1. The 500 mb wind is $V_{500} = -7.07\hat{i} + 7.07\hat{j}$. The 1000-500 mb shear is $V_{\text{shear}} = 7.07\hat{j}$. What are the $u$ and $v$ components of the 1000 mb wind, $V_{1000}$? Draw a picture, and label it completely. (5 pts.)

**Ans:** $\vec{V}_{\text{shear}} = \vec{V}_{500} - \vec{V}_{1000}$, so $\vec{V}_{1000} = \vec{V}_{500} - \vec{V}_{\text{shear}} = (-7.07 - 0)\hat{i} + (7.07 - 7.07)\hat{j} = -7.07\hat{i}$, as illustrated below. So the $u$ and $v$ components of the 1000 mb wind are -7.07 and 0.

\[
\begin{align*}
V_{500} &\quad \downarrow \\
V_{1000} &\quad \downarrow \\
V_{\text{shear}} &\quad \downarrow
\end{align*}
\]

2. A vector flow field is described by $\vec{U} = 3y\hat{i} - 3x\hat{j} + 0\hat{k}$. Compute the three-dimensional vorticity and divergence for this field. Show your equations and steps clearly. (6 pts.)

**Ans:** We have $u = 3y$, $v = -3x$ and $w = 0$. Divergence is

\[
\nabla \cdot \vec{U} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0,
\]

since $u$ is not a function of $x$, $v$ is not a function of $y$, and $w$ is zero.

For vorticity, we can expand out all three directions, or realize that two of them vanish because $w$ is zero and nothing varies with $z$. We are left with the vertical vorticity term

\[
\nabla \times \vec{U} = \hat{k} \left[ \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right] = \hat{k}(-3 - 3) = -6\hat{k}.
\]
3. The wind velocity (m/s) at the location of the star in the figure below is \( \vec{U} = 10\hat{j} \) km/h. There is no vertical wind component. As it travels, air is being cooled at 1 K/h. Compute the local time rate of change of temperature at the location of the star, in K per hour. Set up the equation completely, make your approximations and simplifications clear, and show your work leading to your answer. (6 pts.)

**Ans:** Our equation is

\[
\frac{\partial T}{\partial t} = \frac{dT}{dt} - u \frac{\partial T}{\partial x} - v \frac{\partial T}{\partial y}.
\]

We are given \( \frac{dT}{dt} = -1 \) K/h, \( u = 0 \) and \( v = 10 \) km/h. Since \( u = 0 \), \( \frac{\partial T}{\partial x} \) does not matter. The north-south temperature gradient is

\[
\frac{\partial T}{\partial y} = \frac{273 \text{ K} - 283 \text{ K}}{100 \text{ km}} = -0.1 \text{ K/km},
\]

so \( -v \frac{\partial T}{\partial y} = -10 \) (km/h)(-0.1 K/km) = +1 K/h. Note temperature advection and the cooling rate cancel, so the local time rate change of temperature is zero:

\[
\frac{\partial T}{\partial t} = 0.
\]

4. Moisture decreases westward at 1 g/kg per km. Following the motion, the air is becoming drier at 1 g/kg per hour. The horizontal wind vector is \( \vec{V} = -5\hat{j} \) km/h and there are no vertical motions. Compute the local time rate of change of the mixing ratio at a fixed point, in g/kg per h. Make a sketch and label it completely. (3 pts.)

**Ans:** Our equation, solving for the local derivative, is:

\[
\frac{\partial q}{\partial t} = \frac{dq}{dt} - u \frac{\partial q}{\partial x} - v \frac{\partial q}{\partial y}.
\]

We are given \( \frac{dq}{dt} = -1 \) g/kg per h. Here, \( u = 0 \) and \( v = -5 \), so only the north-south wind can do advection. But, the moisture gradient is west-east, so there is no advection. As a consequence, \( \frac{\partial q}{\partial t} = \frac{dq}{dt} = -1 \) g/kg per h.
5. The temperature at a particular point is 280K. 1 km to the north, the temperature is 282K. 1 km to the south, the temperature is 282K. Compute the second derivative of temperature in the north-south direction at the intermediate point, expressed in K per square km. Draw a picture. Show your work and be very clear with respect to the sign of your answer. Also address: is this exact or an approximation, and why? (5 pts.)

**Ans:** This is a straightforward application of the second derivative:

\[
\frac{f''(y)}{\Delta y^2} = \frac{f(y + \Delta y) - 2f(y) + f(y - \Delta y)}{\Delta y^2}
\]

\[
= \frac{282 - 2 \cdot 280 + 282}{1^2} \text{ K/km}^2
\]

\[
= +4 \text{ K/km}^2
\]

It is likely an approximation because of truncation error. Non-zero higher order terms were neglected in construction of our 2nd derivative formula. But, recall the Mean Value Theorem and Extended Mean Value Theorem, which say that if you compute the last retained term in the Taylor series at an intermediate but unknown point, the higher-order terms become zero and there is no truncation error. So, the only situation in which there is no truncation error is when we’re applying our calculated derivative to that unknown point. Unlikely.