1. The horizontal vector field is $\vec{V} = 2x^2 \hat{i} + x^2 \hat{j}$. There is no vertical motion. Compute the horizontal divergence and vertical vorticity, and evaluate for the point $(x, y) = (1,1)$. Show all work. You do not need to draw a picture. (6 pts.)

**Ans:** Note $u = 2x^2$ and $v = x^2$. The horizontal divergence is:

$$\nabla \cdot \vec{V} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$$

$$= \frac{\partial}{\partial x}(2x^2) + 0$$

$$= 4x$$

At point $(x, y) = (1,1)$, the divergence = 4 per second. The vertical component of vorticity is:

$$\hat{k} \cdot (\nabla \times \vec{V}) = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$$

$$\zeta = \frac{\partial}{\partial x}(x^2) + 0$$

$$= 2x$$

At point $(x, y) = (1,1)$, the vorticity = 2 per second.

2. Air passing over the ocean is being moistened at 1 g/kg per hour. But the local time rate of change of moisture at a fixed point is zero. The wind is westerly at 1 km/h. What is the sign and magnitude of the west-east moisture gradient? Answer in (g/kg)/km. Draw a picture. Double-check your sign. (6 pts.)

**Ans:** We have $\frac{dq}{dt} = +1 \text{ (g/kg)/h}$, but $\frac{\partial q}{\partial t} = 0$. We have $u = +1 \text{ km/h}$. So we need dry advection, i.e., moisture increasing downwind (to the east). Start with the definition of the total derivative, prune the unneeded terms, and solve for the moisture gradient $\frac{\partial q}{\partial x}$:

$$\frac{dq}{dt} = \frac{\partial q}{\partial t} + u \frac{\partial q}{\partial x}$$

$$\frac{\partial q}{\partial x} = \frac{1 dq}{u \ dt}$$

$$= \frac{1}{1 \ \text{km/h}} [1 \ \text{g/kg/h}]$$

$$= 1 \ (\text{g/kg})/\text{km}$$

Note this gradient is positive, directed to the east, as required.
3. The equations of horizontal motion for a unit mass parcel and involving only the Coriolis
accelerations are
\[
\frac{du}{dt} = fv,
\]
and
\[
\frac{dv}{dt} = -fu.
\]
Prove the Coriolis force cannot change kinetic energy following the motion. (6 pts.)

**Ans:** We seek to show that \( \frac{de}{dt} = 0 \), where the energy per unit mass is
\[
e = \frac{1}{2}(u^2 + v^2).
\]

\[
\frac{de}{dt} = \frac{1}{2} \frac{d}{dt} \left[ u^2 + v^2 \right]
\]
\[
= \frac{1}{2} \left[ 2u \frac{du}{dt} + 2v \frac{dv}{dt} \right]
\]
\[
= uf - vfu
\]
\[
= 0.
\]

4. The vertical wind shear vector is westerly. The 1000 mb wind is easterly at 100 m/s. The
500 mb wind is westerly at 100 m/s. Compute the temperature advection in the 1000-500
mb layer, and express in K per h. (4 pt.)

**Ans:** You do have enough information to answer this question. The vertical shear is westerly,
so the isotherms of layer mean temperature are oriented west-east with cold air to the north.
The (geostrophic) wind component that does advection does not change with height through
this layer, and a nonzero north-south wind would be needed in this case for advection. Since
the north-south component is clearly zero at 1000 mb, it must be true through the layer.
Thus, there is no advection, so the answer is 0 K/h.
5. If the rotating Earth were a perfect sphere, at most latitudes apparent gravity would not point directly towards the center of mass. Succinctly explain why. Draw picture(s) and label them completely. I expect to see vector algebra in your answer. (5 pts.)

Ans: The Earth’s rotation vector is $\vec{\Omega}$, with magnitude $\Omega$. An object at latitude $\phi$, at distance $\vec{R}$ from the spin axis, experiences these two forces in its reference frame: true gravity $\vec{g^*}$, pointing directly towards the center of mass, and the centrifugal acceleration $\Omega^2 \vec{R}$, directed outward from the spin axis. The vector sum of these accelerations is apparent gravity $\vec{g} = \vec{g^*} + \Omega^2 \vec{R}$, as illustrated in the picture. Note that if the true gravity vector is pointing towards the center of mass, which is in the local vertical on a perfect sphere, then the apparent gravity cannot be. (The exception is the equator.)

6. Did the quizzes help? Answer: yes, no, not sure. (1 pt. No wrong non-missing answer)

Ans: No wrong non-missing answer.