Review of the Total Derivative

- consider a quantity (scalar or vector) that is a function of space and time:

$$Q = Q(x, y, z, t)$$

- the total differential, dQ, is

$$dQ = \frac{\partial Q}{\partial x} dx + \frac{\partial Q}{\partial y} dy + \frac{\partial Q}{\partial z} dz + \frac{\partial Q}{\partial t} dt$$

- divide both sides by dt to get time derivative of Q:

$$\frac{dQ}{dt} = \frac{\partial Q}{\partial x} \underbrace{\frac{dx}{dt}}_{u} + \frac{\partial Q}{\partial y} \underbrace{\frac{dy}{dt}}_{v} + \frac{\partial Q}{\partial z} \underbrace{\frac{dz}{dt}}_{w} + \frac{\partial Q}{\partial t} \cancelto{1}{\frac{dt}{dt}}$$

$$\rightarrow \frac{dQ}{dt} = \underbrace{u\frac{\partial Q}{\partial x} + v\frac{\partial Q}{\partial y} + w\frac{\partial Q}{\partial z}}_{\vec{V}\cdot\vec{\nabla}Q} + \frac{\partial Q}{\partial t}$$

$$\frac{dQ}{dt} = \frac{\partial Q}{\partial t} + \vec{V}\cdot\vec{\nabla}Q$$

total derivative = local tendency + advection

Eularian framework: $\frac{dQ}{dt}$: rate of change following parcel

Lagrangian framework: $\frac{\partial Q}{\partial t}$: rate of change at a fixed point (local rate of change)

$$\frac{\partial Q}{dt} = \frac{dQ}{dt} - \vec{V} \cdot \vec{\nabla} Q$$