MATH: VECTOR-ALGEBRA FUNDAMENTALS

Vector of dimension n:
n-tuples of real numbers

$$\vec{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{pmatrix}$$

Vector addition:

$$\vec{x} + \vec{y} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{pmatrix} + \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ \vdots \\ y_n \end{pmatrix} = \begin{pmatrix} x_1 + y_1 \\ x_2 + y_2 \\ x_3 + y_3 \\ \vdots \\ x_n + y_n \end{pmatrix}$$

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Vectors in the R-Language

```
> x<-c(1,2,4,8,16,32)
> y<-c(1,2,3,4,5,6)
> x+y
[1]  2  4  7 12 21 38
>
```

MATH: VECTOR-ALGEBRA FUNDAMENTALS

Scaling of a vector: multiplication with a scalar (a real number)

$$a\vec{x} = a \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} ax_1 \\ ax_2 \\ ax_3 \\ \vdots \\ ax_n \end{pmatrix}$$

Vectors in the R-Language

```
>
> x<-c(1,2,4,8,16,32)
> a<-2
> a*x
[1]  2  4  8 16 32 64
> |
```

MATH: VECTOR-ALGEBRA FUNDAMENTALS

The magnitude of a vector:

$$|\vec{x}| = \sqrt{\sum_{i=1}^{n} x_i^2} = \sqrt{x_1^2 + x_2^2 + x_3^2 + \ldots + x_n^2}$$

The dot-product of two vectors (inner product)

$$\vec{x} \cdot \vec{y} = \sum_{i=1}^{n} x_i y_i = x_1 y_1 + x_2 y_2 + x_3 y_3 + \ldots + x_n y_n$$

Vectors in the R-Language

```
> x<-c(1,2,4,8,16,32)
> y<-c(1,2,1,-2,0.25,0.125)
> x*y
[1]   1   4   4 -16   4   4
```

MATH: VECTOR-ALGEBRA FUNDAMENTALS

```
> magnitude(x)
Error: could not find function "magnitude"
>
```

R has not a function for this !?

The magnitude of a vector:

$$|\vec{x}| = \sqrt{\sum_{i=1}^{n} x_i^2} = \sqrt{x_1^2 + x_2^2 + x_3^2 + \ldots + x_n^2}$$

The dot-product of two vectors (inner product)

$$\vec{x} \cdot \vec{y} = \sum_{i=1}^{n} x_i y_i = x_1 y_1 + x_2 y_2 + x_3 y_3 + \ldots + x_n y_n$$

Exercise: Write your own function 'magnitude'

Vectors in the R-Language

```
> x<-c(1,2,4,8,16,32)
> y<-c(1,2,1,-2,0.25,0.125)
> x*y
[1]   1   4   4 -16   4   4
```

x*y is not a dot-product operation in R

MATH: VECTOR-ALGEBRA FUNDAMENTALS

The magnitude of a vector:

$$|\vec{x}| = \sqrt{\sum_{i=1}^{n} x_i^2} = \sqrt{x_1^2 + x_2^2 + x_3^2 + \ldots + x_n^2}$$

Vectors in the R-Language

```
# vector magnitude
magnitude<-function(x){
  n<-length(x)
  res<-0
  for (i in 1:n){
    res<-res+x[i]^2
  }
  return(sqrt(res))
}
```

```
> x<-c(1,1,1)
> magnitude(x)
[1] 1.732051
>
```

MATH: VECTOR-ALGEBRA FUNDAMENTALS

The magnitude of a vector:

$$|\vec{x}| = \sqrt{\sum_{i=1}^{n} x_i^2} = \sqrt{x_1^2 + x_2^2 + x_3^2 + \ldots + x_n^2}$$

Vectors in the R-Language

```
# vector magnitude
magnitude<-function(x){
  n<-length(x)
  res<-0
  for (i in 1:n){
    res<-res+x[i]^2
  }
  return(sqrt(res))
}
```

Another (and faster way)to calculate the magnitude: we use x*x and R's built-in sum() function

```
magnitude2<-function(x){
  x<-x*x
  return(sqrt(sum(x)))
}
```

Note: see scripts/vectorfunctions.R

MATH: VECTOR-ALGEBRA FUNDAMENTALS

The dot-product of two vectors
(inner product)

$$\vec{x} \cdot \vec{y} = \sum_{i=1}^{n} x_i y_i = x_1 y_1 + x_2 y_2 + x_3 y_3 + \ldots + x_n y_n$$

Vectors in the R-Language

```
dotproduct<-function(x,y){
  n<-length(x)
  res<-0
  for (i in 1:n){
    res<-res+x[i]*y[i]
  }
  return(res)
}
```

```
> x<-c(1,2,4,8,16,32)
> y<-c(-1,-1,-1,-1,-1,1)
> dotproduct(x,y)
[1] 1
>
```

Note: see scripts/vectorfunctions.R

MATH: VECTOR-ALGEBRA FUNDAMENTALS

The dot-product of two vectors
(inner product)

$$\vec{x} \cdot \vec{y} = \sum_{i=1}^{n} x_i y_i = x_1 y_1 + x_2 y_2 + x_3 y_3 + \ldots + x_n y_n$$

Vectors in the R-Language

```
dotproduct<-function(x,y){
  n<-length(x)
  n2<-length(y)
  if (n2!=n){
    res<-NA
  } else {
    res<-0
    for (i in 1:n){
      res<-res+x[i]*y[i]
    }
  }
  return(res)
}
```

This is a safer function:
If vectors have different lengths, dot-product is undefined and NA is returned.

Note: see scripts/vectorfunctions.R

CALCULATING ALBANY AIRPORT MONTHLY MEAN TEMPERATURE ANOMALIES

After executing the script albany_climatology.R we have the vector named 'tavgclim'

```
> length(tavgclim)
[1] 12
>
```

```
> tavgclim
 [1] 22.79997 26.12960 35.07461 47.71130
 [5] 58.23065 67.12530 71.66252 70.06865
 [9] 61.88550 49.82184 39.95430 28.62132
>
```

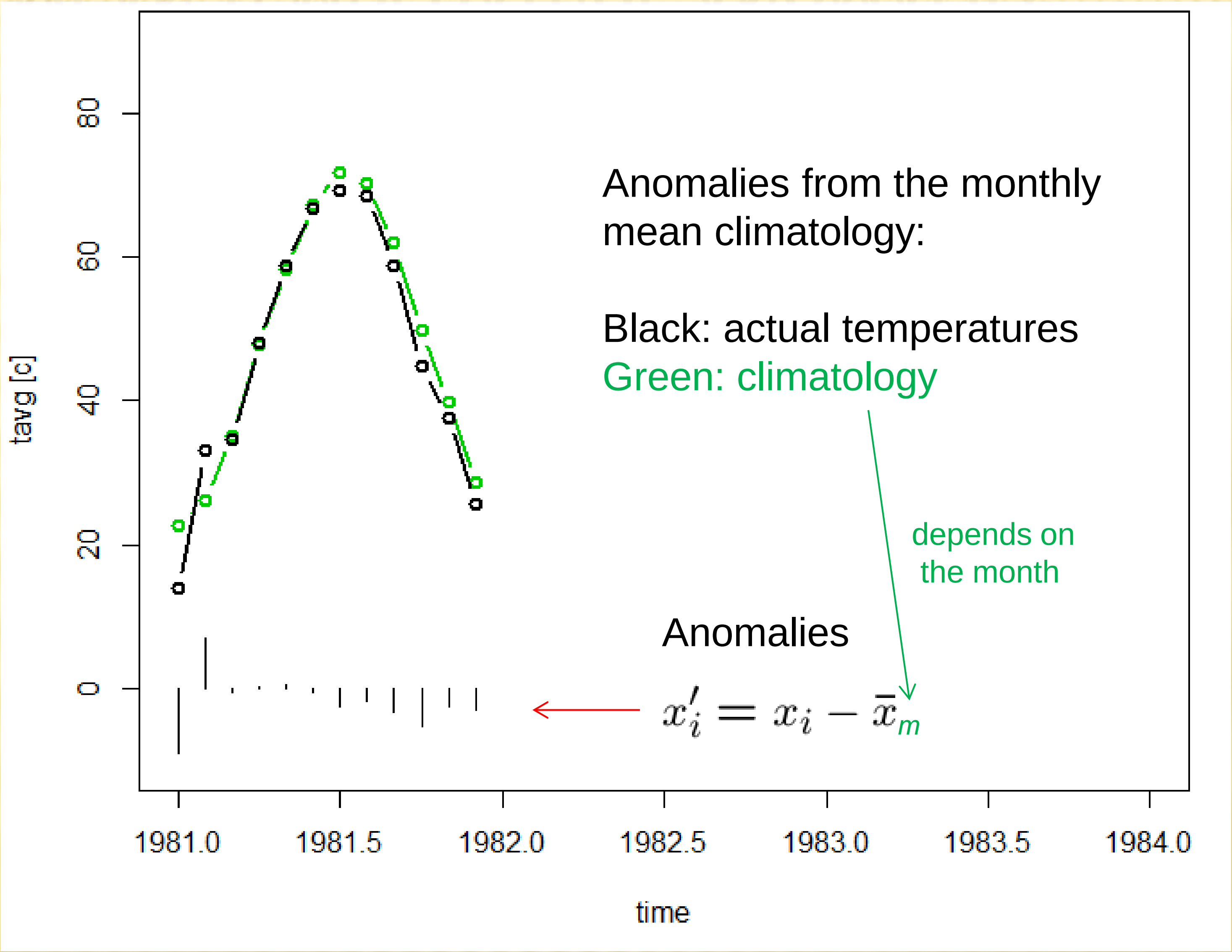
The monthly mean data 1981-2010 are stored in a vector named 'buffer'

```
> length(buffer)
[1] 360
>
```

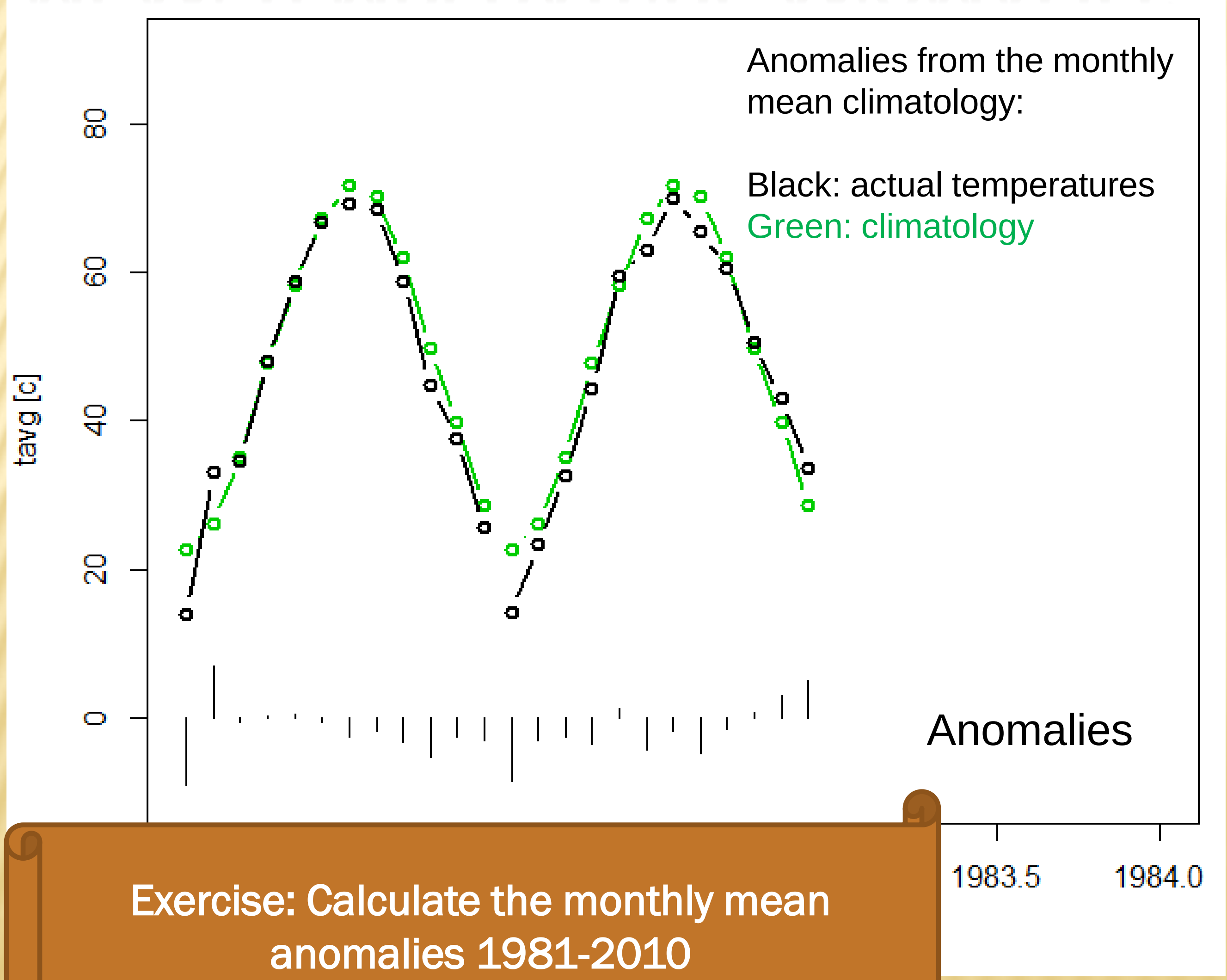
The monthly mean climatology has only 12 values. Anomalies are deviations from the mean

$$x_i' = x_i - \bar{x}$$

CALCULATING ALBANY AIRPORT MONTHLY MEAN TEMPERATURE ANOMALIES

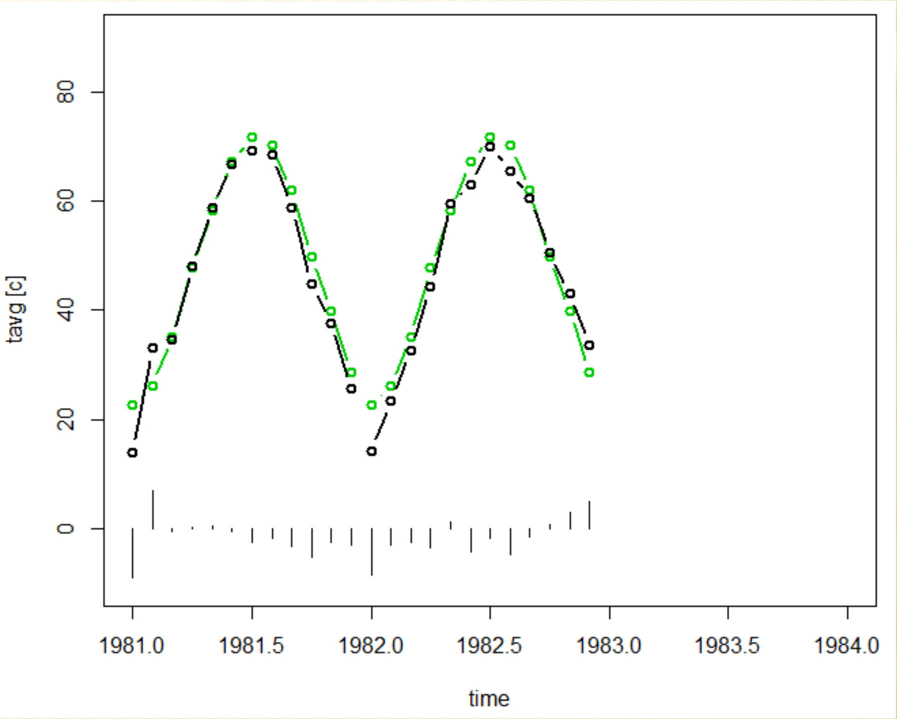


CALCULATING ALBANY AIRPORT MONTHLY MEAN TEMPERATURE ANOMALIES



Exercise: Calculate the monthly mean anomalies 1981-2010

CALCULATING ALBANY AIRPORT MONTHLY MEAN TEMPERATURE ANOMALIES

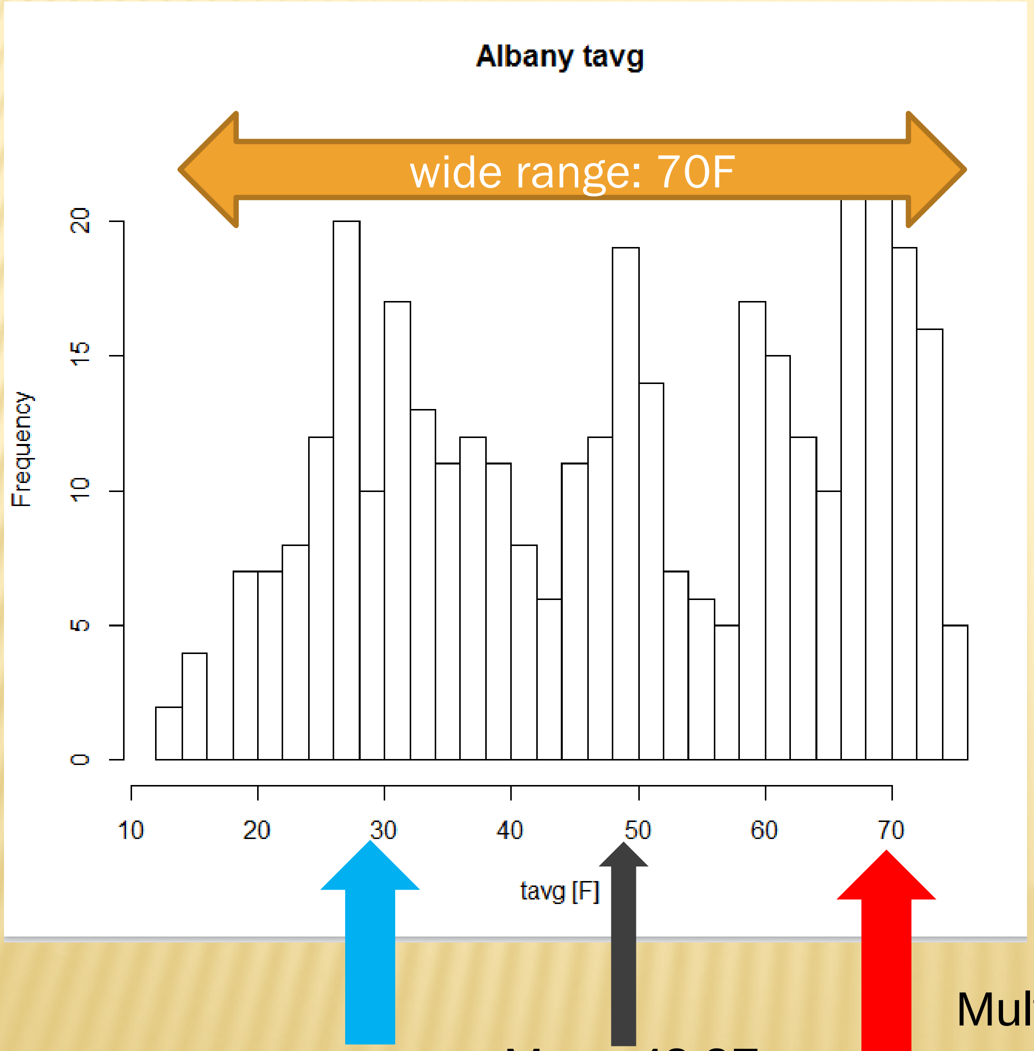


R-commands in the command console

(albany_climatology.R must have been executed
Before)

```
> plot(c(1981,1984),c(-10,90),xlab='time',ylab='tavg [c]',typ='n')
> lines(thelp[1:12],tavgclim[1:12],col=3,typ='b',lwd=2)
> lines(thelp[1:12],buffer[1:12],typ='b',lwd=2)
> lines(thelp[1:12],buffer[1:12]-tavgclim[1:12],typ='h',lwd=1,col=1)
> lines(thelp[13:24],tavgclim[1:12],col=3,typ='b',lwd=2)
> lines(thelp[13:24],buffer[13:24],typ='b',lwd=2)
> lines(thelp[13:24],buffer[13:24]-tavgclim[1:12],typ='h',lwd=1,col=1)
> 
```

MONTHLY MEAN ANOMALIES



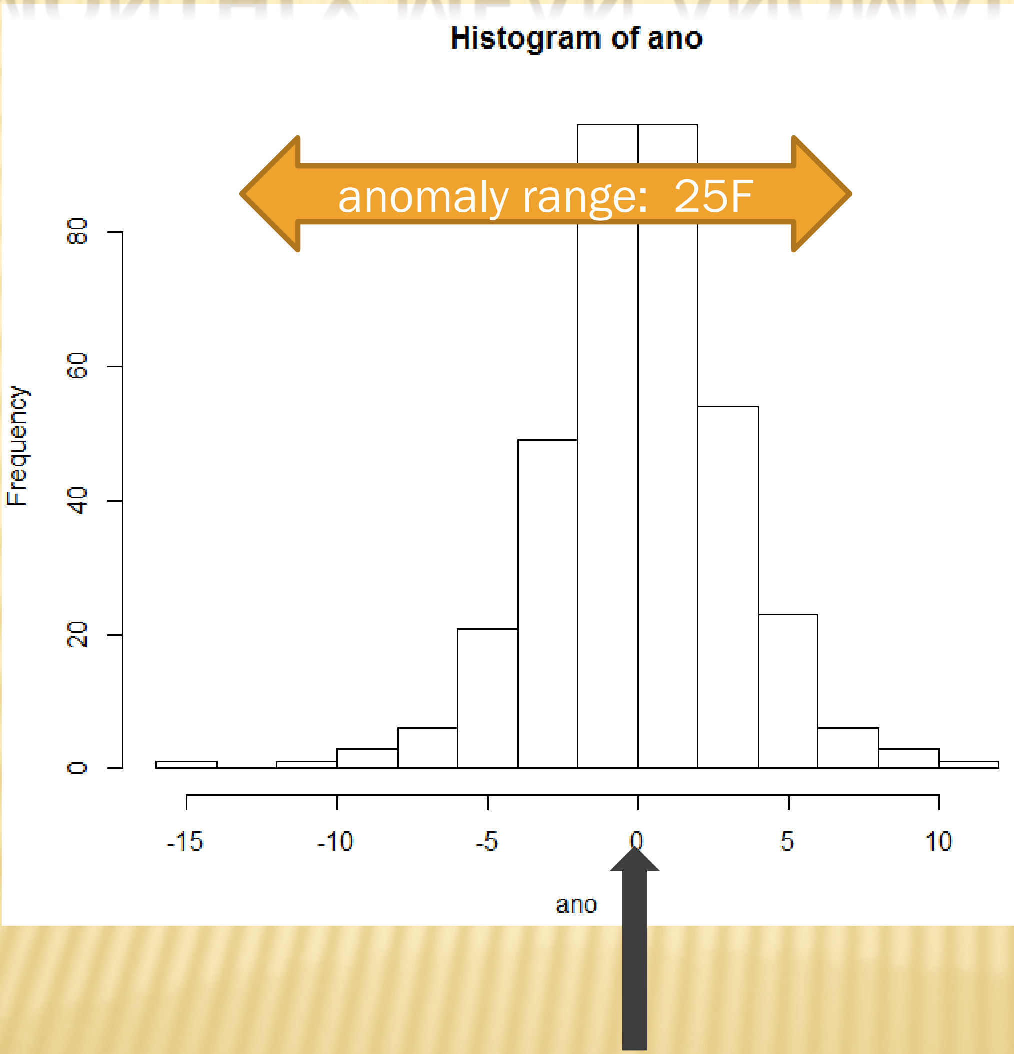
The seasonal temperature cycle makes the data analysis of the random fluctuations around the expected mean climatological cycle difficult.

The standard deviation measures 17.2F

Multiple centers (cold warm seasons)

Mean 48.3F

MONTHLY MEAN ANOMALIES

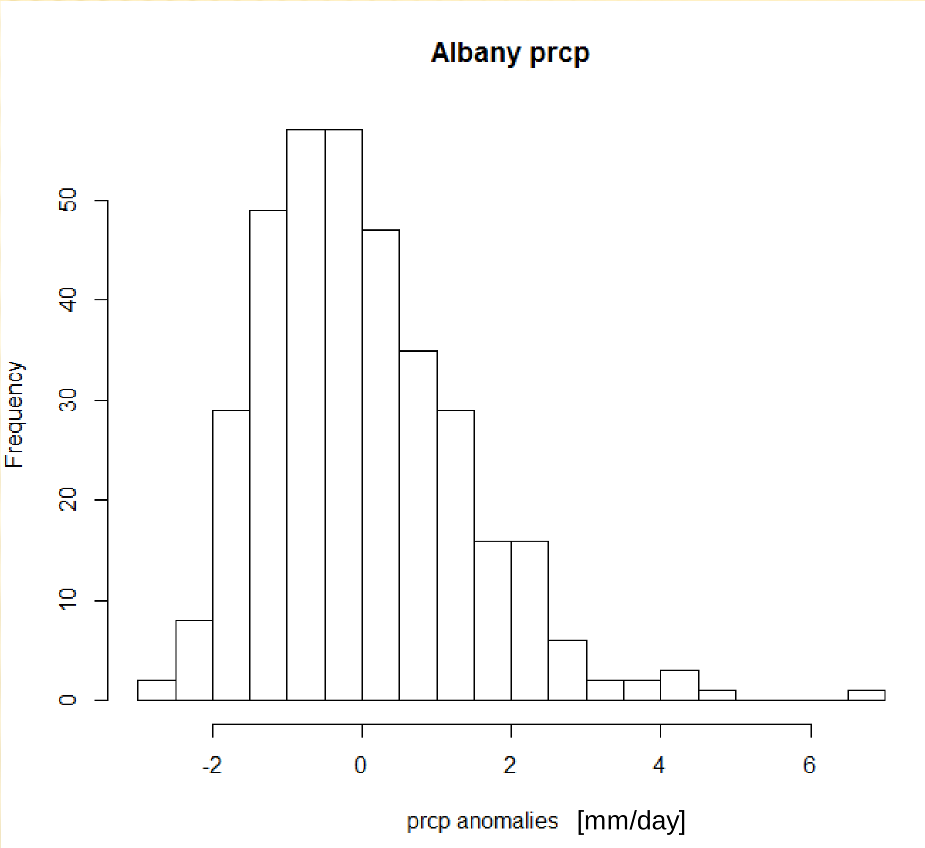


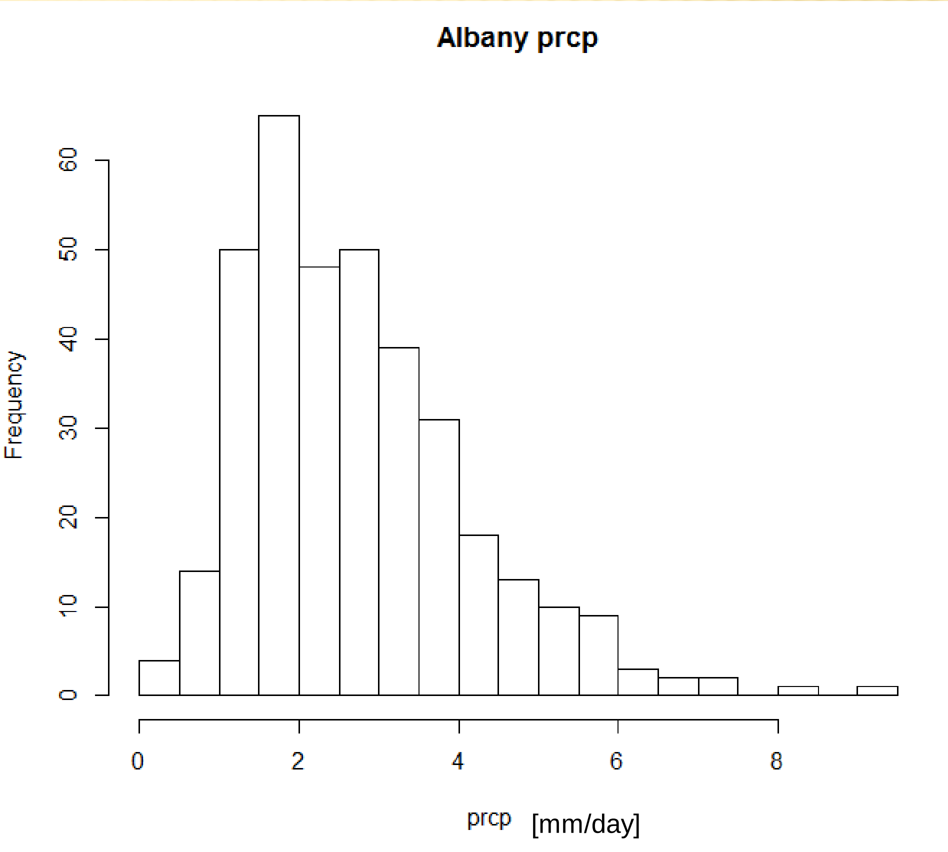
This histogram is showing now the distribution of the monthly mean temperature anomalies of all 360 months 1981-2010 (station Albany Airport).

The standard deviation is 3.1F for the monthly mean anomalies

Centered around 0 (one center in the distribution)

MONTHLY MEAN ANOMALIES





Skewed distribution with long tail to the right

The seasonal precipitation cycle is not large compared with the month to month anomalies. Note the correct units are: mm per day.

PITOT TUBE

Henri Pitot (1695–1771)

Pitot tube on a modern Airbus plane

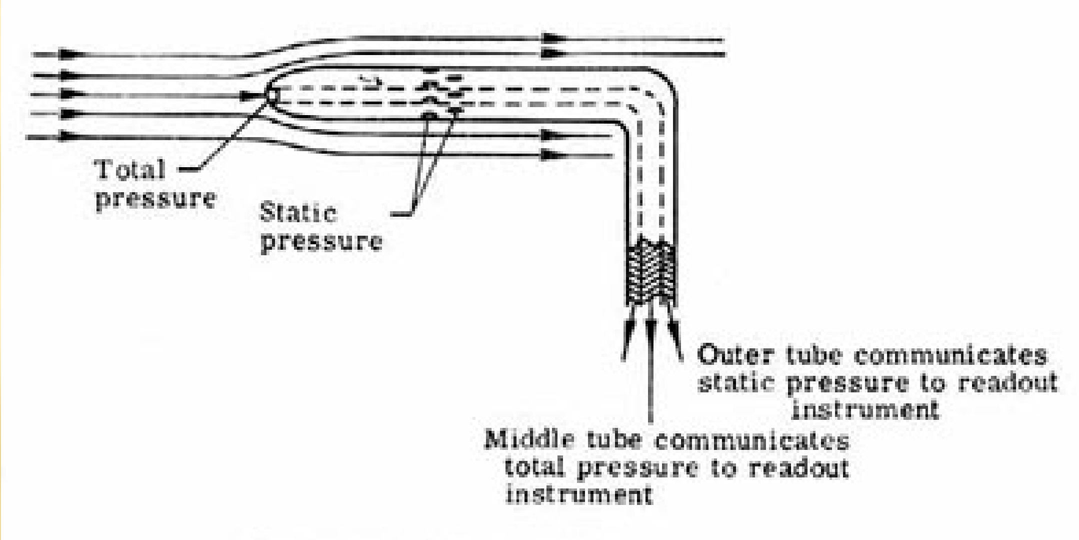
- Measures the speed of a fluid
- Bernoulli's Law:

Stagnation pressure = static pressure + dynamic pressure

$$p_t = p_s + \left(\frac{\rho V^2}{2}\right)$$

$$\Rightarrow V = \sqrt{\frac{2(p_t - p_s)}{\rho}}$$

ρ density of fluid



Source: Wikipedia http://en.wikipedia.org/wiki/Pitot_tube, images retrieved Feb. 2014

http://www.daviddarling.info/encyclopedia/P/pitot_tube.html

WHAT WAS THE AVERAGE WIND SPEED IF WE HAD AVERAGED PRESSURE READINGS?

Consider you had 3 readings from the Pitot Tube of the pressure difference between dynamic and static pressure, but only the average was reported.

$\Delta p = g\rho\Delta h$

Observations: every minute one reading

Variable	Obs. 1	Obs. 2	Obs. 3	mean
Delta p	No information			8
V				???

If the instrument returned only the 3-minute average

WHAT WAS THE AVERAGE WIND SPEED IF WE HAD AVERAGED PRESSURE READINGS?

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$\Delta p = g\rho\Delta h$

Observations: every minute one reading

Variable	Obs. 1	Obs. 2	Obs. 3	mean
Delta p	No information			8
C*V				sqrt(8) =2.83

Note: the conversion factor C from pressure to wind-speed units is a constant instrument/fluid-specific factor

WHAT WAS THE AVERAGE WIND SPEED IF WE HAD PRESSURE READINGS?

Consider you had 3 readings from the Pitot Tube of the pressure difference between dynamic and static pressure

$$\Delta p = g\rho\Delta h$$

Observations every minute, one reading

Variable	Obs. 1	Obs. 2	Obs. 3	mean
Delta p	4	16	8	9.33
C*V	No direct measurements			sqrt(9.33) =3.06

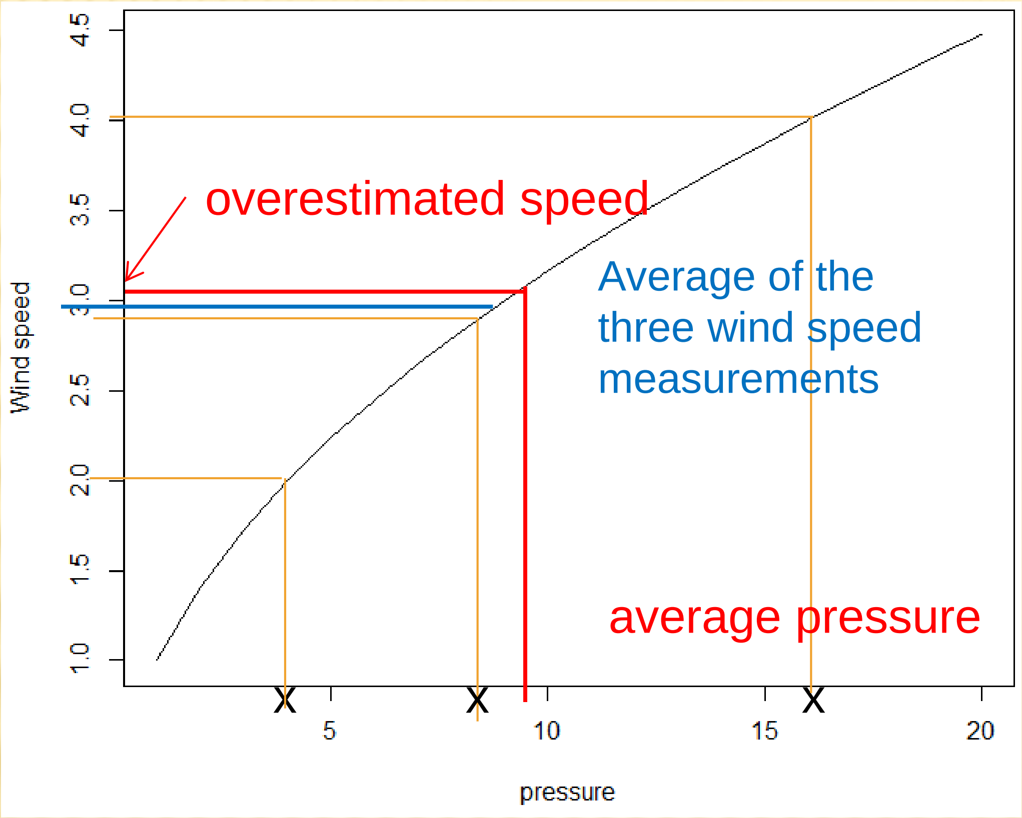
WHAT WAS THE AVERAGE WIND SPEED IF WE HAD PRESSURE READINGS?

Consider you had 3 readings from the Pitot Tube of the pressure difference between dynamic and static pressure

$$\Delta p = g\rho\Delta h$$

Variable	Obs. 1	Obs. 2	Obs. 3	mean
Delta p	4	16	8	9.3
V	2	4	2.83	2.94

Non-linear relationship between observed variable and variable of interest:



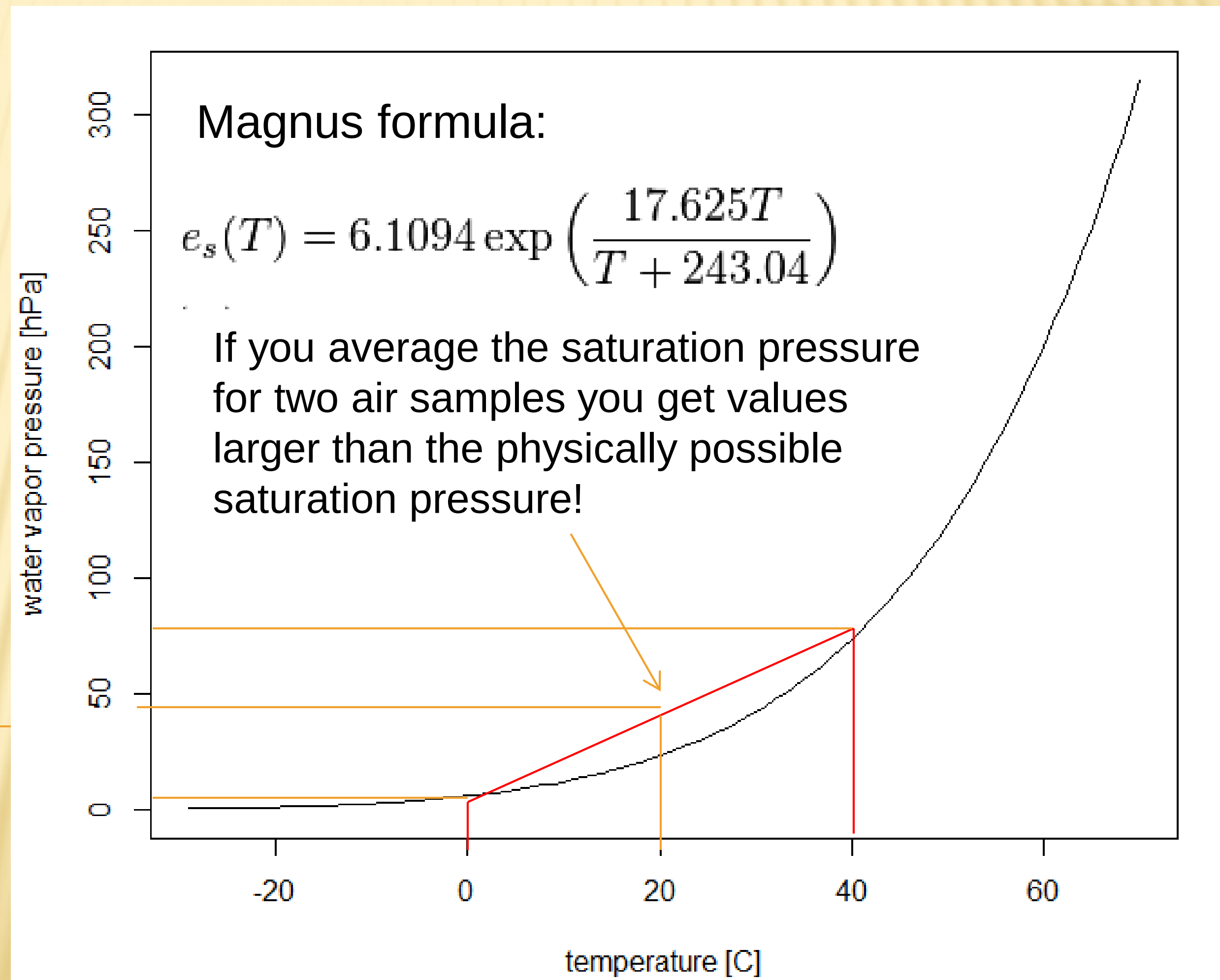
We would overestimate the 3-minute average wind speed if we averaged the pressure observations.

Non-linear transformation of Observations change the statistical Estimates such as the mean, standard deviation, and actually the histogram and shape of the sample distributions

Not always are the differences in the mean so subtle!

Non-linear relationship between observed variable and variable of interest:

Based on the Clausius–Clapeyron equation the saturation pressure of water vapor in the atmosphere is an exponential function of the air temperature (T in Celsius):



Non-linear relationship between observed variable and variable of interest: