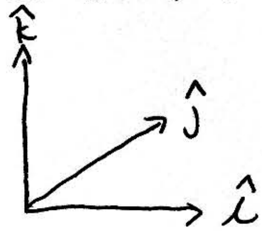
Vector Review

$\vec{V}$: 3D vector $\rightarrow u\hat{i} + v\hat{j} + w\hat{k}$ (components of $\vec{v}$ in $\hat{i}, \hat{j}, \hat{k}$)

$\vec{V}_H$: 2D vector $\rightarrow u\hat{i} + v\hat{j}$



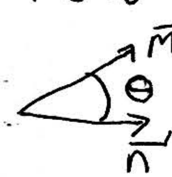
Vector Math

Scalar multiplication of a vector:

$$s\vec{v} = su\hat{i} + sv\hat{j} + sw\hat{k}$$

dot product: $\vec{m} \cdot \vec{n} = |\vec{m}||\vec{n}| \cos\theta$

θ: angle between $\vec{m}, \vec{n}$



- $\vec{m} \cdot \vec{n} = 0$ if $\vec{m} \perp \vec{n}$

$\rightarrow \theta = 90°$, $\cos 90° = 0$

$\vec{m} = m_1\hat{i} + m_2\hat{j} + m_3\hat{k}$ $\qquad$ $\vec{n} = n_1\hat{i} + n_2\hat{j} + n_3\hat{k}$

$\rightarrow \vec{m} \cdot \vec{n} = m_1 n_1 + m_2 n_2 + m_3 n_3 \rightarrow$ a scalar

cross product:
$$\vec{m} \times \vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ m_1 & m_2 & m_3 \\ n_1 & n_2 & n_3 \end{vmatrix}$$

$$= +(m_2 n_3 - m_3 n_2)\hat{i} - (m_1 n_3 - m_3 n_1)\hat{j} + (m_1 n_2 - m_2 n_1)\hat{k}$$

$\rightarrow$ a vector

Right hand Rule

$\hat{i} \times \hat{j} = \hat{k}$	$\hat{j} \times \hat{i} = -\hat{k}$	$\hat{k} \times \hat{i} = \hat{j}$
$\hat{i} \times \hat{k} = -\hat{j}$	$\hat{j} \times \hat{k} = \hat{i}$	$\hat{k} \times \hat{j} = -\hat{i}$

gradient: $\vec{\nabla} = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$

$\vec{\nabla}_H = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j}$

Laplacian: $\vec{\nabla}^2 = \frac{\partial^2}{\partial x^2}\hat{i} + \frac{\partial^2}{\partial y^2}\hat{j} + \frac{\partial^2}{\partial z^2}\hat{k} \rightarrow \vec{\nabla}\cdot\vec{\nabla}$

$\vec{\nabla}_H^2 = \frac{\partial^2}{\partial x^2}\hat{i} + \frac{\partial^2}{\partial y^2}\hat{j} \rightarrow \vec{\nabla}_H\cdot\vec{\nabla}_H$

In meteorology

$\vec{V} = \langle u, v, w \rangle \rightarrow$ 3D wind

$\vec{V}_H = \langle u, v \rangle \rightarrow$ horizontal wind

divergence: $\vec{\nabla}\cdot\vec{V} = \langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \rangle \cdot \langle u, v, w \rangle$

$$= \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$$

advection of a quantity (scalar or vector), Q, by 3D wind:

$$-(\vec{V}\cdot\vec{\nabla})Q = -\left(u\frac{\partial Q}{\partial x} + v\frac{\partial Q}{\partial y} + w\frac{\partial Q}{\partial z}\right)$$

curl: $\vec{\nabla}\times\vec{V} = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right) \times (u\hat{i} + v\hat{j} + w\hat{k})$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix} = +\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)\hat{i} - \left(\frac{\partial w}{\partial x} - \frac{\partial u}{\partial z}\right)\hat{j} + \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\hat{k}$$

- the curl of the 3D wind is the 3D vorticity vector

  - on synoptic scale, the vertical component of vorticity is >> the horiz. component

    → vertical component of vorticity vector is

    ζ = rel. vorticity

    $\rightarrow \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$