Development of the QG Vorticity Equation

- Start w/ the full vorticity eqn developed in class:

$$\frac{\partial \zeta}{\partial t} = -\vec{V}\cdot\nabla\zeta - v\beta - (\nabla\cdot\vec{V})f - (\nabla\cdot\vec{V})\zeta - \hat{k}\cdot\nabla w \times \frac{\partial \vec{v}}{\partial z} + \hat{k}\cdot(\nabla p \times \nabla \alpha) + \hat{k}\cdot\nabla\times\vec{F}$$

QG Assumptions:

- $f \equiv f_o + \beta y$
- $\vec{V}_{ag} << \vec{V}_g$, so $\vec{V}$ is replaced by $\vec{V}_g$ in advection terms
- $\nabla\cdot\vec{V} \equiv \nabla\cdot\vec{V}_{ag} = -\frac{\partial w}{\partial z}$ (recall that $\nabla\cdot\vec{V}_g = 0$)

  in pressure coordinates:

$$\nabla\cdot\vec{V}_{ag} = -\frac{\partial \omega}{\partial p} \qquad \left[\omega \equiv \frac{dp}{dt}\right]$$

- with the above assumptions and some scale analysis, the following terms are eliminated:

$$\boxed{-(\nabla\cdot\vec{V})\zeta} \qquad \boxed{-\hat{k}\cdot\nabla w \times \frac{\partial \vec{v}}{\partial z}} \qquad \boxed{\hat{k}\cdot(\nabla p \times \nabla\alpha)}$$

→ **QG Vorticity Eqn** (in pressure coordinates) is:

$$\underbrace{\frac{\partial \zeta_g}{\partial t}}_{\text{local tend. of geo. relative vorticity}} = \underbrace{-\vec{V}_g\cdot\nabla(\zeta_g + f)}_{\text{advection of absolute geostrophic vorticity by geo. wind}} + \underbrace{f_o \frac{\partial \omega}{\partial p}}_{\text{stretching of planetary vorticity}} + \underbrace{\hat{k}\cdot\vec{\nabla}\times\vec{F}}_{\text{vertical component of the curl of frictional force (important only in boundary layer)}}$$

- geostrophic relative vorticity can be written as $\frac{1}{f_o}\nabla_p^2\Phi$

  (note: $\nabla_p^2\Phi \propto -\Phi$)

  → sub in $\frac{1}{f_o}\nabla_p^2\Phi$ for ζ_g and multiply by f_o to get an alternative form of the QG Vorticity Eqn:

$$\nabla_p^2\left[\frac{\partial \Phi}{\partial t}\right] = -f_o\vec{V}_g\cdot\nabla\left[\frac{1}{f_o}\nabla_p^2\Phi + f\right] + f_o^2\frac{\partial \omega}{\partial p} + f_o\hat{k}\cdot\nabla\times\vec{F}$$