Lab 2: Hadley Circulation

ATM 622: General Circulation of the Atmosphere
October 30, 2019
DUE: November 11, 2019 (Theory) and December 6, 2019 (Lab)

I. Theory

Please hand in this section individually.

1) Derive an expression for the tangential velocity at the top of the tank, $v(z = d)$, using the following equations for gradient wind balance and hydrostatic balance for a Boussinesq fluid in cylindrical coordinates:

$$2\Omega v + \frac{v^2}{r} = \frac{\partial \Phi}{\partial r}, \tag{1}$$

$$\frac{\partial \Phi}{\partial z} = g\beta_T \left(T - T_o\right), \tag{2}$$

where Ω is the rotation rate of the tank, v is the tangential velocity, r is the radius, z is the height, $\Phi = p/\rho_o$, p is the pressure, ρ_o is a reference density, g is the gravitational acceleration, β_T is the expansion coefficient of water, T is the temperature, and T_o is a reference temperature. Assume that the radial temperature gradient is constant:

$$\frac{\partial T}{\partial r} = \frac{\Delta T}{b - a}, \tag{3}$$

where ΔT is the temperature difference across the tank, b is the outer radius of the tank, and a is the inner radius of the tank. Also assume that

$$v(z = 0) = 0. \tag{4}$$

2) Rewrite your expression for the tangential velocity at the top of the tank in terms of a thermal Rossby number. In this framework, the thermal Rossby number is

$$\mathrm{Ro_T} = \frac{g\beta_T d\Delta T}{\Omega^2 r(b - a)}. \tag{5}$$

How does the tangential velocity vary as a function of $\mathrm{Ro_T}$?

3) The Eady model for baroclinic instability has a shortwave cutoff, λ_c, given by

$$\lambda_c = \frac{1.31 N d}{\Omega}, \tag{6}$$

where N is the Brunt-Vaisala frequency, which may be expressed as

$$N = \sqrt{g\beta_T \frac{\Delta T}{d}}. \tag{7}$$

We have assumed that the vertical temperature difference from the top to the bottom of the tank is the same as the radial temperature difference across the tank. Any wave mode with $\lambda < \lambda_c$ cannot grow via linear instability. The wavelength of a particular mode is given by

$$\lambda = \frac{2\pi r}{m} \quad (m \neq 0), \tag{8}$$

where m is the azimuthal wavenumber. Beginning with the condition that $\lambda < \lambda_c$, derive a stability constraint for $\mathrm{Ro_T}$ that is a function of r and m, i.e., $\mathrm{Ro_T} >?$.

For a given m, if $\mathrm{Ro_T}$ is above the critical value such that the stability constraint is true, instability cannot exist. For $m = 1$, this means that only a Hadley circulation in thermal wind balance is possible. Once $\mathrm{Ro_T}$ drops below the critical value, baroclinic instability is possible.

II. Lab

As a group, answer the questions and turn in one report.

Materials: white baseboard, square tank, circular insert, glass beaker, two digital thermometers.

If not already setup, use two binder clips to modify the circular insert such that it has a diameter of about 26 cm. The binder clips should be both on the top of the insert. Record a, the radius of the trunk of the beaker (not the lip of the beaker), and b, the approximate radius of the circular insert.

Tape one of the digital thermometers to the outer side of the beaker such that the sensor is flush with the bottom. This will be your temperature measurement at a. Tape the other digital thermometer to the inner side of the circular insert, such that the sensor is 7 cm above the bottom. This will be your temperature measurement at b.

Position the beaker in the center of the tank. Fill the **beaker** about half way with water. Position the circular insert so that it is symmetric around the beaker as best you can.

Fill the volume outside the beaker with warm water to a depth of about 10 cm. To do this, fill the tank with water from the faucet to about a depth of 6–7 cm. Use the kettle to fill the remaining 3–4 cm with boiling water. You will have to fill the kettle about 3–4 times. After filling the tank, take your ruler and stir the water around to make sure the temperature is homogenized. If the circular insert moved any during this process, readjust it so that it is again symmetric around the beaker. Record d, the depth of the water.

Add ice to the beaker at this point in time.

Set the rotation rate of the tank to the slowest possible value by pulling the motor position all the way out and turning the dial slowly until the tank starts to rotate. This should result in a rotation rate around 2.5–2.7 rpm. Record Ω, the rotation rate of the tank.

Solid body rotation will take about 15 minutes to reach. Simultaneously, ΔT will increase between a and b. You want a ΔT of at least 10 K. Keep adding ice to the beaker if the ice completely melts during the spin up to solid body rotation, but not thereafter. After 15 minutes, you will begin collecting data. Place two drops of food coloring in the annulus between the beaker and circular insert.

Every five minutes, record ΔT, v, and the dominant wavenumber m. ΔT is the difference in temperature between the outer and inner thermometers. To estimate v, drop a few stars in the annulus and time one revolution of the innermost (fastest moving) particle in the rotating frame of reference. You will also need to take a picture of the TV screen in order to estimate the radius of the particle as well, which is needed to calculate v[1]. Visually estimate m by assessing the number of troughs and ridges in the dye pattern. Note that there may be combinations of wavenumbers present ($m = 0$ and $m = 2$, for instance). Trough/ridge patterns may be subtle, but there should be a dominant wavenumber present with perhaps a secondary wavenumber at times. As the food coloring becomes diffuse, add another one or two drops of food coloring. It may help to alternate colors as well. Continue recording data every five minutes until ΔT decreases to 0.5–0.8 K, when you do not see any additional change occurring.

4) Use your data to calculate a theoretical tangential velocity based on your expression from problem 1). Note that $\beta_T = 2 \times 10^{-4}$ K^{-1}. How do your theoretical and observed tangential velocities compare? If there are large discrepancies, what may be possible sources of error?

5) Create a contour plot of $\mathrm{Ro_T}$ as a function of r and ΔT, using ranges based on your experiment. On the same figure, plot a trajectory of your recorded ΔT values at fixed $r = 0.5(a+b)$. Along the trajectory, use different symbols or labels for the observed wavenumber(s). Create a separate plot of critical values of $\mathrm{Ro_T}$ as a function of m, with $r = 0.5(a+b)$. Explain why lower wavenumbers manifest first as ΔT decreases. Do your observed m coincide with the stable/unstable regimes for each respective wavenumber? If not, hypothesize why. (*Hint: Viscosity (diffusive processes) suppress instability. What else is lacking in the theory that might be important in the tank?*)

6) Physically explain why baroclinic instability is suppressed when the thermal Rossby number is too large (*Hint: Think about the Eady model.*) How does the thermal Rossby number relate to the amount of mean available potential energy and consequently the vigor of the overturning circulation?

[1]We are not using the particle tracker for this experiment because of the apparatus clutter and dye.