The Counter-Gradient Heat Flux in the Lower Atmosphere and in the Laboratory

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ABSTRACT

Observations in the lower atmosphere and of laboratory convection which demonstrate the existence of upward heat flux, $\overline{w'\theta'}>0$, with vanishing or counter potential temperature gradient, $\partial\bar{\theta}/\partial z \geqslant 0$, are reviewed. Data of Telford and Warner (1964) are utilized to explain the magnitude of the counter gradient within the framework of the thermal variance equation. The diffusion or triple-correlation term, which can allow a counter-gradient flux to exist, is interpreted qualitatively.

For theoretical studies of the planetary boundary layer which employ an eddy coeffiicent for sensible heat, the modified definition

$$\overline{w'\theta'} \equiv -K_H'(\partial\bar{\theta}/\partial z - \gamma_c)$$

is suggested, with $\gamma_c \approx 6.5\times10^{-6}$C cm^{-1} in clear air.

1. Introduction

The occurrence of an upward directed, sensible heat flux when the lapse rate is neutral or slightly stable is not predicted by simple mixing-length theory (Sutton, 1953) which concludes that the flux must be directed down the gradient of potential temperature. This theory was amended slightly by Priestley and Swinbank (1947) to allow for the fact that a parcel of air not moving vertically at some instant may have a potential temperature differing from that of its environment at the same level, and may subsequently start to rise or sink because of buoyancy effects. A new term was then added to the mixing-length equation which could allow for a counter-gradient heat flux. In this manner, Priestley and Swinbank qualitatively explained how the upward directed heat flux could be passed from a superadiabatic layer near the ground to levels of 200 m and above where superadiabatic lapse rates are seldom observed. Although this addition to the mixing-length theory prevented it from being violated, the theory has nevertheless become irrelevant in this case since the additional buoyancy term is not susceptible to direct measurement or to accurate theoretical prediction.

For some time after the appearance of the revised mixing-length theory, meteorologists seemed uncertain if the original theory needed amendment. Above a heated surface, large amounts of sensible heat could be transported upwards at levels where the lapse rate is only very slightly superadiabatic if the eddy coefficient for heat were very large. Measurements showing a slightly stable lapse rate needed to be only slightly in error in order that the heat flux still be directed down the gradient.

However, as more and better measurements appeared, this argument became untenable. The average of a large number of measurements of local Richardson number centered at 100 m and higher, when the heat flux was unquestionably upward at these levels, disclosed only positive values in the Great Plains Turbulence Field Program (Lettau and Davidson, 1957, Table 7.5.2). With weak winds and strong thermal convection, Webb (1958) found that the level of vanishing gradient could be as low as about 25 m.

The aircraft-based measurements of Bunker (1956) disclosed slightly positive potential temperature gradients with upward heat flows, from levels of 150 m to about 550 m above the western Atlantic. More refined aircraft measurements by Telford and Warner (1964) have similarly revealed counter-gradient heat flows from about 150 m above the ground to about 350 and 1250 m, respectively, on two occasions. In these aircraft measurements, the averaged kinematic heat flux, $\overline{w'T'}$, was measured directly, and the potential temperature measurements were obtained over sufficiently large vertical distances that systematic relative errors of 0.3C or more would have had to be present to explain the stable lapse rate as being due to instrumental inaccuracy.

Estimates of the eddy coefficient for heat K_H, based upon temperatures measured at the Cedar Hill tower in Texas, have been made by Wong and Brundidge (1966). The analysis disclosed frequent negative values of K_H, especially between the hours of 0300 and 0700 local time. Unfortunately, all K_H values derived were in relation to an unknown value at the lowest level which was obtained by assumption, and radiative cooling was ignored even during periods of smallest turbulent heat flux. The upward heat flux they frequently obtained in the early morning hours even at low levels is inconsistent with all other observations known to the writer, and may have been a spurious result of these assumptions, or may have been associated with terrain inhomogeneities.

Turning now to laboratory measurements, the study of free convection between horizontal plates by Thomas and Townsend (1957) has shown that the central 50 per cent or so of the region between plates is isothermal, or adiabatic, within the accuracy of measurement, with a Rayleigh number of 3.8×10^5. The same result has recently been obtained by Deardorff and Willis (1966) in air for the larger Rayleigh numbers of 6.3×10^5, 2.5×10^6 and 1.0×10^7. At these Rayleigh numbers the motions are highly turbulent. As the Rayleigh number is increased, the percentage of the region between plates occupied by the approximately adiabatic lapse rate increases.

2. Understanding the counter-gradient heat flux

Fortunately, there is a much more rigorous and relevant framework than mixing-length theory with which to understand how an upward heat flux can persist with a vanishing or counter potential temperature gradient. The equation which predicts the local change of temperature variance (Townsend, 1959) is also the relevant one to describe, in the horizontally homogeneous and steady state, the conditions under which the turbulent heat flux will either be directed down the gradient, or counter gradient (Deardorff, 1961). This equation is, approximately,

$$\frac{\partial}{\partial t}\left(\frac{\overline{\theta'^2}}{2}\right)=-\overline{w'\theta'}\frac{\partial\bar{\theta}}{\partial z}-\frac{\partial}{\partial z}\left(\frac{\overline{w'\theta'^2}}{2}\right)+\left(\frac{\overline{\theta}}{\overline{T}}\right)\times[-\kappa\overline{(\nabla T')^2}+\overline{R'\theta'}],\quad(1)$$

where the overbar represents a horizontal average, the prime the deviation from this average, θ is potential temperature, T is absolute temperature, w is the vertical velocity component, z is the vertical coordinate (positive upward), κ is the thermal diffusivity of air (molecular value), and R' is the net rate of temperature change due to radiative heating or cooling relative to the average radiative change at the level z. In addition to horizontal homogeneity, the assumptions leading to (1) are

$$\overline{\rho w}=0,\quad(2a)$$

$$(\rho w)'\approx\bar{\rho}w',\quad(2b)$$

$$\frac{|p'|}{\bar{p}}\ll\frac{|T'|}{\bar{T}},\quad(2c)$$

$$\overline{\bar{w}\theta'}=0=\bar{\theta}',\text{ etc. (Reynolds averaging assumptions)},\quad(2d)$$

No liquid water present. (2e)

The first term on the right of (1) tends to increase the thermal variance when the kinematic heat flux $\overline{w'\theta'}$ is directed down the gradient. It is thus usually called a "production" term. The last term on the right tends to decrease the variance because the molecular portion is negative definite, and because

$$\overline{R'\theta'}<0\quad(3)$$

under most conceivable conditions. It may, therefore, be called a "smoothing" term. The second term on the right is a triple-correlation or diffusion term. It is seen that when this term is negligible, the heat flux must be directed down the gradient in nearly steady conditions. Consequently, only when the diffusion term, $-(\partial/\partial z)(\overline{w'\theta'^2}/2)$, is positive and exceeds the smoothing term in magnitude is a counter-gradient heat flux to be expected. Only then is

$$\overline{w'\theta'}\partial\bar{\theta}/\partial z>0.\quad(4)$$

It has been suggested (Deardorff, 1961) that in the atmospheric counter-gradient region the diffusion term alone supports the counter-gradient flux, and that the smoothing term in (1) is negligible. However, measurements of $\overline{w'\theta'^2}$ or $\overline{w'T'^2}$ in the counter-gradient region were not then available. A single set of such measurements by Telford and Warner (1964) above uniform terrain is now available to test this idea. Values of the production and diffusion terms are given in Table 1 for their case of 12 June 1961, which is the only case with sufficiently great vertical resolution. The heat flux decreased with height, but was positive up to about 1250 m on this occasion. The close cancellation of the two terms at the lower two levels indeed tends to verify the hypothesis that the smoothing term in (1) is negligible at these levels. Unfortunately, the agreement at $z=578$ m may be accidental because of a reported uncertainty here in $\partial\bar{\theta}/\partial z$. At $z=997$ m the lack of agreement could have been caused by insufficient vertical resolution over the interval of 533 m, especially since an inversion base would be expected near the level of negligible upward heat flux at 1263 m. Although more aircraft measurements are needed, these results do support the view that the counter-gradient

TABLE 1. Values of production and diffusion terms of Eq. (1) obtained by Telford and Warner (1964) on 12 June 1961 above uniform terrain.

Average z (m)	Δz (m)	Production $-\overline{w'T'}\partial\bar{\theta}/\partial z$ [(°C)^2 sec^{-1}]	Diffusion $-\frac{\partial}{\partial z}(\overline{w'T'^2}/2)$ [(°C)^2 sec^{-1}]
350	152	-4.7×10^{-5}	4.6×10^{-5}
578	204	-0.76×10^{-5}	0.81×10^{-5}
997	533	-2.2×10^{-5}	0.28×10^{-5}

heat flux in the atmosphere is supported by the diffusion term.

In laboratory thermal convection, the vertical distances involved are too small to establish reliably if the potential temperature gradient is slightly positive, negative or exactly zero in the central region. However, measurements have been made by Deardorff and Willis (1966) of the terms on the right of (1), except for the radiation term which appeared to be entirely negligible. It was found at all three Rayleigh numbers previously mentioned that the production term was negligible in the central region. The diffusion and molecular smoothing terms were not negligible, but approximately balanced each other. Hence, at these Rayleigh numbers it may be considered that the upward heat flux occurs with a vanishing potential temperature gradient.

The small scales involved in laboratory convection apparently allow molecular smoothing to be sufficiently large to balance the diffusion term. In the atmospheric case, this smoothing is not nearly large enough and a counter potential temperature gradient must occur for balance. The intermediate case when all three terms on the right of (1) are important occurs in laboratory convection closer to the boundaries, and must occur also in the atmosphere near and below the base of a counter-gradient region.

3. Interpretation of the diffusion term

Since the term $-(\partial/\partial z)(\overline{w'\theta'^2}/2)$ is crucial to a rigorous explanation for vanishing and counter-potential temperature gradients, it will be qualitatively interpreted here.

This term arises from the vertical advection of $\theta'^2/2$ in the equation for $(d/dt)(\overline{\theta'^2})/2$. We may examine $\overline{w'\theta'^2}=\overline{(w'\theta')\theta'}$ and note from observations of Priestley (1959), Bunker (1956), and others, that with free convection $w'\theta'$ is generally large when $\theta'>0$, and quite small in magnitude otherwise. Hence, $\overline{w'\theta'^2}$ is positive but decreases with height in the vanishing or counter-gradient region because $\overline{\theta'^2}$ and $\overline{w'\theta'}$ both decrease with height there. Hence, the diffusion term, $-(\partial/\partial z)(\overline{w'\theta'^2}/2)$, is positive in this region.

An alternate interpretation stems from the fact that the predominate convective elements are separated by distances considerably greater than their widths. Hence, $\overline{T'^3}=\overline{T'(T'^2)}>0$, and positive skewness exists both below and within the counter-gradient region. Because of the positive correlation between w' and T', some positive correlation then occurs between w' and T'^2 also such that $\overline{w'T'^2}>0$ again. The measurement by Telford and Warner of $\overline{T'^3}$ confirm this interpretation although sampling errors were sufficiently great that the generally positive values of $\overline{T'^3}$ decreased erratically with height. In laboratory convection, negative skewness of the temperature structure was observed by Deardorff and Willis near the upper boundary which corresponds to positive skewness near the lower boundary. From symmetry considerations, this skewness must be zero midway between horizontal plates of a convection chamber.

This interpretation of the diffusion term and its role in the counter-gradient flux is consistent with, and supplements, the earlier viewpoint of Priestley (1954) and Bunker (1956). This viewpoint is that the counter-gradient heat flux consists of eroding parcels or plumes of relatively warm air which form in the underlying superadiabatic layer and penetrate a slightly stable region above.

4. K_H in the counter-gradient region

Obviously, an eddy coefficient for heat defined by

$$\overline{w'\theta'}-\equiv K_H\partial\bar{\theta}/\partial z, \tag{5}$$

has no usefulness in a counter-gradient region where it is negative, or at the base or top of such a region where it jumps from $\pm\infty$ to $\mp\infty$. However, a coefficient K_H' defined by

$$\overline{w'\theta'}=-K_H'(\partial\bar{\theta}/\partial z-\gamma_c), \tag{6}$$

may be useful. Here γ_c is a positive parameter designating an upper limit for a counter potential temperature gradient in clear air. From (1), this upper limit is seen to be

$$\gamma_c\equiv\left(\frac{\partial\bar{\theta}}{\partial z}\right)_{\text{upper limit}}=-\frac{\partial}{\partial z}\left(\frac{\overline{w'\theta'^2}}{2}\right)\Big/\overline{w'\theta'}. \tag{7}$$

From the 350-m data of Table 1, and from the measurements of $\overline{w'T'}$, the right-hand side of (7) is

$$\gamma_c=6.5\times10^{-6}\text{C cm}^{-1}. \tag{8}$$

More detailed aircraft measurements of $\overline{w'T'^2}$, $\overline{w'\theta'}$ and $\bar{\theta}$ are necessary to obtain a better value for γ_c and to estimate its variability in the vertical. However, if the constant value in (8) had been used along with Eq. (6) in the planetary boundary studies of Estoque (1963) and Wu (1965), for example, their potential temperature profiles calculated for the afternoon hours would have been in considerably better agreement with those observed in the lowest kilometer or half kilometer. Formulation (6) could also be safely used outside of the counter-gradient region in most studies since usually $\gamma_c\ll|\partial\bar{\theta}/\partial z|$ in either the superadiabatic layer adjacent to the surface, or in stable regions of downward heat flux.

It should be mentioned that at levels where cloudiness prevails, γ_c will be larger than the value given here, and will approach the value appropriate to the moist adiabatic lapse rate for fully saturated air.

REFERENCES

Bunker, A. F., 1956: Measurements of counter-gradient heat flux in the atmosphere. *Austr. J. Phys.*, **9**, 133–143.

Deardorff, J. W., 1961: On the direction and divergence of the small-scale turbulent heat flux. *J. Meteor.*, **18**, 540–548.

——, and G. E. Willis, 1966: Investigation of turbulent thermal convection between horizontal plates. National Center for Atmospheric Research, Boulder, Colo. Prepublication review manuscript No. 140, 58 pp, available from authors.

Estoque, M. A., 1963: A numerical model of the atmospheric boundary layer. *J. Geophys. Res.*, **68**, 1103–1113.

Lettau, H. H., and Ben Davidson, 1957: *Exploring the Atmosphere's First Mile*, Vol. 1. New York, Pergamon Press, p. 343.

Priestley, C. H. B., 1954: Vertical heat transfer from impressed temperature fluctuations. *Austr. J. Phys.*, **7**, 202–209.

——, 1959: *Turbulent Transfer in the Lower Atmosphere.* Chicago, Univ. of Chicago Press, 130 pp.

——, and W. C. Swinbank, 1947: Vertical transport of heat by turbulence in the atmosphere. *Proc. Roy. Soc. London*, **A189**, 543–561.

Sutton, O. G., 1953: *Micrometeorology.* New York, McGraw-Hill Book Co., p. 148.

Telford, J. W., and J. Warner, 1964: Fluxes of heat and vapor in the lower atmosphere derived from aircraft observations. *J. Atmos. Sci.*, **21**, 539–548.

Thomas, D. B., and A. A. Townsend, 1957: Turbulent convection over a heated horizontal surface. *J. Fluid Mech.*, **2**, 473–492.

Townsend, A. A., 1959: Temperature fluctuations over a heated horizontal surface. *J. Fluid Mech.*, **5**, 209–241.

Webb, E. K., 1958: Vanishing potential temperature gradient in strong convection. *Quart. J. R. Meteor. Soc.*, **84**, 118–125.

Wong, E. Y. J., and K. C. Brundidge, 1966: Vertical and temporal distributions of the heat conductivity and flux. *J. Atmos. Sci.*, **23**, 167–178.

Wu, S., 1965: A study of heat transfer coefficients in the lowest 400 meters of the atmosphere. *J. Geophys. Res.*, **70**, 1801–1807.