# The Concept of Thermal Vorticity



- thermal vorticity is analogous to thermal wind
- thermal wind: geostrophic wind at some upper level minus geostrophic wind at lower level

$$\vec{V}_T = \vec{V}_{g2} - \vec{V}_{g1}$$

- thermal vorticity: geostrophic vorticity at some upper level minus geostrophic vorticity at lower level

$$\zeta_T = \zeta_{g2} - \zeta_{g1}$$

Proof: start with $\zeta_g = \frac{\partial v_g}{\partial x} - \frac{\partial u_g}{\partial y}$

- take $-\frac{\partial}{\partial p}$

$$-\frac{\partial \zeta_g}{\partial p} = -\frac{\partial}{\partial p}\left(\frac{\partial v_g}{\partial x}\right) + \frac{\partial}{\partial p}\left(\frac{\partial u_g}{\partial y}\right)$$

$$= \frac{\partial}{\partial x}\left(-\frac{\partial v_g}{\partial p}\right) - \frac{\partial}{\partial y}\left(-\frac{\partial u_g}{\partial p}\right)$$

- recall that $V_T = \frac{-\partial \vec{V}_g}{\partial p} = \vec{V}_{g2} - \vec{V}_{g1}$

$$\rightarrow \zeta_T = -\frac{\zeta_g}{\partial p} = \frac{\partial v_T}{\partial x} - \frac{\partial u_T}{\partial y} = \zeta_{g2} - \zeta_{g1}$$

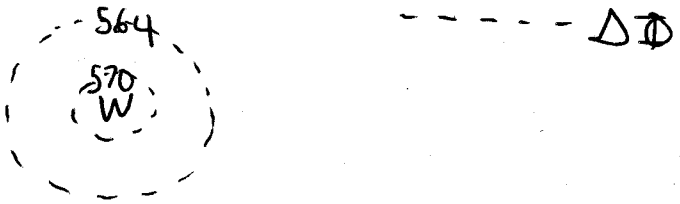
→ $\zeta_T$ is the vorticity of the thermal wind

$$V_T = \frac{1}{f}\hat{k} \times \nabla(\Phi_2 - \Phi_1)$$

## concept of cold/warm core systems

warm core: warmest air is at center of system
cold core: coldest air "

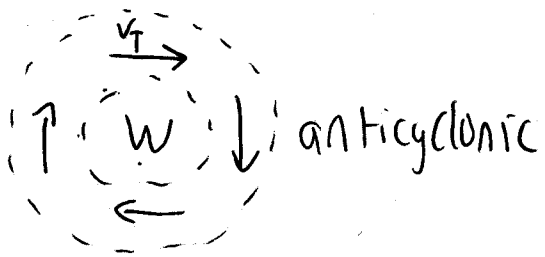
### warm core



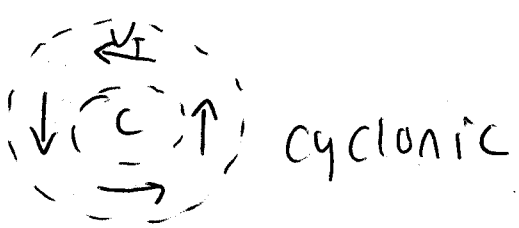

### cold core


• can draw $\vec{V}_T$ for each to determine whether thermal vorticity (vorticity of thickness field) is cyclonic or anticyclonic

### warm core




### cold core



warm core: $V_T$ is anticyclonic

$\rightarrow \zeta_T < 0$ (in NH)

cold core: $V_T$ is cyclonic

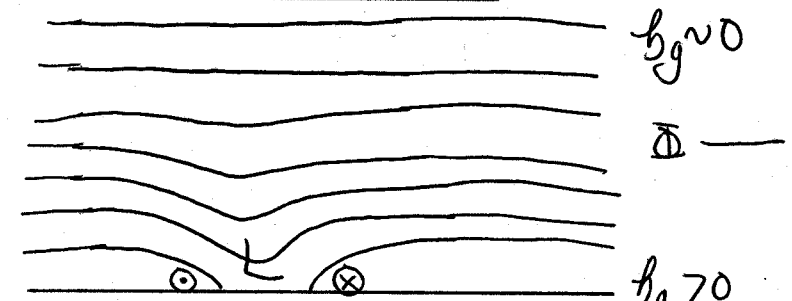
$\rightarrow \zeta_T > 0$ (in NH)

- for all warm core systems, $\zeta_T < 0$

  $\rightarrow$ means $\zeta_{g_2} - \zeta_{g_1} < 0$

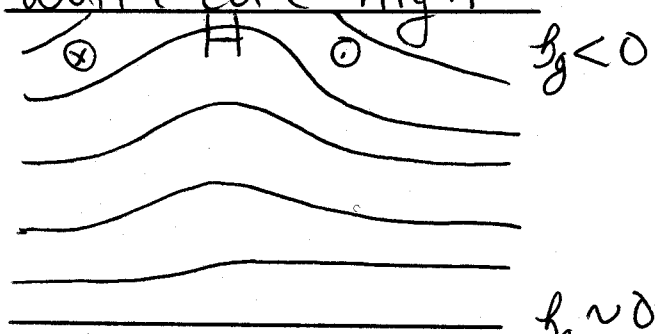
  $\rightarrow$ $\zeta_g$ must decrease with height!

warm core low




$\zeta_g$ decreases with height

warm core high



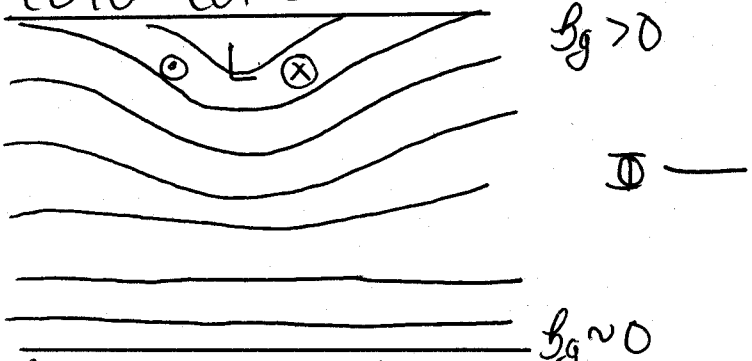

$\zeta_g$ decreases with height

- for all cold core systems, $\zeta_T > 0$

  $\rightarrow$ means $\zeta_{g_2} - \zeta_{g_1} > 0$

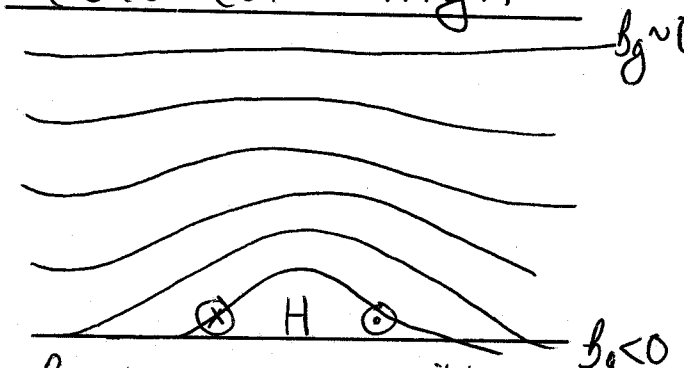
  $\rightarrow$ $\zeta_g$ must increase with height!

cold core low




$\zeta_g$ increases with height

cold core high




$\zeta_g$ increases with height