Midterm Review

ATM 622: General Circulation of the Atmosphere
October 21, 2019

Top of Atmosphere Radiation

Meridional energy transport required to compensate for top of atmosphere net radiative imbalance.

$$\int_{-\frac{\pi}{2}}^{\phi_o} \int_0^{2\pi} \overline{F_{net}} a^2 \cos\phi d\lambda d\phi = \int_0^{2\pi} \overline{vE} a \cos\phi d\lambda \Big|_{\phi_o}$$

Annual mean net northward energy flux at Equator. Net imbalance due to ocean heat uptake.

Decomposition of Circulation

$$A = \overline{A} + A'$$
$$A = [A] + A^*$$
$$\left[\overline{AB}\right] = \left[\overline{A}\right]\left[\overline{B}\right] + \left[\overline{A}^*\overline{B}^*\right] + \left[\overline{A'B'}\right]$$

In the annual mean, there can be no net meridional mass flux across a latitude circle.

$$\int_{p_s}^{0} [\overline{v}]\, dp = 0$$

Angular Momentum Budget

$$M = \Omega a^2 \cos^2\phi + ua\cos\phi$$

Integrated budget over atmosphere

$$\frac{\partial}{\partial t}\int_m M dm = -\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} a^2\cos\phi \int_0^{2\pi} p_s \frac{\partial z_s}{\partial\lambda} d\lambda d\phi - \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_0^{2\pi} \rho a^3 \cos^2\phi c u d\phi d\lambda$$

Mountain torques result of pressure differences between west and east sides of mountains. In the annual mean, areas of surface easterlies and westerlies must approximately balance.

Zonal and time averaged budget

$$\frac{\partial\left[\overline{M}\right]}{\partial t} = -\frac{1}{a\cos\phi}\frac{\partial}{\partial\phi}\left(\left[\overline{vM}\right]\cos\phi\right) - \frac{\partial}{\partial p}\left(\left[\overline{\omega M}\right]\right) - g\left[\frac{\partial\overline{z}}{\partial\lambda}\right] + a\cos\phi\left[\overline{F_u}\right]$$

At long time scales, no net poleward transport of planetary angular momentum

$$\left[\overline{vM_r}\right] = a\cos\phi\left(\left[\overline{u}\right]\left[\overline{v}\right] + \left[\overline{u}^*\overline{v}^*\right] + \left[\overline{u'v'}\right]\right)$$

Mean meridional northward transport of M_r when mean southerlies (northerlies) associated with mean westerlies (easterlies). Eddy northward transport of M_r when eddies positively tilted.

$$-\frac{\partial\psi_M}{\partial p} = 2\pi a\cos\phi\left[\overline{vM_r}\right]$$
$$\frac{\partial\psi_M}{\partial\phi} = 2\pi a^2\cos\phi\left(\left[\overline{\omega M}\right] + a\cos\phi\left[\overline{\tau}\right]\right)$$

Schematic of angular momentum sources/sinks and transport

Energy Budget

$$E = I + \Phi + L + K = c_v T + gz + L_v q + \frac{1}{2}\left(u^2 + v^2 + w^2\right)$$

$$\frac{dI}{dt} = -\alpha\nabla\cdot\mathbf{F}_{rad} + c_p D_T + L_v(c-e) - \mathbf{u}\cdot\mathbf{F}_r - p\alpha\nabla\cdot\mathbf{u}$$
$$\frac{d\Phi}{dt} = gw$$
$$\frac{dL}{dt} = L_v(e-c) + L_v D_q$$
$$\frac{dK}{dt} = -\alpha\mathbf{u}\cdot\nabla p - gw + \mathbf{u}\cdot\mathbf{F}_r$$
$$\frac{dE}{dt} = -\alpha\nabla\cdot(\mathbf{u}p) - \alpha\nabla\cdot\mathbf{F}_{rad} + D_h$$

Integrated budget over atmosphere

$$\frac{\partial}{\partial t}\int_m E dm = -\int_S \mathbf{F}_{rad}\cdot\mathbf{n} ds + \int_{sfc}\rho D_h ds$$

$$\mathcal{E} = S + \Phi + L + K = c_p T + gz + L_v q + \frac{1}{2}\left(u^2 + v^2 + w^2\right)$$

$$\frac{dS}{dt} = -\alpha\nabla\cdot\mathbf{F}_{rad} + c_p D_T + L_v(c-e) - \mathbf{u}\cdot\mathbf{F}_r + \alpha\frac{\partial p}{\partial t} + \alpha\mathbf{u}\cdot\nabla p$$

$$\frac{d\mathcal{E}}{dt} = -\alpha\nabla\cdot\mathbf{F}_{rad} + D_h + \alpha\frac{\partial p}{\partial t}$$

Schematic of sources/sinks and conversions between different energies

Zonal and time averaged budget

$$\frac{\partial \left[\overline{\mathcal{E}}\right]}{\partial t} = -\frac{1}{a\cos\phi}\frac{\partial}{\partial\phi}\left(\left[\overline{v\mathcal{E}}\right]\cos\phi\right) - \frac{\partial}{\partial p}\left(\left[\overline{\omega\mathcal{E}}\right]\right) + g\left[\frac{\partial \overline{F}_{rad}}{\partial p}\right] + \left[\overline{D_h}\right]$$

Features of the decomposition of meridional flux of $\mathcal{E}$ into four components and mean/eddy terms

$$-\frac{\partial\psi_{\mathcal{E}}}{\partial p} = 2\pi a\cos\phi\left[\overline{v\mathcal{E}}\right]$$
$$\frac{\partial\psi_{\mathcal{E}}}{\partial\phi} = 2\pi a^2\cos\phi\left(\left[\overline{\omega\mathcal{E}}\right] + g\left[\overline{F}_{rad}\right] + \left[\overline{\tau}_h\right]\right)$$

Available Potential Energy

Reference state defined to be minimization of enthalpy ($H = c_pT + L_vq$) through adiabatic, inviscid rearrangement of parcels.

Available potential energy is the difference in enthalpy between current state and reference state. Variance of pressure on isentropic surfaces or variance of (potential) temperature on pressure surfaces scaled by the inverse of the static stability.

$$A = \frac{Ra^2}{2gp_o^\kappa}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_0^{2\pi}\int_0^{\infty}\widetilde{p}^{1+\kappa}\frac{\widetilde{p_\theta'^2}}{\widetilde{p}^2}\cos\phi d\theta d\lambda d\phi \approx \frac{a^2}{2g}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_0^{2\pi}\int_0^{p_{sfc}}\widetilde{\Gamma}\frac{T_p'^2}{\widetilde{T}_p}\cos\phi dp d\lambda d\phi$$

Lorenz Energy Cycle

$$K_m = \frac{1}{2}\int_m \left([u]^2 + [v]^2\right) dm$$
$$K_e = \frac{1}{2}\int_m \left(\left[u^{*2}\right] + \left[v^{*2}\right]\right) dm$$
$$A_m = \frac{1}{2}\int_m \widetilde{\Gamma}\frac{[T'^2]}{\widetilde{T}} dm$$
$$A_e = \frac{1}{2}\int_m \widetilde{\Gamma}\frac{[T'^{*2}]}{\widetilde{T}} dm$$

$$\frac{\partial}{\partial t}K_m = \int\left([v^*u^*]\frac{\partial}{a\partial\phi}[u] + [\omega^*u^*]\frac{\partial}{\partial p}[u] + \left[v^{*2}\right]\frac{\partial}{a\partial\phi}[v] + [\omega^*v^*]\frac{\partial}{\partial p}[v]\right)$$
$$-\int\frac{g[v]}{a}\frac{\partial[z]}{\partial\phi} + \int\left([u][F_u] + [v][F_v]\right)$$

$$\frac{\partial}{\partial t}K_e = -\int\left([v^*u^*]\frac{\partial}{a\partial\phi}[u] + [\omega^*u^*]\frac{\partial}{\partial p}[u] + \left[v^{*2}\right]\frac{\partial}{a\partial\phi}[v] + [\omega^*v^*]\frac{\partial}{\partial p}[v]\right) - \int\left(\frac{g}{a\cos\phi}\left[u^*\frac{\partial z^*}{\partial\lambda}\right] + \frac{g}{a}\left[v^*\frac{\partial z^*}{\partial\phi}\right]\right) + \int\left([u^*F_u^*] + [v^*F_v^*]\right)$$

$$\frac{\partial}{\partial t}A_m = \int\left([v^*T^*]\frac{\partial}{a\partial\phi}\left(\widetilde{\Gamma}\frac{[T']}{\widetilde{T}}\right) + [\omega^*T^*]\frac{\partial}{\partial p}\left(\widetilde{\Gamma}\frac{[T']}{\widetilde{T}}\right)\right) + \int[\alpha'][\omega] + \int\frac{\widetilde{\Gamma}}{c_p\widetilde{T}}[T']\left[\dot{Q}\right]$$

$$\frac{\partial}{\partial t}A_e = -\int\left([v^*T^*]\frac{\partial}{a\partial\phi}\left(\widetilde{\Gamma}\frac{[T']}{\widetilde{T}}\right) + [\omega^*T^*]\frac{\partial}{\partial p}\left(\widetilde{\Gamma}\frac{[T']}{\widetilde{T}}\right)\right) + \int[\alpha'^*\omega^*] + \int\frac{\widetilde{\Gamma}}{c_p\widetilde{T}}\left[T'^*\dot{Q}^*\right]$$

Note that $\int\frac{g[v]}{a}\frac{\partial[z]}{\partial\phi} = \int[\alpha'][\omega]$ and similarly for eddy component.

Conversion of A_m to K_m when mean flow down meridional pressure gradient or when thermally direct meridional circulation. Conversion from K_e to K_m when eddy momentum transport up mean gradient (eddies tilted with shear). Dissipation of K_m and K_e due to friction on respective flows. Conversion of A_e to K_e when eddy flow down pressure gradients or when thermally direct zonal circulation. Conversion of A_m to A_e when eddy heat transport down mean gradient (eddies enhancing warm and cold sectors). Generation of A_m through heating of warm latitudes and cooling of cold latitudes. Generation of A_e through heating of relatively warm longitudes and cooling of relatively cold longitudes.

Schematic of Lorenz energy cycle