km. The wind is westerly at 5 m/s. There is no north-south or vertical wind, and no west-east temperature gradient. The local temperature change at our location is +1°C per hour (warming). What is the heating or cooling rate, in K/hr, a parcel must be experiencing as it travels in our direction?

**Ans:** The equation we need (removing terms that are zero) is

\[
\frac{dT}{dt} = \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y}.
\]

We have \( u = +5 \text{ m s}^{-1} \), but there is no temperature gradient in the west-east direction. There is a temperature gradient in the north-south direction, but \( v = 0 \). So the only nonzero term is \( \frac{\partial T}{\partial t} = +1 \text{ K h}^{-1} \). Therefore, \( \frac{dT}{dt} = +1 \text{ K h}^{-1} \).

2. The temperature at a point 50 km north of a station is 3°C cooler than at the station. If the wind is northeasterly at 10 m/s and the air is being heated by radiation at a rate of 1.0°C/hr, what is the local time rate of change of temperature at the station? Express the answer in °C per day.

**Ans:** The wind is from the northeast, but since the temperature only varies in the north-south direction, only the north-south component of the wind is doing any temperature (here, cold) advection. The \( v \) component is \(-7.07 \text{ m s}^{-1} = -25452 \text{ m h}^{-1} \), negative since it is directed southward. The cold advection is being opposed by some amount by radiative heating.

Again, the Lagrangian temperature tendency is (with zero terms removed):

\[
\frac{dT}{dt} = \frac{\partial T}{\partial t} + \frac{\partial T}{\partial y}
\]

We need to compute the local temperature derivative at the station, so solve for that term. The total derivative describes the radiative heating following the parcel; the other term is temperature advection. **Watch the minus signs.** Thus:

\[
\frac{\partial T}{\partial t} = \frac{dT}{dt} - \frac{\partial T}{\partial y}
\]

\[
= +1.0 \text{ K hr}^{-1} - (-25452 \text{ m h}^{-1}) \left[ \frac{-3 \text{ K}}{50000 \text{ m}} \right]
\]

\[
= +1.0 \text{ K hr}^{-1} - 1.53 \text{ K hr}^{-1}
\]

\[
= -0.53 \text{ K hr}^{-1}
\]

\[
= -12.7 \text{ K day}^{-1}
\]

If you got the sign wrong on the advection, you would end up with +60.72 K/day instead, a very different answer. But, you should have anticipated the advection would be negative, and checked your signs.
3. A perfectly dry, insulated air parcel would cool at the dry adiabatic lapse rate (DALR) upon ascent, owing to expansion cooling. For this problem, we can round the DALR to 10°C/km of vertical displacement. Suppose at every point, air is being warmed by radiation at 2.5°C/hr. You observe an air parcel ascending at constant vertical velocity, but its temperature following the motion is not changing. What is the ascent rate necessary for expansion cooling to offset radiation warming? Express the answer in meters per second.

**Ans:** Again, we start with the total derivative for temperature, removing terms that are zero. In this case, that leaves the vertical term:

\[
\frac{dT}{dt} = \frac{\partial T}{\partial t} + w \frac{\partial T}{\partial z}.
\]

The total derivative on the left-hand side describes the temperature change following the air parcel. For this problem, it is zero. The local time rate of change of temperature owing to radiation is specified to be +2.5 K/hr. The only “tricky” bit (if it is even that) is to recognize that the concept of lapse rate presumes a minus sign, so \( \frac{\partial T}{\partial z} = -10 \text{ K/km} \). So, we have

\[
\frac{\partial T}{\partial t} = -w \frac{\partial T}{\partial z}
\]

\[
+2.5 \text{ K/hr} = -w[-10 \text{ K/km}]
\]

\[
w = \frac{2.5 \text{ K}}{3600 \text{ s}} \frac{1000 \text{ m}}{10 \text{ K}} \approx 0.07 \text{ m/s}
\]

If you missed the minus sign on the vertical temperature gradient, you should realize you had a sign error by your understanding of the problem. The air parcel is experiencing both radiative warming and expansion cooling, such that the two contributions cancel and its temperature is not changing. To experience expansion cooling in this situation, the parcel must be rising, towards lower pressure. Thus, \( w \) has to be positive.

Suppose you get a negative value for \( w \) and you do not realize where you missed the sign. In this case, for maximum credit you should provide an explanation or justification for why the sign you calculated must be incorrect.

4. The horizontal geostrophic wind is \( \vec{V}_g = u_g \hat{i} + v_g \hat{j} \). Work backwards from

\[
\vec{V}_g = \hat{k} \times \frac{1}{\rho f} \nabla p
\]

and recover \( u_g = -\frac{1}{\rho f} \frac{\partial p}{\partial y} \) and \( v_g = \frac{1}{\rho f} \frac{\partial p}{\partial x} \).

**Ans:**

\[
\vec{V}_g = \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \end{bmatrix} = \left[ -\frac{1}{\rho f} \frac{\partial p}{\partial y} \right] \hat{i} + \left[ \frac{1}{\rho f} \frac{\partial p}{\partial x} \right] \hat{j}
\]
5. Troy is 10 km due east from Albany airport. Let the point halfway between Albany and Troy be called “X”. The temperatures at Albany, X, and Troy are 10, 8, and 12°C, respectively.

(a) Compute the first derivative at point X. Express in degrees per kilometer.

Ans: In the figure, \( \Delta x = 5 \) km, representing the distance between Albany and X, or between X and Troy, so the distance between Albany and Troy is \( 2\Delta x = 10 \) km as required.

\[
f'(X) = \frac{f(x_0 + \Delta x) - f(x_0 - \Delta x)}{2\Delta x} = \frac{12 - 10}{10} = 0.2 \text{ K/km}.
\]

(b) Compute the second derivative at point X. Express in degrees per square kilometer.

Ans: This can be done by computing the first derivative at the locations indicated by the black dots on the figure, and then computing the derivative of that.

\[
f''(X) = \frac{f(x_0 + \Delta x) - f(x_0) - (f(x_0) - f(x_0 - \Delta x))}{\Delta x} = \frac{12 - 8 - (8 - 10)}{5} = \frac{6}{25} \text{ K/km}^2 = 0.24 \text{ K/km}^2
\]

(c) Briefly discuss why your calculations might contain error.

Ans: At the very least, both formulae are based on truncated Taylor series. The Mean Value Theorem states that they would be exactly correct (to within roundoff) somewhere in the interval, but we do not know where that point is. Truncation results in error if any of the neglected terms (representing higher order derivatives) are nonzero. In part (b), for example, you demonstrated that the second derivative neglected in part (a) is definitively nonzero.