Lab 1: Angular Momentum

ATM 622: General Circulation of the Atmosphere
September 11, 2019
DUE: September 18, 2019 (Theory) and September 27, 2019 (Lab)

I. Theory

Please hand in this section individually in class.

1) Show that the angular momentum of a fluid in cylindrical coordinates is

$$M = \Omega r^2 + rv \tag{1}$$

where Ω is the rotation rate of the tank, r is the radius, and v is the tangential velocity.

2) Derive a conservation equation for the angular momentum, i.e., $\frac{dM}{dt}$. The tangential momentum equation is

$$\frac{dv}{dt} = -\frac{1}{\rho r}\frac{\partial p}{\partial \theta} - \frac{uv}{r} - 2\Omega u + F_\theta \tag{2}$$

where ρ is the density, p is the pressure, θ is the azimuthal angle, u is the radial velocity, F_θ is the friction, and the total derivative is given by:

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \mathbf{u}\cdot\nabla = \frac{\partial}{\partial t} + u\frac{\partial}{\partial r} + \frac{v}{r}\frac{\partial}{\partial \theta} + w\frac{\partial}{\partial z} \tag{3}$$

Qualitatively describe what the terms represent in the conservation equation for the angular momentum.

3) Convert the advection term into flux form, using the fact that for an incompressible fluid $\nabla \cdot \mathbf{u} = 0$. Take the zonal and time average of the conservation equation. Assume a steady state and the divergence of the vertical flux of angular momentum is negligible. Under these set of assumptions what is the balance in the zonal and time mean angular momentum budget?

II. Lab

As a group, answer the questions and turn in one report via e-mail.

Materials: white baseboard, square tank, circular insert, and inner metal cylinder.

Fill the tank and inner metal cylinder about halfway with water. Adjust the rotation rate based on that assigned to your group. Spin up to solid body rotation. Adjust the settings in the particle tracker.

Place the ice pack in the inner cylinder to initiate the experiment and wait 10 minutes. Then evenly scatter about 6 tracers in the tank. Track these tracers.

Save good tracks[1]. **You want approximately the same number of cyclonic and anti-cyclonic trajectories as to not bias the averaging. You also want to sample the flow over the full range of radii.** Approximately 15-20 total trajectories should suffice.

After you have collected your trajectories, **sprinkle a small amount** of permanganate crystals at a few radii and azimuths. Qualitatively note regions of surface westerlies and easterlies as the permanganate spreads out on the bottom. Take a few pictures.

4) Plot your trajectories.

5) Convert your Cartesian data into radius, azimuth, radial velocity, and tangential velocity. You will likely have to quality control your data to eliminate unreasonable velocities (>1 cm/s). Conduct a zonal (azimuthal) and time average by binning and averaging your velocities by radius in intervals of 2 cm. Construct plots of $[\overline{u}]$,$[\overline{v}]$, $\left[\overline{M}\right]$, and $\left[\overline{uM}\right]$ as a function of radius. Describe what you observe from your data in terms of the location or zonal jets, the radial distribution of angular momentum, and the radial flux of angular momentum.

6) Over what radii is there a flux convergence/divergence of angular momentum? What is the sign of the frictional torque against the metal cylinder? What does this imply about the frictional torque and the flow elsewhere in the tank if the total volume integrated angular momentum of the fluid is constant? Do your observations of the surface flow or your velocity data support your assertion?

7) We can define a decomposition that allows us to calculate the mean radial and combined eddy components:

$$\left[\overline{uM}\right] = [\overline{u}]\left[\overline{M}\right] + \left[\overline{u'^*M'^*}\right] \tag{4}$$

where the first term on the RHS is the mean radial component (analogous to the mean meridional component) and the second term on the RHS is the combined eddy component.

Construct plots of both components and comment on their relative contributions. Sketch a diagram of how the mean radial and eddy components redistribute angular momentum and how the frictional torques along the sides and bottom of the tank are acting as sources/sinks of angular momentum.

8) Lastly, make the connection with the real atmosphere. How does this lab mimic the zonal and time average meridional transport of angular momentum in the atmosphere?

[1] You may have trouble with the tracers sticking to the sides and clumping. If all the tracers misbehave, you may need to blow on them to get them unstuck, or carefully move them with the eraser end of a pencil. If so, wait about a minute before tracking them to let any turbulence dissipate.